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Non-stationary Hours in a DSGE Model

The time series fit of dynamic stochastic general equilibrium (DSGE) models often suffers from restrictions on the long-run dynamics that are at odds with the data. Using Bayesian methods we estimate a stochastic growth model in which hours worked are stationary and a modified version with permanent labor supply shocks. If firms can freely adjust labor inputs, the data support the latter specification. Once we introduce frictions in terms of labor adjustment costs, the overall time series fit improves and the model specification in which labor supply shocks and hours worked are stationary is preferred.

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DYNAMIC STOCHASTIC GENERAL equilibrium (DSGE) models have become a workhorse for studying various aggregate economic phenomena. Since these models generate both business cycle fluctuations as well as long-run growth paths, they should ultimately be able to match data across all frequencies. Despite the significant progress in developing empirically viable models (e.g., Christiano, Eichenbaum, and Evans 2005, Smets and Wouters 2003), the time series fit

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of DSGE models is typically inferior to the fit of vector autoregressions (VAR) that are estimated with well-calibrated shrinkage methods, as documented in Del Negro et al. (2007). One reason for the poor time series fit is the restrictions imposed by the so-called balanced growth path. Along the balanced growth path, (i) the "big ratios" (investment-output, consumption-output, capital-output, and real wage-output) are stable as output, consumption, investment, capital stock, and real wages grow at the same rate, and (ii) the real rates of return to capital and per capita hours worked are stationary.[1] As pointed out, for instance, by Canova, Finn, and Pagan (1994), these model-implied co-trending relationships are often rejected by the data. Modifications to the probabilistic structure of the exogenous shocks that generate fluctuations in DSGE models can be used to generalize trend structures. For instance, in a two-sector model Edge, Laubach, and Williams (2003) introduce trends in sector-specific productivity processes such that the relative price of investment becomes non-stationary and real investment and consumption can grow at different rates.

This paper focuses on the stationarity of hours worked. Many researchers doubt that hours worked are stationary as we have observed apparent changes in labor-supply patterns over recent decades (e.g., McGrattan and Rogerson 2004, Galí 2005). Usual suspects responsible for persistent shifts in per capita hours are structural changes in demography, government purchases, tax codes, household production technology, or preferences itself. Recently, business cycle theorists have been particularly concerned with this issue because assumptions about the persistence of hours has far reaching implications for our understanding of propagation mechanisms as well as the sources of economic fluctuations. Shapiro and Watson (1988) report that about half of the cyclical variation in output can be accounted for by the stochastic trend in labor supply. In response to a provocative finding by Galí (1999) that hours worked decrease after a favorable technology shock, Christiano, Eichenbaum, and Vigfusson (2003), Chari, Kehoe, and McGrattan (2004), and Basu, Fernald, and Kimball (2004) show that the statistical inference in a structural VAR crucially depends on the treatment of low frequency components of hours worked.

This paper makes two contributions. First, we present a modified stochastic growth model in which hours worked have a stochastic trend, generated by a non-stationary labor supply shock. In terms of properly detrended variables the model has a well-defined steady-state and can be solved, for instance, by a log-linear approximation around this steady state. Since this specification implies that the technology shock is the only source for permanent shifts in average labor productivity, the popular long-run VAR identification scheme for technology shocks remains consistent with our model. The modification proposed in this paper can be easily incorporated into large-scale DSGE models with real and nominal rigidities and potentially improve their empirical performance.

Second, based on output and hours data we compute posterior odds for four versions of the stochastic growth model obtained by using either a stationary or non-stationary

1. See King, Plosser, and Rebelo (1988) for the restrictions on technology and preferences that satisfy the balanced growth path property.

labor supply shock. Since the absence of strong endogenous dynamics in DSGE models (e.g., Cogley and Nason 1995) might lead us to favor a model specification with a non-stationary labor supply shock in our empirical analysis, we also consider model specifications with richer internal dynamics by introducing adjustment costs in employment.[2] We find that without adjustment costs the specification with a non-stationary labor supply shock is preferred for three alternative data sets that have been used in the literature. Posterior odds range from 8:1 to 100:1.[3] However, as the prior distribution for the autocorrelation of the labor supply shock in the model with stationary hours is shifted toward more persistence, the evidence in favor of the non-stationary hours specification deteriorates. This reflects a well-known difficulty in distinguishing unit root from highly persistent yet stationary dynamics.

Once labor adjustment costs are included in the stochastic growth model, the specification with a stationary labor supply shock is preferred. According to our posterior distributions there is a negative correlation between the adjustment cost parameter and the persistence of labor supply shock. With an estimated autocorrelation of 0.8 for the labor supply shock, the adjustment cost model can essentially reproduce the observed sample autocorrelation and variance of hours worked. Depending on the data sets, the posterior odds in favor of the stationary hours worked specification range from 2:1 to 9:1. Overall, the model specifications with adjustment costs are strongly preferred to those without. Given the weak and partially conflicting evidence on the stationarity of hours from univariate tests as, for instance, documented by Christiano, Eichenbaum, and Vigfusson (2003), it is in our view preferable to conduct a multivariate specification analysis directly in the context of the model of interest. Cross-coefficient restrictions and the careful specification of prior distributions can help to sharpen inference and conclusions depend on auxiliary assumptions about frictions in the labor market.

The remainder of this paper is organized as follows. Section 1 presents the stochastic growth model and discusses its long-run dynamics. Section 2 explains our estimation procedure. The results from the empirical analysis are presented in Section 3, and Section 4 concludes.

