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Exchange rate flexibility under the zero lower bound[☆]

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ABSTRACT

An independent monetary policy and a flexible exchange rate generally help a country in adjusting to macroeconomic shocks. But recently in many countries, interest rates have been pushed down close to the lower bound, limiting the ability of policy-makers to accommodate shocks, even with flexible exchange rates. This paper argues that when the zero bound constraint on nominal interest rates is binding and policy lacks an effective 'forward guidance' mechanism, a flexible exchange rate system may be inferior to a single currency area. With monetary policy constrained by the zero bound, a flexible exchange rate exacerbates the impact of shocks. Remarkably, this may hold true even if only a subset of countries are constrained by the zero bound, and other countries optimally adjust their interest rate targets. For a regime of multiple currencies to dominate a single currency in a zero bound environment, it is necessary to have effective forward guidance in monetary policy.

1. Introduction

Optimal currency area theory (e.g. Kenen 1969, Mundell 1961) states that a country subject to idiosyncratic macro shocks should have its own independent monetary policy and a flexible exchange rate.A country with flexible exchange rates can reduce interest rates in the face of a negative demand shock, allowing an exchange rate depreciation, which ensures faster adjustment in relative prices. This adjustment mechanism is absent within a single currency area. Much of the criticism of the Eurozone is built on optimal currency area logic. When one country in the Eurozone goes into recession, it cannot offset this through exchange rate depreciation. The lack of independent monetary policy has been identified as one of the biggest hindrances to a faster recovery of economic activity of Southern European countries.

An important feature of the recent crisis in both Europe and elsewhere, however, is that the normal functioning of monetary policy has been severely circumscribed by the zero bound constraint. In the Eurozone, and many other countries, interest rates have been at historically low levels and have been unable to respond adequately to the scale of the downturns in the real economy. Arguably, the Eurozone and many other regions have been stuck in a liquidity trap.

The main aim of this paper is to show that the standard reasoning in favor of multiple currencies and flexible exchange rates may be incorrect in a liquidity trap. When monetary policy is constrained by the zero bound on interest rates and policy-makers lacks effective forward guidance, it may be better to have a single region-wide currency than a regime of multiple floating currencies. Remarkably, this conclusion may still hold even if only a subset of countries in the region are constrained by the zero bound, and the other countries are free to follow optimal monetary policy rules. Equivalently, our analysis says that when a region experiences large a negative demand shock which leads to policy rates being constrained by the zero bound, then it may in fact be better inside the single currency area than if it had its own independent currency and a floating exchange rate.

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To clarify the logic of these results, take a simple New Keynesian open economy model and assume there are country-specific demand shocks. Then under 'normal' times, when nominal interest rates are positive and monetary policy follows an active inflation targeting rule, a disinflationary shock in one region is followed by a relative decline in real interest rates in that region accompanied by an exchange rate depreciation, which limits the impact of the shock.

Now however, take a negative demand shock in the case where monetary policy is constrained by the zero bound. Then, a disinflationary shock raises the real interest rate and leads to an exchange rate appreciation in the most affected region. Rather than off-setting the effect of the shock, the exchange rate moves in the 'wrong direction', *exacerbating* the effects of the shock.

By contrast, a single currency area eliminates the possibility of perverse exchange rate adjustment, and achieves a superior sharing of macro risk among regions. Under flexible exchange rates, unconventional monetary policy which offers guidance about the future path of monetary policy can potentially be used to prevent an undesirable response of the exchange rate at the zero bound. However, this unconventional monetary policy requires central banks to have the ability to credibly commit to future interest rate paths. In a sense, the elimination of independent currencies acts as a commitment technology, removing the possibility of *perverse* adjustment of exchange rates following country specific shocks, whether the zero bound constraint on nominal interest rates is binding or not.

We present the argument in three stages. First we use a stylized 'canonical' two country New Keynesian model where countries may be subject to demand shocks arising from temporary changes in the rate of time preference (savings shocks). In the first case, monetary policy is governed by a simple Taylor rule, which applies so long as nominal interest rates are positive. In a multiple currency, flexible exchange rate version of the model, when the Taylor rule is operative, a country-specific savings shock elicits a compensating nominal and real exchange rate depreciation for the affected country. If, in the same circumstance, the region were governed by a single currency area, a real depreciation would require a relative domestic price deflation, which would be more costly and prolonged.