1. MODEL

The model economy is a one-sector stochastic growth model with technology and labor supply shocks. We consider four versions of the model: in $\mathcal{M}_0$ and $\mathcal{M}_1$ firms can choose the employment level at the given wage rate without any adjustment cost. In $\mathcal{A}_0$ and $\mathcal{A}_1$, on the other hand, it is costly for firms to adjust the employment level. In $\mathcal{A}_0$ and $\mathcal{M}_0$ the labor supply shock is a stationary AR(1) process, whereas it is modeled as random walk in $\mathcal{A}_1$ and $\mathcal{M}_1$. The presence of adjustment costs will

2. We are grateful to one of the referees for suggesting this extension.

3. All statements in this paper involving posterior odds or posterior model probabilities assume that the specifications have equal prior probability.

influence our conclusions regarding the persistence of labor supply shocks as they generate an endogenous propagation.

The representative household maximizes the expected discounted lifetime utility from consumption C_t and hours worked H_t:

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^{t+s}\left(\ln C_{t+s}-\frac{(H_{t+s}/B_{t+s})^{1+1/\nu}}{1+1/\nu}\right)\right]. \tag{1}$$

The log utility in consumption implies a constant long-run labor supply in response to a permanent change in technology. The short-run (Frisch) labor supply elasticity is ν. The labor supply shock is denoted by B_t. An increase of B_t raises aggregate labor supply. This may reflect permanent shifts in per capita hours of work due to demographic changes, tax reforms, shifts in the marginal rate of substitution between leisure and consumption, or (non-neutral) technological changes in household production technology. The household supplies labor at the competitive equilibrium wage W_t and rents capital K_t to the firms at the competitive rental rate R_t. The capital stock depreciates at the rate δ, and the per period budget constraint faced by the household is

$$C_t+K_{t+1}-(1-\delta)K_t=W_tH_t+R_tK_t. \tag{2}$$

Firms rent capital, hire labor services, and produce final goods according to the following Cobb-Douglas technology:

$$Y_t=(A_tH_t)^{\alpha}K_t^{1-\alpha}\left[1-\varphi\cdot\left(\frac{H_t}{H_{t-1}}-1\right)^2\right]. \tag{3}$$

The stochastic process A_t represents the exogenous labor augmenting technical progress. The last term captures the cost of adjusting labor inputs: $\varphi\geq 0$. In models $\mathcal{M}_0$ and $\mathcal{M}_1$, there is no adjustment cost: $\varphi=0$. Despite various types of adjustment costs in the labor market—for example, search (Andolfatto 1996), learning (Chang, Gomes, and Schorfheide 2002), time non-separable utility in leisure (Kydland and Prescott 1982)—we use a simple reduced-form quadratic cost to firms without taking a particular stand on the microfoundations of the nature of friction. The firms maximize expected discounted future profits

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^{t+s}\lambda_{t+s}\left(Y_{t+s}-W_{t+s}H_{t+s}^d-R_{t+s}K_{t+s}^d\right)\right], \tag{4}$$

where λ_t is the marginal value of a unit consumption to a household, which is treated as exogenous to the firm. In equilibrium $\lambda_t=1/C_t$ and the goods, labor, and capital markets clear:

$$Y_t=C_t+K_{t+1}-(1-\delta)K_t,\qquad H_t^d=H_t,\quad\text{and}\quad K_t^d=K_t.$$

We assume that the log production technology evolves according to a random walk with drift:

$$\ln A_t = \gamma + \ln A_{t-1} + \epsilon_{a,t}, \quad \epsilon_{a,t} \sim iid\mathcal{N}\big(0, \sigma_a^2\big). \tag{5}$$

The level of technology in period 0 is denoted by A_0. We consider two specifications for the stochastic process of B_t. In models $\mathcal{M}_0$ and $\mathcal{A}_0$, the labor supply shock follows a stationary AR(1) process:

$$\mathcal{M}_0\colon \quad \ln B_t = \rho_b \ln B_{t-1} + (1 - \rho_b) \ln B_0 + \epsilon_{b,t}, \quad \epsilon_{b,t} \sim iid\mathcal{N}\big(0, \sigma_b^2\big), \tag{6}$$

where $0 \leq \rho_b < 1$ and $\ln B_0$ is the unconditional mean of $\ln B_t$. In model $\mathcal{M}_0$ and $\mathcal{A}_0$ the innovation $\epsilon_{b,t}$ only has a transitory effect. Alternatively, in models $\mathcal{M}_1$ and $\mathcal{A}_1$ the labor supply shock evolves according to a random walk:

$$\mathcal{M}_1\colon \quad \ln B_t = \ln B_{t-1} + \epsilon_{b,t}, \quad \epsilon_{b,t} \sim iid\mathcal{N}\big(0, \sigma_b^2\big) \tag{7}$$

and we use $\ln B_0$ to denote the initial level of $\ln B_t$. In both specifications, the innovations $\epsilon_{a,t}$ and $\epsilon_{b,t}$ are assumed to be uncorrelated at all leads and lags.

It is well known that in models $\mathcal{M}_0$ and $\mathcal{A}_0$ hours are stationary and that output, consumption, and capital grow according to the technology process A_t. Hence, one can induce stationarity with the following transformation:

$$\mathcal{M}_0\colon \quad \widetilde{Y}_t = \frac{Y_t}{A_t}, \quad \widetilde{C}_t = \frac{C_t}{A_t}, \quad \widetilde{K}_{t+1} = \frac{K_{t+1}}{A_t}.$$

In models $\mathcal{M}_1$ and $\mathcal{A}_1$, on the other hand, the labor supply shock B_t induces a stochastic trend into hours as well as output, consumption, and capital. To obtain a stationary equilibrium these variables have to be detrended according to:

$$\mathcal{M}_1\colon \quad \widetilde{H}_t = \frac{H_t}{B_t}, \quad \widetilde{Y}_t = \frac{Y_t}{A_t B_t}, \quad \widetilde{C}_t = \frac{C_t}{A_t B_t}, \quad \widetilde{K}_{t+1} = \frac{K_{t+1}}{A_t B_t}.$$

With these transformations, we obtain a system of rational expectations equations that characterizes the equilibrium dynamics of the endogenous variables in the neighborhood of the steady state. It can be solved by standard log-linearization methods (e.g., King, Plosser, and Rebelo 1988, Sims 2002).

We note two important aspects of the model specification. First, a permanent labor supply shock raises both hours worked and output permanently in models $\mathcal{M}_1$ and $\mathcal{A}_1$. However, one can show that it does not have a permanent effect on labor productivity Y_t/H_t. Thus, all four versions of the stochastic growth model are consistent with the following popular identification assumption: technology shocks are the only source for a stochastic trend in labor productivity. Second, in models $\mathcal{M}_1$ and $\mathcal{A}_1$ there is a positive probability that hours worked exceed a given threshold $\bar{H}$, for example, 24 hours per day. Our log-linear approximation ignores this bound and provides an accurate characterization of the local dynamics only if hours worked are well below this threshold.[4]

2. ECONOMETRIC APPROACH

We fit the DSGE models to observations on the log level of real per capita output and hours worked, denoted by the 2 × 1 vector y_t. Let $\epsilon_t = [\epsilon_{a,t}, \epsilon_{b,t}]'$ and define the vector of structural model parameters as $\theta = [\alpha, \beta, \gamma, \delta, \nu, \varphi, \ln A_0, \ln B_0, \rho_b, \sigma_a, \sigma_b]'$. It is well known that log-linearized DSGE models have a state space representation:

$$y_t = \Gamma_0 + \Gamma_1 s_{1,t} + \Gamma_2 s_{2,t} + \Gamma_3 t \tag{8}$$

$$s_{1,t} = \Phi_1 s_{1,t-1} + \Psi_1 \epsilon_t \tag{9}$$

$$s_{2,t} = s_{2,t-1} + \Psi_2 \epsilon_t. \tag{10}$$

The system matrices of this state space representations are functions of the structural parameters θ. The deterministic trend in (8) captures the effect of the drift in the random walk technology process A_t. Equation (9) represents the law of motion for the state variables of the detrended model, and (10) describes the evolution of stochastic trends: $s_{2,t} = \ln A_t - \gamma t$ in models $\mathcal{M}_0$ and $\mathcal{A}_0$ and $s_{2,t} = [\ln A_t - \gamma t, \ln B_t]'$ in $\mathcal{M}_1$ and $\mathcal{A}_1$.

The Kalman filter can be used to compute the likelihood function $\mathcal{L}(\theta \mid Y^T)$ for the state space system (8)–(10). To initialize the Kalman filter a distribution for the state vector in period $t = 0$ has to be specified. If all state variables are stationary, a natural choice is the unconditional distribution of s_t. In our model, however, a part of the state vector, $s_{2,t}$, is non-stationary. Hence, we factorize the initial distribution as $p(s_{1,0})$ $p(s_{2,0})$ and set the first component equal to the unconditional distribution of $s_{1,t}$, whereas the second component, composed of the distribution of $\ln A_0$ (for $\mathcal{M}_0, \mathcal{A}_0$) and $[\ln A_0, \ln B_0]'$ (for $\mathcal{M}_1, \mathcal{A}_1$), respectively, is absorbed into the specification of our prior $p(\theta)$. According to Bayes Theorem the posterior distribution of θ is given by

$$p(\theta \mid Y^T) = \mathcal{L}(\theta \mid Y^T) p(\theta) / p(Y^T). \tag{11}$$

The fit of models can be assessed based on the marginal data densities:

$$p(Y^T) = \int \mathcal{L}(\theta \mid Y^T) p(\theta)\, d\theta. \tag{12}$$

4. A similar issue arises when modeling nominal interest rates, which often appear to be locally non-stationary but at the same time are bounded from below by zero. While linear time series models cannot explain apparent unit root behavior of interest rates between, say 4% and 12%, and mean-reverting behavior elsewhere, non-linear models can. For instance, Aït-Sahalia (1996) estimates a diffusion model with a non-linear drift function that is consistent with interest rates appearing to be non-stationary processes over extended time periods while being overall stationary.

If the prior odds of two models are equal to one, then the ratio of marginal data densities provides the posterior odds. Log marginal data densities penalize the maximized log likelihood function by a measure of model complexity and can be interpreted as a measure of one-step-ahead out-of-sample predictive performance. The Bayesian analysis is implemented with Markov Chain Monte Carlo methods described in Schorfheide (2000).

3. EMPIRICAL ANALYSIS

We use three different data sets composed of quarterly U.S. real per capita GDP and hours worked from 1954:Q2 to 2001:Q4. The observations from 1954:Q2 to 1958:Q4 are treated as pre-sample to quantify prior distributions. Since we are comparing the fit of the DSGE model specifications to that of a VAR with four lags, we reserve the observations from 1959:Q1 to 1959:Q4 for the initialization of lags. Since the VAR likelihood function is conditional on the observations from the year 1959, we adjust the DSGE model likelihood function accordingly.[5] For Data Set 1 we use real GDP from the DRI-Global Insight database (GDPQ) and divide it by population of age 20 or older (PM20 + PF20). Hours worked is measured as average weekly hours of all people in the non-farm business sector compiled by the Bureau of Labor Statistics (EEU00500005). We multiply the hours series by the employment ratio, which is the number of people employed (LHEM, DRI-Global Insight) divided by population (PM20 + PF20). Data Set 2 is obtained from CEV. Per capita output is obtained by dividing GDPQ by civilian population age 16 or older (P16, DRI-Global Insight). Hours worked are measured as total hours (LBMN, DRI-Global Insight) divided by P16. Data Set 3 has been used by Galí and Rabanal (2004) and is extracted from Haver Analytics' USECON database. Output is defined as non-farm business sector output (LXNFO) divided by civilian non-institutional population age 16 or older (LNN). Hours are measured as non-farm business sector hours (LXNFH) divided by the same population measure. All series are seasonally adjusted and transformed by taking natural logs.[6] The observations on hours worked, in percentage deviations from their respective sample means, are depicted in Figure 1. An informal inspection of the plots suggests that hours worked are highly persistent in all three data sets.[7] Hence, the specifications with non-stationary labor supply shocks may provide an empirically plausible alternative to $\mathcal{M}_0$ and $\mathcal{A}_0$.