Now, however, assume that interest rates are constrained by the zero bound. In this case, the country experiencing the large savings shock will experience relative price deflation, pushing up its relative real interest rate[1] , and generating a nominal and real exchange rate *appreciation* . This appreciation exacerbates the effect of the original shock. By contrast, the *relative* real interest rate and real exchange rate adjustment process under a single currency area is the same, whether or not the zero bound constraint applies. As a result, in a zero bound environment, adjustment to country-specific shocks is more efficient in a single currency area than under multiple currencies with flexible exchange rates. With flexible exchange rates, the endogenous movement in the exchange rate acts as a destabilizing mechanism at the zero lower bound.

We then extend this analysis to the case where monetary policy is chosen optimally in a cooperative framework, and some countries may not be constrained by the zero bound. Remarkably, we find that the same argument applies. That is, it may be better to have a single currency area than a system of multiple currencies with flexible exchange rates, even when only one of the two countries is in a liquidity trap, and the other country follows an optimal monetary policy to maximize a weighted sum of each country's welfare. The logic here is, in fact, the same as in the previous case. While an optimal monetary policy can alleviate the impact of perverse movements in the exchange rate, it may still be better not to have had any exchange rate adjustment at all, when the affected country is at the zero bound.

Finally, we extend the model to allow for 'forward guidance' in monetary policy. Here, both countries have full commitment to determine the path of interest rates both during the life of the shock and after the expiry of the shock. In this case, we find that the traditional logic is restored. Optimal forward guidance can ensure that the country affected by the shock promises highly accommodative monetary policy in the future, after the shock ends, and if this promise is credible, it achieves an immediate contemporaneous movement of exchange rates in the right direction. By doing so, it can improve the adjustment process, compared with that in a single currency area. An optimal policy, with effect forward guidance, multiple currencies, and flexible exchange rates, is in general better than an equivalent policy under a single currency area.

Hence, a key message of the paper is that forward guidance is a particularly critical element in monetary policy making in open economies with flexible exchange rates, when the zero bound constraint is likely to be binding. By contrast, without effective forward guidance, a single currency area acts as an in-built commitment mechanism guaranteeing that a country pushed into a liquidity trap will experience future inflation, reducing the impact of the shock on current inflation. With multiple currencies, flexible exchange rates, and no commitment, there is no such ability to guarantee future inflation for the affected country.

The commitment potential of pegged exchange rates is highlighted in a previous paper by Corsetti et al. (2011). While their paper is concerned with the effects of fiscal policy in a small open economy, their mechanism is similar to the one implicit in our paper. They note that if exchange rates are fixed, and temporary shocks do not affect the long run real exchange rate, any current disinflation must be matched by future inflation as relative prices return to PPP. Therefore, a fixed exchange rate is a form of price level of targeting. It has been noted in previous literature that price level targeting is in fact a way to establish a degree of commitment at the zero lower bound (see Eggertsson and Woodford, 2003).

The paper is also closely related to the recent literature on monetary and fiscal policy in a liquidity trap. In particular, with the experience of Japan in mind Krugman (1998), Eggertsson and Woodford (2003, 2005), Jung et al. (2005), Svensson (2003), Auerbach and Obstfeld (2005) and many other writers explore how monetary and fiscal policy could be usefully employed even when the authorities have no further room to reduce short term nominal interest rates. Recently, a number of authors have revived this literature in light of the very similar problems recently encountered by the economies of Western Europe and North America. Papers by Christiano et al. (2011), Devereux (2010), Eggertsson (2011), Cogan et al. (2009) have explored the possibility for using government spending expansions, tax cuts, and monetary policy when the economy is in a liquidity trap. Bodenstein et al. (2009) is an example of a fully specified two country DSGE model which examines the international transmission of standard business cycle shocks when one country is in a liquidity trap. In addition, Werning (2012) explores optimal monetary and fiscal policy in a continuous time model in face of zero lower bound constraints. Correia et al. (2013) explore a set of alternative fiscal instruments that can be used as a substitute for monetary policy in a zero lower bound situation.

The counterintuitive implications of the zero lower bound outlined in this paper parallel in part the surprising results that in a closed economy, some typically expansionary policies may be contractionary. An example is given of the contractionary effects of tax cuts in Eggertsson (2011).