The benchmark prior distribution of the parameters is summarized in Table 1. We assume all parameters to be *a priori* independent. By and large, the prior means are

5. This adjustment can be easily implemented by calculating $\mathcal{L}(\theta \mid y_{-3}, \dots, y_0, Y^T)/\mathcal{L}(\theta \mid y_{-3}, \dots, y_0)$, where y_0 corresponds to 1959:Q4 and Y^T denotes to sample 1960:Q1 to 2001:Q4.

6. We use the X-12 filter to adjust the BLS hours series EEU0050005.

7. The t-statistics for a standard augmented Dickey-Fuller test (4 lags, constant, no trend) are -2.80, -2.55, and -2.44 for the three data sets, respectively. The critical values for the rejection of the unit root hypothesis are -2.57 (10%), -2.86 (5%), and -3.46 (1%).

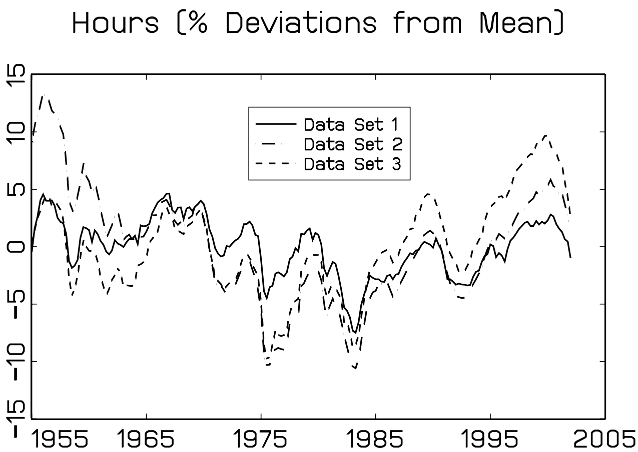

FIG. 1. Hours Worked Data.

TABLE 1
BENCHMARK PRIOR DISTRIBUTIONS

Parameter	Domain	Density	Data Set	Model	Para (1)	Para (2)
α	[0, 1)	Beta	all	all	0.660	0.020
β	[0, 1)	Beta	all	all	0.995	0.002
γ	$\mathbb{R}$	Normal	all	all	0.005	0.005
δ	[0, 1)	Beta	all	all	0.025	0.005
ν	$\mathbb{R}^+$	Gamma	all	all	1.000	0.500
ρ_b	[0, 1)	Beta	all	$\mathcal{M}_0, \mathcal{A}_0$	0.900	0.050
σ_a	$\mathbb{R}^+$	InvGamma	all	$\mathcal{M}_0, \mathcal{M}_1$	0.010	1.000
			all	$\mathcal{M}_1, \mathcal{A}_1$	0.015	1.000
σ_b	$\mathbb{R}^+$	InvGamma	all	$\mathcal{M}_0, \mathcal{M}_1$	0.010	1.000
			all	$\mathcal{M}_1, \mathcal{A}_1$	0.015	1.000
$\ln A_0$	$\mathbb{R}$	Normal	1	$\mathcal{M}_0, \mathcal{A}_0$	5.647	0.200
			1	$\mathcal{M}_1, \mathcal{A}_1$	5.674	0.200
			2	$\mathcal{M}_0, \mathcal{A}_0$	2.346	0.200
			2	$\mathcal{M}_1, \mathcal{A}_1$	2.394	0.200
			3	$\mathcal{M}_0, \mathcal{A}_0$	−1.857	0.200
			3	$\mathcal{M}_1, \mathcal{A}_1$	−1.821	0.200
$\ln B_0$	$\mathbb{R}$	Normal	1	$\mathcal{M}_0, \mathcal{A}_0$	3.236	0.200
			1	$\mathcal{M}_1, \mathcal{A}_1$	3.209	0.200
			2	$\mathcal{M}_0, \mathcal{A}_0$	6.453	0.200
			2	$\mathcal{M}_1, \mathcal{A}_1$	6.405	0.200
			3	$\mathcal{M}_0, \mathcal{A}_0$	6.346	0.200
			3	$\mathcal{M}_1, \mathcal{A}_1$	6.309	0.200
φ	$\mathbb{R}^+$	Gamma	all	$\mathcal{A}_0, \mathcal{A}_1$	33.00	15.00

NOTES: In the non-stationary models $\mathcal{M}_1$ and $\mathcal{A}_1$, ρ_b is fixed at 1. Para (1) and Para (2) list the means and the standard deviations for beta, gamma, and normal distributions; s and ν for the inverse gamma distribution, where $p_{\mathcal{IG}}(\sigma \mid \nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$.

chosen based on a pre-sample of observations from 1954:Q2 to 1958:Q4. The prior mean of the labor share α is 0.66 and that for the quarter-to-quarter growth rate of productivity, γ, is 0.5%. The prior for β is centered at 0.995. Combined with the prior mean of γ, this corresponds to an annualized real return of about 4%. The depreciation rate δ lies between 1.8% and 3.3% per quarter. The 90% probability interval for the Frisch labor supply elasticity ν ranges from 0.3 to 1.8.