Some recent papers consider international dimensions of optimal policy in a liquidity trap. Jeanne (2009) examines whether either monetary policy or fiscal policy can implement an efficient

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[1] This response of real interest rates is very similar to those identified in the closed economy literature on the zero bound constraint (see in particular, Christiano et al., 2011, and Eggertsson, 2011).

equilibrium in a 'global liquidity trap' in a model of one-period ahead pricing similar to that of Krugman (1998). Fujiwara et al. (2009) use numerical results to describe optimal monetary policy responses to asymmetric natural interest rate shocks. Fujiwara et al., (2013) examine optimal policy responses to technology shocks in a model without home bias. Our model incorporates home bias in a way that implies that demand shocks require relative price changes. Cook and Devereux (2011) and Fujiwara and Ueda (2010) examine the fiscal policy multiplier in an open economy subject to the zero lower bound constraint. Farhi and Werning (2013) provide a general comparison of fiscal multipliers in a currency union both at and away from the zero lower bound. Erceg and Linde (2010) compare the effects of fiscal consolidation in a single currency area with that in a multi-currency model where the zero lower bound may apply. They find that the comparison depends on the degree of price flexibility (we discuss their paper further in Section 4 below). Two more recent papers analyze the effect of 'deleveraging' shocks in multi-country models. Benigno and Romei (2014) explore the effect of a increase in the borrowing constraint on one country in a two-country model. They explore alternative monetary policies that can be used to alleviate the impact of deleveraging. Fornaro (2014) constructs a multi-country 'Bewley model' of a monetary union and examines the effect of a tightening of the leverage constraint. Our paper differs from both these papers essentially in that we do not have any borrowing constraints. We look at the effect of savings shocks in a world of fully open capital mobility. Finally, a more directly related paper is Cook and Devereux (2013), which looks at optimal monetary and fiscal policy in a flexible exchange rate version of a model similar to the one in the present paper. But Cook and Devereux (2013) does not examine the implications of exchange rate policy, nor allow for the possibility of a single currency area. In addition, they do not discuss the implications of forward guidance.

Finally, we stress that we see this paper principally as a theoretical exploration in open-macro policy. That is, we raise some questions about the standard set of policy prescriptions in international macroeconomics in an environment where policy is constrained by the zero bound, and when problems of time consistency make policy commitment hard to establish. Nevertheless, in a final section of the paper, we present some evidence on the movement of the Japanese Yen and the Swiss Franc around the time that these countries moved to a zero interest rate policy. This evidence suggests that the joint process of relative deflation and zero nominal interest rates may in fact be associated with exchange rate *appreciation*.

The paper is structured as follows. The next section sets out the two country basic model. Section 3 shows some properties of the model solution. Then Section 4 shows the main result of the paper in a simple setting with arbitrary monetary rules that may be constrained by the zero bound. Section 5 extends the argument to a situation where monetary policy is chosen optimally to maximize a weighted sum of each country's welfare, but again constrained by the zero lower bound, and without commitment. Section 6 extends the analysis to allow for monetary policy commitment. Section 7 provides a brief discussion of the empirical relevance of the results, with reference to experience in Japan and Switzerland. Some conclusions then follow.

2. A two country model

Take a standard two country New Keynesian model, denoting the countries as 'home' and 'foreign'. Utility of a representative infinitely-lived home household evaluated from date 0 is :

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(U\left(C_t, \xi_t\right) - V\left(N_t\right) \right) \tag{1}$$

where U and V represent the utility of the composite home consumption bundle C_t, and disutility of labor supply N_t. The variable ξ_t represents a shock to preferences or a 'demand' shock . We assume that $U_{12} > 0$. A positive ξ_t shock implies that agents become temporarily more anxious to consume today rather than in the future. A negative ξ_t shock implies that agents wish to defer consumption to the future, and so will increase their desired savings.

Composite consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} \left(1 - \left(\frac{v}{2}\right)\right)^{\frac{v}{2}}$, C_H is the home country composite good, and C_F is the foreign composite good. If $v > 1$ then there is a preference bias for domestic goods (home bias). Consumption aggregates C_H and C_F are composites, defined over a range of home and foreign differentiated goods, with elasticity of substitution θ between goods. Price indices for home and foreign consumption are:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, P_F = \left[\int_0^1 P_F(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_{Ht}^{v/2} P_{Ft}^{1-v/2}$ and with identical home bias for the foreign country, the foreign CPI is $P^* = P_F^{*v/2} P_H^{*1-v/2}$

Demand for each differentiated good ($j = H, F$) is

$$\frac{C_j(i)}{C_j} = \left(\frac{P_j(i)}{P_j} \right)^{-\theta}.$$

The law of one price holds for each good so $P_j(i) = SP_j^*(i)$ where S_t is the nominal exchange rate (home price of foreign currency).