We specify a prior for the adjustment cost parameter φ as follows. The adjustment costs of labor can be viewed as the total expenditure spent for recruiting new workers. In order to increase the amount of labor input by ΔH, firms incur the adjustment cost of $\varphi(\frac{\Delta H}{H})^2 Y$ in our model. It is known that the average recruiting cost is about 50% of quarterly salary of a worker recruited.[8] This implies: $\varphi(\frac{\Delta H}{H})^2 Y = \zeta W \Delta H$, where ζ is a fraction of the recruiting cost in terms of wage. With a labor share of $2/3(=\frac{WH}{Y})$, $\zeta = 0.5$, and a 1% increase of employment ($\frac{\Delta H}{H} = 1\%$), we obtain $\varphi = 33$. We use a fairly diffuse prior distribution that is centered at 33 and has a standard deviation of 15.

The presence of adjustment costs dampens the effect of technology and labor supply shocks on output and hours worked. In order to guarantee that the adjustment cost specifications have *a priori* similar implications for the volatility of the endogenous variable as $\mathcal{M}_0$ and $\mathcal{M}_1$, we use slightly different priors for the standard deviations of the structural shocks. Under $\mathcal{M}_0$ and $\mathcal{M}_1$ the priors for σ_a and σ_b are centered at 0.010, whereas under $\mathcal{A}_0$ and $\mathcal{A}_1$ they are centered at 0.015.

For $\mathcal{M}_0$ and $\mathcal{A}_0$ the prior mean of $\ln B_0$ is constructed by matching average hours worked over the pre-sample period with the steady state level of hours worked $\widetilde{H}^*$, evaluated at the prior mean values of the remaining structural parameters. For $\mathcal{M}_1$ and $\mathcal{A}_1$ the prior mean of $\ln B_0$ is obtained by equating hours worked in 1958:Q4 with the steady state level $B_0\widetilde{H}^*$. Similarly, we select the prior mean of $\ln A_0$ by matching $A_0\widetilde{Y}^*$ and $A_0 B_0\widetilde{Y}^*$, respectively, with the level of output in 1958:Q4. The prior standard deviations for $\ln A_0$ and $\ln B_0$ are 0.2. Finally, for $\mathcal{M}_0$ and $\mathcal{A}_0$ the 90% probability interval for the autoregressive parameter ρ_b ranges from 0.825 to 0.977, implying a fairly persistent labor supply process.

The posterior means and 90% probability intervals based on Data Set 1 are reported in Table 2. For convenience, we also report means and probability intervals for the prior distribution. The estimates of α, β, δ, and γ are very similar across model specifications. While the data are not very informative about α, β, and δ, the probability interval of γ shrinks by a factor of 4. The posterior means of the labor supply elasticity ν range from 0.15 to 0.53. The estimated standard deviation of the technology shock is essentially the same across all four model specifications. The specifications $\mathcal{A}_0$ and $\mathcal{A}_1$ tend to generate larger estimates of σ_b because adjustment costs dampen the effect of labor supply shocks on hours worked. We re-estimated[9] the four models based on Data Sets 2 and 3 and obtained very similar results with one exception: for Data Set 3 the estimates of the labor supply elasticity range from 0.92 ($\mathcal{A}_0$) to 1.05 ($\mathcal{M}_0$) and are somewhat larger than the microlevel estimates. However, these numbers are roughly

8. According to the headhunting service website www.staffing.org/recruitingefficiency/calculator2004.aspx, the nation-wide average recruiting efficiency (total recruiting cost divided by the annual salary of the recruited) is 87.6%. This corresponds to the recruiting cost of 12.4% of an annual salary and 49.6% of a quarterly salary of the worker recruited.

9. Detailed estimates are provided in a technical appendix that is available from the authors upon request.

TABLE 2

Posterior Distribution—Benchmark Prior, Data Set 1

	Prior		Posterior			
			Stationary B_t		Non-stationary B_t	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval
			No Adjustment Costs ($\varphi = 0$)			
α	0.660	[0.627, 0.693]	0.652	[0.624, 0.681]	0.654	[0.624, 0.685]
β	0.995	[0.992, 0.998]	0.995	[0.993, 0.998]	0.995	[0.992, 0.998]
γ	0.005	[−0.003, 0.013]	0.004	[0.002, 0.006]	0.004	[0.003, 0.006]
δ	0.025	[0.017, 0.033]	0.023	[0.016, 0.030]	0.024	[0.016, 0.031]
ν	1.000	[0.233, 1.735]	0.527	[0.179, 0.865]	0.474	[0.160, 0.783]
ρ_B	0.900	[0.826, 0.978]	0.951	[0.919, 0.981]		
σ_A	N/A	[0.003, 0.081]	0.011	[0.010, 0.013]	0.011	[0.010, 0.013]
σ_B	N/A	[0.003, 0.081]	0.006	[0.005, 0.006]	0.006	[0.006, 0.007]
$\ln A_0$	5.647	[5.316, 5.970]	5.708	[5.474, 5.949]		
	5.674	[5.342, 6.000]			5.717	[5.444, 5.990]
$\ln B_0$	3.236	[2.905, 3.562]	3.176	[3.150, 3.204]		
	3.209	[2.878, 3.535]			3.166	[2.894, 3.430]
			With Adjustment Costs ($\varphi \geq 0$)			
α	0.660	[0.627, 0.693]	0.658	[0.631, 0.687]	0.661	[0.633, 0.689]
β	0.995	[0.992, 0.998]	0.995	[0.992, 0.998]	0.995	[0.992, 0.998]
γ	0.005	[−0.003, 0.013]	0.004	[0.002, 0.006]	0.004	[0.003, 0.006]
δ	0.025	[0.017, 0.033]	0.023	[0.016, 0.031]	0.024	[0.016, 0.031]
ν	1.000	[0.234, 1.736]	0.433	[0.024, 0.921]	0.153	[0.009, 0.337]
ρ_B	0.900	[0.825, 0.977]	0.800	[0.657, 0.940]	1.000	[1.000, 1.000]
σ_A	N/A	[0.004, 0.121]	0.011	[0.010, 0.012]	0.011	[0.010, 0.012]
σ_B	N/A	[0.004, 0.120]	0.034	[0.009, 0.071]	0.012	[0.009, 0.015]
$\ln A_0$	5.647	[5.321, 5.979]	5.748	[5.510, 5.981]		
	5.674	[5.342, 5.999]			5.754	[5.521, 5.979]
$\ln B_0$	3.235	[2.905, 3.563]	3.171	[3.147, 3.197]		
	3.209	[2.879, 3.537]			3.194	[2.870, 3.518]
φ	33.00	[9.636, 55.40]	11.36	[1.145, 22.90]	8.054	[0.623, 16.69]