The household's implicit labor supply at nominal wage W_t is:

$$U_C(C_t, \xi_t) W_t = P_t V'(N_t). \tag{2}$$

Optimal risk sharing implies

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*) \frac{S_t P_t^*}{P_t} = U_C(C_t^*, \xi_t^*) T_t^{(v-1)}, \tag{3}$$

where $T = \frac{SP_F^*}{P_H}$ is the home country terms of trade.

Nominal bonds pay interest R. Then the home consumption-Euler equation is:

$$\frac{U_C(C_t, \xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}}. \tag{4}$$

Foreign household preferences and choices can be defined exactly symmetrically.

2.1. Firms

Each firm i employs labor to produce a differentiated good, so that its output is

$$Y_t(i) = N_t(i).$$

Profits are $\Pi_t(i) = P_{Ht}(i) Y_t(i) - W_t H_t(i) \frac{\theta-1}{\theta}$ indicating a subsidy financed by lump-sum taxation to eliminate steady state first order inefficiencies. Each firm re-sets its price according to Calvo pricing with probability $1-\kappa$. Firms that adjust set a new price given by $\tilde{P}_{Ht}(i)$:

$$\widetilde{P}_{Ht}(i)=\frac{E_t\sum_{j=0}m_{t+j}\kappa^j\frac{W_{t+j}}{A_{t+j}}Y_{t+j}(i)}{E_t\sum_{j=0}m_{t+j}\kappa^jY_{t+j}(i)} \tag{5}$$

where the stochastic discount factor is $m_{t+j}=\frac{P_t}{U_C(C_t,\xi_t)}\frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}}$. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht}=[(1-\kappa)\widetilde{P}_{Ht}^{1-\theta}+\kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

The behavior of foreign firms and the foreign good price index may be described analogously.

2.2. Market clearing

Equilibrium in the market for good i is

$$Y_{Ht}(i)=\left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta}\left[\frac{v}{2}\frac{P_t}{P_{Ht}}C_t+(1-\frac{v}{2})\frac{S_tP_t^*}{P_{Ht}}C_t^*\right].$$

Aggregate market clearing in the home good is:

$$Y_{Ht}=\frac{v}{2}\frac{P_t}{P_{Ht}}C_t+(1-\frac{v}{2})\frac{S_tP_t^*}{P_{Ht}}C_t^*. \tag{7}$$

Here $Y_{Ht}=V_t^{-1}\int_0^1 Y_{Ht}(i)di$ is aggregate home country output, where we have defined $V_t=\int_0^1\left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta}di$. It follows that home country employment (employment for the representative home household) is given by $N_t=\int_0^1 N(i)di=Y_{Ht}V_t$.

An equilibrium in the world economy with positive nominal interest rates may be described by the Eqs. (2), (4), (5), (6), and (7) for the home country, and the analogous equations for the foreign economy. Together with Eq. (3), and for given values of V_t and V_t^*, given monetary rules (to be discussed below), these equations determine an equilibrium sequence for the variables $C_t, C_t^*, W_t, W_t^*, S_t, P_{Ht}, P_{Ft}^*, \tilde{P}_{Ht}, \tilde{P}_{Ft}^*, R_t, R_t^*$, and N_t, N_t^*.

3. The effects of savings shocks

Define $\sigma\equiv-\frac{U_{CC}\overline{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption; and $\phi\equiv-\frac{V''\overline{H}}{V'}$ as the elasticity of the marginal disutility of hours worked. Assume that $\sigma>1$. In addition, $\varepsilon_t=\frac{U_{C\xi}}{U_C}\ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country.