consistent with the estimates obtained by Chang and Kim (2006) and estimates from an experimental survey by Kimball and Shapiro (2003), who report a value of about 1.

For the stationary model without adjustment costs, we observe that the estimated autocorrelations of the labor supply shocks, ρ_b, are near unity. The posterior means are 0.95 (Data Set 1), 0.97 (Data Set 2), and 0.98 (Data Set 3). However, once we allow for non-zero adjustment costs and enrich the internal propagation mechanism of the DSGE model the estimates of ρ_b drop to 0.80 (Data Set 1), 0.85 (Data Set 2), and 0.89 (Data Sets 3), respectively. In Figure 2, we plot draws from the joint prior and posterior distributions of ρ_b and φ for model $\mathcal{A}_0$ and Data Set 1. Even though the two parameters are independent *a priori*, the posterior clearly exhibits a negative correlation: large values of the adjustment cost parameters are associated with relatively small values of ρ_b and persistence of hours is generated endogenously.

To assess overall time series fit of the stochastic growth models, we report marginal data densities in Table 3. If one assigns equal prior probabilities to the

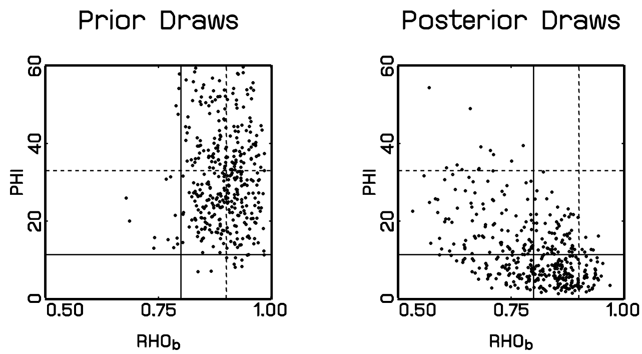

FIG. 2. Joint Distribution of φ and ρ_b.

NOTES: Figure depicts 400 draws from prior (posterior) distribution. Intersection of dashed (solid) lines indicates prior (posterior) mean. Results are based on Benchmark Prior and Data Set 1.

TABLE 3

LOG MARGINAL DATA DENSITIES

Data set	Prior	$\mathcal{M}_0$	$\mathcal{M}_1$	$\mathcal{A}_0$	$\mathcal{A}_1$	VAR(4)
1	B	1176.33	1178.45	1182.10	1180.21	1180.49
	P1		1176.81			
	P2		1178.61			
	P3	1177.64				
	P4	1174.85				
2	B	1161.35	1165.81	1187.39	1185.26	1163.75
	P1		1164.62			
	P2		1165.83			
	P3	1164.15				
	P4	1159.73				
3	B	1114.65	1119.26	1145.26	1144.29	1136.41
	P1		1117.29			
	P2		1119.61			
	P3	1117.92				
	P4	1112.97				

NOTES: B denotes the benchmark prior in Table 1 whereas P1 through P4 refer to the alternative priors in Table 4. Posterior odds of, say, $\mathcal{M}_1$ versus $\mathcal{M}_0$ can be obtained by multiplying prior odds with the Bayes factor $\exp[\ln p(Y^T \mid \mathcal{M}_1) - \ln p(Y^T \mid \mathcal{M}_0)]$.

model specifications, exponentiated differences of log marginal data densities can be interpreted as posterior odds.[10] Without adjustment costs the non-stationary model $\mathcal{M}_1$ has a higher marginal data density than the stationary model $\mathcal{M}_0$. The posterior odds in favor of $\mathcal{M}_1$ range from 8:1 (Data Set 1) to 100:1 (Data Set 3). However, once adjustment costs are introduced the ranking changes. Now the model with the stationary labor supply shock, $\mathcal{A}_0$, is preferred to the non-stationary model, $\mathcal{A}_1$. The

10. According to the somewhat arbitrary but often cited classification in Appendix B of Jeffreys (1961), a posterior odds ratio between 3 and 10, 10 and 32, 32 and 100, above 100 provides “substantial,” “strong,” “very strong,” “decisive” evidence, respectively.