For this and the next section, we make a simplifying assumption about the nature of preference shocks. We assume that the shock is unanticipated, remains constant in each time period with probability μ, and reverts back to zero with probability $1-\mu$. This assumption implies that under independent monetary policy and flexible exchange rates, there are no predetermined state variables in the model. Hence, all endogenous real variables and inflation rates in the world economy will inherit the same persistence as the shock itself, in expectation. Thus, for any such endogenous variable x_t, we may write $E_t(x_{t+1})=\mu x_t$. After the shock expires, all variables will then revert to their zero initial equilibrium. This property does not carry over to the single currency area, since in that case, the lagged terms of trade becomes an independent state variable (as shown below).

3.1. The world and relative economy

We derive a log-linear approximation of the model as in Clarida et al., (2002) and Engel (2010). Let $\hat{x}_t$ be the percentage deviation of a given variable x_t from its non-stochastic steady state level. In the analysis below, each variable will be described in this way, except for the nominal interest rate and inflation rates, which are defined in levels, and ε_t, which is already defined in terms of deviation from the steady state value of zero. We define the term $D\equiv\sigma v(2-v)+(1-v)^2\geq1$. In addition, define $\zeta\equiv\frac{(v-1)}{D}$. The parameter, $0\leq\zeta\leq1$, measures the intensity of home bias. In the absence of home bias, $\zeta=0$; under full home bias $\zeta=1$.

We express the approximations in terms of world *averages* and world *relatives*. Thus, the world average (relative) value for variable x is given by $x^W=\frac{1}{2}(x+x^*)$ $(x^R=\frac{1}{2}(x-x^*))$.

From Eqs. (3) and (7), as well as the equivalent condition for the foreign country, the partial solutions for the terms of trade and relative consumption are:

$$\widehat{\tau}_t=2\frac{\sigma}{D}\widehat{y}_t^R-2\zeta\varepsilon_t^R \tag{8}$$

$$\widehat{c}_t^R=2\zeta\widehat{c}_t^R+\frac{2v(2-v)}{D}\varepsilon_t^R \tag{9}$$

Given relative income, and assuming $v>1$, a negative ε_t^R shock reduces relative demand for the home good, causing a terms of trade deterioration. Given ε_t^R, a rise in relative income also causes a terms of trade deterioration, and a rise in relative home consumption.

Using these conditions in combination with a linear approximation of Eqs. (2), (4), (5), and (6), we can derive the following forward looking inflation equations and open economy IS relationships, expressed in terms of world averages and world relatives. We write this system in 'gap' terms, where we define the variable $\tilde{x}=x-\bar{x}$ as the gap between the log of a variable and the log of its flexible price analogue ($\bar{x}$). The only exceptions are for inflation, which is written in logs, since its flexible price value is zero, and the nominal interest rate, which is expressed in levels[2] . The world average equations are:

$$\pi_t^W=k(\phi+\sigma)\widetilde{y}_t^W+\beta E_t\pi_{t+1}^W \tag{10}$$

$$\sigma E_t(\widetilde{y}_{t+1}^W-\widetilde{y}_t^W)=r_t^W-E_t\pi_{t+1}^W-\bar{r}_t^W \tag{11}$$

The coefficient k depends on the degree of price rigidity. The term $\bar{r}_t^W$ is the world average interest rate that would apply in a flexible price equilibrium, which we term the 'natural' interest rate. In the online Appendix we show that $\bar{r}_t^W$ is expressed as:

$$\bar{r}_t^W=\rho+\frac{(1-\mu)\phi}{\sigma+\phi}\varepsilon_t^W. \tag{12}$$

A negative shock to ε_t^W leads to a fall in the world natural interest rate when $\mu<1$.

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[2] Note that $\pi^W=0.5(\pi+\pi^*)$, where π and π^* are the home and foreign PPI inflation rates.

Now let $\sigma_D \equiv \frac{\sigma}{D}$,where $\sigma \geq \sigma_D \geq 1$. Then the world 'relative' equations are written as:

$$\pi_t^R = k(\phi + \sigma_D)\tilde{y}_t^R + \beta E_t \pi_{t+1}^R \tag{13}$$

$$\sigma_D E_t(\tilde{y}_{t+1}^R - \tilde{y}_t^R) = r_t^R - E_t \pi_{t+1}^R - \bar{r}_t^R \tag{14}$$

where $\bar{r}_t^R$ represents the natural world relative interest rate, defined in the Appendix as:

$$\bar{r}_t^R = \frac{(1-\mu)\zeta\phi}{\sigma_D + \phi}\varepsilon_t^R. \tag{15}$$

A negative ε_t^R shock reduces the world relative natural interest rate when $\mu < 1$ and when there is home bias in the consumption bundle.

Eqs. (10)–(11) and (13)–(14) describe the response of the world economy to the savings shock through the world average inflation rate and output gap movements, and the world relative inflation rate and output gap movements. Note that the degree of home bias does not affect aggregate average outcomes, so that there is a dichotomy in the solution of aggregate and relative models. The effect of home bias on relative outcomes is summarized by the two parameters ζ and σ_D. [3]

The solutions to Eqs. (10)–(11) and (13)–(14) will depend on the rules for monetary policy, captured by the world average and relative nominal interest rates, given by r_t^W and r_t^R. We now turn to this question.

4. The simplest case: comparing interest rate rules with a zero bound constraint

4.1. *Separate currencies with independent monetary policies*

We begin with a simple case to show the essence of the argument. Assume that outside of the zero lower bound, monetary policy is characterized by an interest rate rule.[4] Under separate currencies, each country sets its own interest rate. We assume a simple Taylor rule described (for the home economy) as:

$$r_t = \rho + \gamma\pi_{t.} \tag{16}$$

Here, monetary policy targets the rate of PPI inflation. With separate currencies, using Eq. (16) and the analogous foreign condition we have $r_t^W = \rho + \gamma\pi_t^W$, and $r_t^R = \gamma\pi_t^R$. Combining these two expressions with Eqs. (10)–(14), we can derive the solutions for the four variables π^W, y^W, π^R, and y^R. Then using Eqs. (8) and (9) we can obtain solutions for the terms of trade and relative consumption. The nominal exchange rate s_t may be then obtained from the condition[5]

$$s_t - s_{t-1} = \pi_t^R + \tau_t - \tau_{t-1}. \tag{17}$$

In this example, the solution for world averages is the same under multiple currencies or a single currency area. We therefore focus only on the characteristics of world relatives. Also, with multiple currencies, there are no predetermined state variables, and under the assumed stochastic characteristics of the ε_t shock, the endogenous variables $\tilde{y}_t^R, \pi_t^R$ and τ_t take on the persistence characteristics of the ε_t shock.[6] From Eq. (13), we can then describe a relationship between relative PPI inflation and the relative output gap as:

$$\pi_t^R = \frac{k(\phi + \sigma_D)}{(1-\beta\mu)}\tilde{y}_t{}^R. \tag{18}$$

A rise in the relative home output gap leads to a rise in relative home country inflation.

Likewise, from Eq. (14), we obtain a relationship between relative inflation and the output gap, conditional on the relative natural interest rate $\bar{r}_t^R$. We have to be careful here however, since Eq. (14) depends on the policy rule, and we want to take account of the possibility that the policy rule may be constrained by the zero lower bound. Using the definitions of world averages and relatives, this implies that we impose the conditions:

$$r_t = r_t^W + r_t^R = Max(0, \rho + \gamma\pi_t) \tag{19}$$

$$r_t^* = r_t^W - r_t^R = Max(0, \rho + \gamma\pi_t^*) \tag{20}$$

4.1.1. *Outside the zero bound constraint*

Assume first that neither conditions (19) nor (20) is binding. Then we can substitute $r_t^R = \gamma\pi_t^R$ into Eq. (14), substituting also for $\bar{r}_t{}^R$ and take expectations, obtaining:

$$\pi_t^R = -\frac{(1-\mu)}{(\gamma-\mu)}\left(\sigma_D\tilde{y}_t{}^R - \frac{\zeta\phi}{\sigma_D + \phi}\varepsilon_t^R\right). \tag{21}$$

Fig. 1 illustrates the determination of relative inflation and output gaps when monetary policy is not constrained by the zero bound. Eq. (18) is upward sloping in π^R and $\tilde{y}^R$ space, and Eq. (21) is downward sloping, since a rise in relative output, which is temporary, leads to a fall in the natural real interest rate, and under the monetary rule in Eq. (16) relative inflation must fall so as to accommodate this.

Now we see the impact of a relative savings shock, given by a negative ε_t^R. For $v > 1$, this shifts Eq. (21) to the left, leading to a fall in both inflation and the output gap.