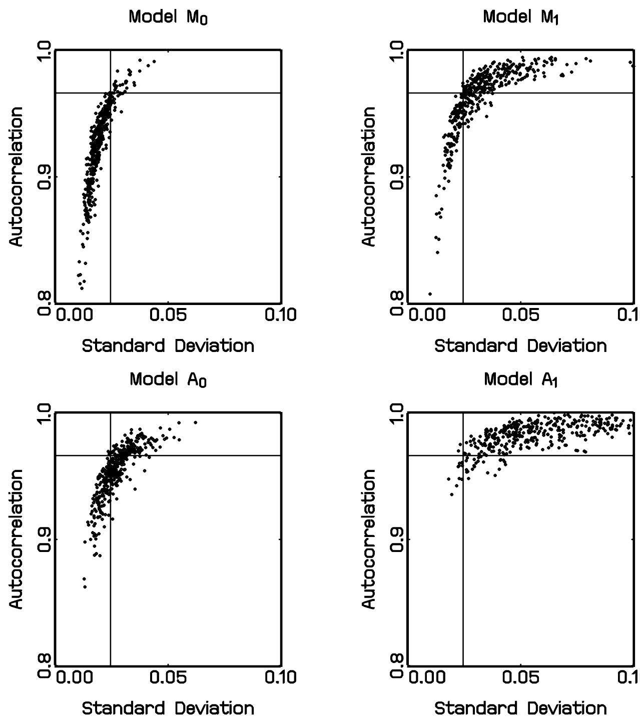

FIG. 3. Sample Moments of Hours—Posterior Predictive Distribution.

NOTES: Figure depicts 400 draws from posterior predictive distribution of sample moments. Intersection of dashed lines indicates the actual (Data Set 1) sample standard deviation and first-order autocorrelation. Results are based on benchmark prior and Data Set 1.

posterior odds range from 2:1 to 9:1. Overall, the models with adjustment cost are preferred to those without. In addition to the DSGE models we estimate a VAR in log levels of output and hours

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + u_t, \quad u_t \sim iid\mathcal{N}(0, \Sigma) \tag{13}$$

with $p = 4$ lags using a Minnesota prior.[11] This prior shrinks the VAR estimates toward univariate random walk representations. While the VAR dominates both $\mathcal{M}_0$ and $\mathcal{M}_1$ in terms of marginal data density, the adjustment cost specifications tend to fit better than the VAR(4), in particular for Data Sets 2 and 3.

In Figure 3, we plot draws from the posterior predictive distribution of the sample autocorrelation and standard deviation of hours worked. All calculations are based on

11. See Doan, Litterman, and Sims (1984). Our version is implemented via dummy observations based on MATLAB code provided by Chris Sims. A description can be found in Appendix C of Lubik and Schorfheide (2006). We use the following hyperparameters: $d = 0.5$, $\lambda = 5$, $\mu = 2$, $\tau = 1$. Mean and standard observations of y_t are calculated based on the pre-sample.

TABLE 4
ALTERNATIVE PRIOR DISTRIBUTIONS

Parameter	Domain	Density	Data set	Model	Para (1)	Para (2)
Alternative Prior P1						
$\ln B_0$	$\mathbb{R}$	Normal	1	$\mathcal{M}_1$	3.209	2.000
			2	$\mathcal{M}_1$	6.405	2.000
			3	$\mathcal{M}_1$	6.309	2.000
Alternative Prior P2						
$\ln B_0$	$\mathbb{R}$	Normal	1	$\mathcal{M}_1$	3.209	0.020
			2	$\mathcal{M}_1$	6.405	0.020
			3	$\mathcal{M}_1$	6.309	0.020
Alternative Prior P3						
ρ_b	[0, 1)	Beta	all	$\mathcal{M}_0$	0.980	0.005
Alternative Prior P4						
ρ_b	[0, 1)	Beta	all	$\mathcal{M}_0$	0.800	0.100

NOTES: Para (1) and Para (2) list the means and the standard deviations for beta and normal distributions.

Data Set 1 and the benchmark prior. Each point in the plot is generated as follows: we take a draw from the posterior distribution of the DSGE model parameters, simulate T artificial observations from the linearized model, and compute sample moments of hours worked. The intersection of the solid lines indicates the sample moments calculated from the actual U.S. data. If the estimated model fits well and is able to explain the salient features of the data, the actual sample moment should not lie to far in the tails of the posterior predictive distribution.[12] The top left panel of Figure 3 indicates that the model with stationary labor supply shocks ($\mathcal{M}_0$) has difficulties reproducing the persistence of hours worked observed in the data. If we make the labor supply shock non-stationary ($\mathcal{M}_1$), the predicted autocorrelation of hours worked rises and is consistent with the data. The lower left panel suggests that once adjustment costs have been introduced, the stationary labor supply shock ($\mathcal{A}_0$) is sufficient to generate realistic sample moments. In fact, non-stationary labor shocks ($\mathcal{A}_1$) lead to too much serial correlation and volatility in hours worked.[13]

We conduct a number of robustness checks by re-estimating $\mathcal{M}_0$ and $\mathcal{M}_1$ under alternative prior distributions presented in Table 4. Prior P1 uses a more diffuse distribution for $\ln B_0$ in the non-stationary model $\mathcal{M}_1$, whereas Prior P2 is more concentrated than the benchmark prior. Not surprisingly, the marginal data density deteriorates under the less informative Prior P1 for all three data sets. However, the change is small because our analysis is conditioned on four initial

12. The notion of Bayesian posterior predictive checks dates back at least to Box (1980) and is explained in detail, for instance, in recent textbooks by Lancaster (2004) and Geweke (2005).