Given independent monetary policies and the monetary rule in Eq. (16), the fall in relative inflation is associated with exchange rate and terms of trade adjustment. From the Euler equations for bond pricing Eq. (4), expressed in relative terms, combined with the risk sharing condition (3), we obtain the following relationship between relative inflation and the terms of trade:

$$\gamma\pi_t^R = E_t(\pi_{t+1}^R + \tau_{t+1} - \tau_t). \tag{22}$$

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[3] The first parameter measures the size of home bias, and represents the direct impact of relative demand shocks on relative inflation and output. When home bias is absent, $(v-1) = \zeta = 0$, and relative demand shocks have no impact on relative demand allocations. The second parameter, σ_D, the intertemporal elasticity of relative demand, governs how intensely relative demand responds to adjustments in the relative interest rates. In the presence of home bias, $\sigma_D < \sigma$, so relative demand responds more to interest rate changes than average demand, since relative interest rate movements result in real exchange rate adjustments and expenditure switching across countries.

[4] In Sections 5 and 6 below, we allow for monetary policy to be set optimally.

[5] Note we approximate around an initial steady state where the level of the terms of trade equals unity, so that hereafter, $\widehat{\tau}_t = \tau_t$ applies.

[6] Note that this is not true of the nominal exchange rate, which displays a unit root, as is clear from Eq. (17).

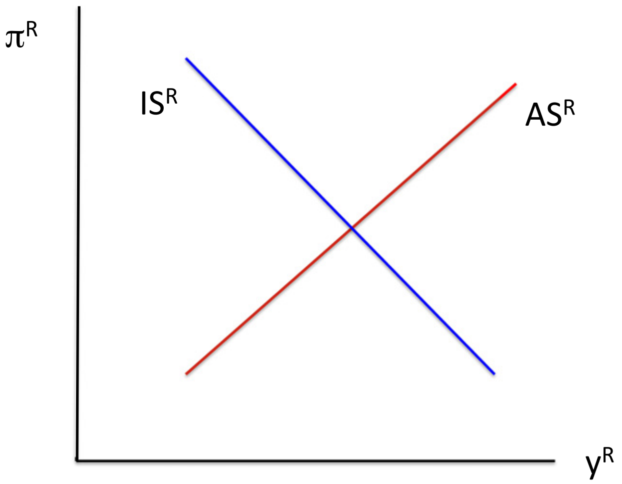

Fig. 1. Relative inflation and relative output under a Taylor rule.

The right hand side of this equation is simply the expected change in the nominal exchange rate, so that Eq. (22) is just the uncovered interest rate parity condition. Imposing the stationarity condition gives us the solution for the terms of trade:

$$\tau_t = -\frac{\gamma-\mu}{1-\mu}\pi_t^R. \tag{23}$$

Since $\gamma > \mu$, the fall in inflation implies a terms of trade deterioration. In response to the savings shock, home relative inflation falls, so that when $\gamma > \mu$, the home relative interest rate falls, which facilitates a terms of trade deterioriation. We also get a nominal exchange rate depreciation, since from Eqs. (17) and (23) we have

$$s_t - s_{t-1} = -\frac{\gamma-1}{1-\mu}\pi_t^R.$$

The full solution for the terms of trade can be derived as:

$$\tau_t = \frac{-k\phi(\gamma-\mu)}{\sigma_D(1-\beta\mu)(1-\mu)+(\gamma-\mu)k(\sigma_D+\phi)}2\zeta\varepsilon_t^R. \tag{24}$$

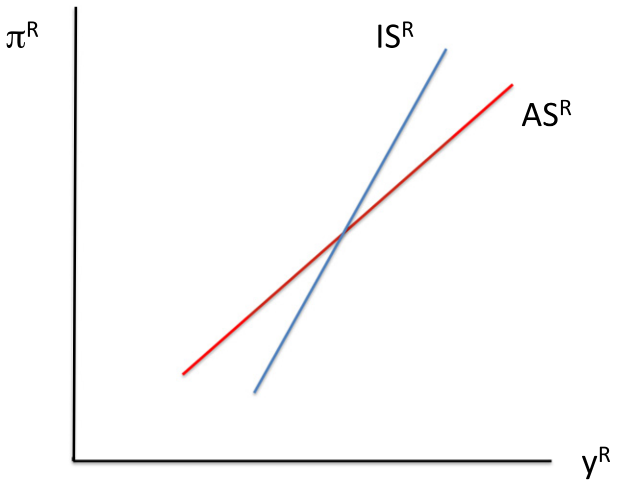

Fig. 2. Relative inflation and relative output at the zero bound with multiple currencies.