13. We also used the Kalman smoother to back out the exogenous processes $\ln A_t$ and $\ln B_t$. Without adjustment costs the smoothed exogenous processes under $\mathcal{M}_0$ and $\mathcal{M}_1$ are virtually identical. The NBER recessions tend to be associated with adverse movements of the labor supply process, which in Chang and Schorfheide (2003) is interpreted as increase in the home production technology relative to the market technology. Since in specification $\mathcal{A}_0$ due to the presence of adjustment costs the estimated ρ_b is less than 0.9, the smoothed $\ln B_t$ process exhibits much more high frequency movements than under the non-stationary hours specification $\mathcal{A}_1$.

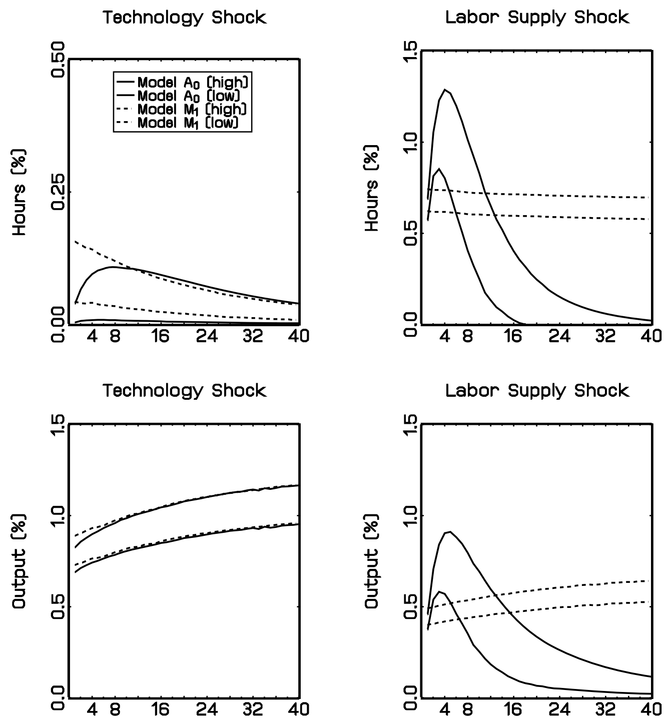

Fig. 4. Impulse Response Functions: $\mathcal{A}_0$ versus $\mathcal{M}_1$.

Notes: Figure depicts impulse response functions to one-standard-deviation shocks: pointwise 90% intervals. Results are based on benchmark prior and Data Set 1.

observations. Tightening the prior for ln B_0 leaves the marginal data density virtually unchanged.

Priors P3 and P4 modify the distribution of ρ_b in $\mathcal{M}_0$ by increasing (P3) and decreasing (P4) the implied persistence of the labor supply shock. For all three data sets P3 raises the marginal data density of $\mathcal{M}_0$ and hence narrows the gap between $\mathcal{M}_0$ and $\mathcal{M}_1$. For instance, based on Data Set 1 the odds in favor of the non-stationary specification drop from 8:1 to 2:1. If one compares $\mathcal{M}_0$ with Prior P3 to $\mathcal{M}_1$ with Prior P1, then $\mathcal{M}_0$ slightly dominates. Under Prior P4, on the other hand, the marginal data density of $\mathcal{M}_0$ falls relative to the benchmark prior. Without adjustment costs the non-stationary specification $\mathcal{M}_1$ is preferred to $\mathcal{M}_0$. However, the magnitude of the posterior odds is sensitive to the prior, reflecting the difficulty of distinguishing unit-root from stationary yet highly persistent dynamics in finite samples.

Based on Data Set 1, 90 percent posterior intervals for impulse responses of output and hours are depicted in Figure 4. The estimated impulse response functions for Data Set 2 and 3 are similar to those obtained from Data Set 1 and hence omitted. Based on the evidence from the marginal data densities we restrict our attention to the specifications $\mathcal{M}_1$ and $\mathcal{A}_0$. In general, the responses of $\mathcal{M}_1$ (dotted lines) are monotonic because without adjustment costs there is little internal propagation. After a permanent rise in technology, hours worked increase initially and return to the steady state level (top left-hand panel). The hours response in $\mathcal{A}_0$ (solid lines) exhibits a slight hump shape due to the adjustment costs. The output responses to a positive technology shock in the two models are very similar: output monotonically approaches a new steady state over time (bottom left-hand panel). In response to a labor supply shock, the two models exhibit striking differences (right-hand panels). After a mean-reverting labor supply shock both hours and output dynamics have clear humps in the adjustment cost model ($\mathcal{A}_0$). A non-stationary labor supply shock, on the other hand, generates monotonic responses in hours and output in the model without adjustment costs ($\mathcal{M}_1$).

4. CONCLUSION

In order to account for the joint movement of observed hours and consumption over the business cycle, signified by a strongly pro-cyclical movement of the consumption-leisure ratio without corresponding variations in the real wage, DSGE models are often augmented with shifts in the marginal rate of substitution between leisure and consumption. These labor supply shocks are typically assumed to be stationary. The balanced-growth-path property of the standard neoclassical growth model, which serves as basis for most DSGE models, implies that hours worked are stationary. This implication, however, appears to be at odds with the persistent movements of per capita hours in the data.

In this article we illustrate that DSGE models can be easily modified to incorporate non-stationary labor supply shocks which generate permanent shifts in hours worked. We then ask whether non-stationary labor supply shocks are required to account for the persistence of per capita hours in the data. According to our time series analysis, the model with permanent labor supply shocks provides a better fit than the specification with transitory shocks, if one abstracts from labor adjustment frictions. However, once one allows for adjustment costs in labor, the ranking is reversed and the model with transitory labor supply shocks is preferred. Overall, the model specifications with adjustment costs fit the data better than those without. The adjustment cost model with stationary labor supply shocks, and hence stationary yet highly persistent labor fluctuations, is able to reproduce the observed sample autocorrelation and variance of hours worked.

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