4.1.2. Shocks at the zero bound constraint

Now contrast this outcome to that constrained by the zero interest bound. In this case, nominal interest rates are equal, and zero, in both the home and foreign country. Then Eq. (14) must reflect this additional constraint. Again, imposing stationarity, we derive the relationship:

$$\pi_t^R = -\frac{1-\mu}{\mu}\left(\sigma_D\tilde{y}_t^R - \frac{\zeta\phi}{\sigma_D+\phi}\varepsilon_t^R\right). \tag{25}$$

This is a upward sloping locus in π^R-$\tilde{y}^R$ space. A rise in current relative inflation raises anticipated future relative inflation, when $\mu > 0$. This reduces the home relative real interest rate, and raises the relative home output gap[7] . We further make the assumption that the stability condition $\frac{1-\mu}{\mu}\sigma_D > k(\phi+\sigma_D)(1-\beta\mu)$ holds, so that the slope of Eq. (25) exceeds that of Eq. (18).

Fig. 2 shows the effects of a savings shock when the zero bound constraint binds. Again, we have a fall in both relative home inflation and the relative home output gap. In contrast to the case where Eq. (16) applies, at the zero bound, the fall in relative inflation tends to magnify the fall in relative output. The fall in relative inflation causes a rise in home relative real interest rates, which causes a secondary fall in home relative demand. So long as the above stability condition holds, this process converges at a lower relative inflation and relative output gap. But for a given ε_t shock, the fall in relative inflation and the relative output gap is larger.

What does this imply for the exchange rate and the terms of trade? Note that, although the zero bound is binding in both countries, the arbitrage conditions (3) and (4) still apply. This means that Eq. (22) still holds, but with the left hand side equal to zero. Under the stationarity condition, then we have

$$\tau_t = \frac{\mu}{1-\mu}\pi_t^R. \tag{26}$$

Then, the terms of trade must *appreciate*. The full solution is written as:

$$\tau_t = \frac{k\phi\mu}{\sigma_D(1-\beta\mu)(1-\mu)-\mu k(\sigma_D+\phi)}2\zeta\varepsilon_t^R.$$

The difference with the previous case is that the fall in relative home inflation leads to a rise in the home relative real interest rate, which leads to an appreciation in the terms of trade. From Eqs. (17) and (26), there is also a nominal exchange rate *appreciation*. The initial impact of the shock on the nominal exchange rate satisfies:[8]

$$s_t - s_{t-1} = \frac{1}{1-\mu}\pi_t^R.$$

This is the essence of the argument over the merits of flexible exchange rates at the zero bound. Because movements in relative inflation lead to perverse movements in relative real interest rates at the zero bound, relative prices move in the 'wrong direction'. The

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[7] Note that in Eggertsson and Woodford (2003), it is critical for this argument that the liquidity trap will expire in expectation . This ensures that the current inflation rate is pinned down by the expectation that prices and inflation will be uniquely determined along a stable manifold in the future.

[8] Because nominal interest rates are equal, the effect of the shock is to cause a one-time appreciation in the nominal exchange rate, and the subsequent anticipated movement in the exchange rate is zero, so that uncovered interest rate parity is still satisfied.

appreciation in the terms of trade exacerbates rather than ameliorates the impact of the initial relative savings shock.[9]

4.2. *Single currency area*

Now assume that both countries are part of a single currency area. To investigate this, we can use the results of Benigno (2004). In a single currency area monetary policy for the whole region is governed by the condition $r_t^W = \rho + \gamma \pi_t^W$, and by definition, both countries face the same nominal interest rate, so that $r_t^R \equiv 0$. At first glance, it looks as if the solution should be the same as the multiple currency case under the zero lower bound. However, Benigno et al. (2007) point out that the condition $r_t^R \equiv 0$ is consistent with multiple equilibrium. Benigno (2004) shows that a fixed exchange rate imposes another initial condition on the dynamics of the terms of trade in a single currency area. This is simply given by Eq. (17), but setting the left hand side to zero so that

$$\tau_t = \tau_{t-1} - \pi_t^R. \tag{27}$$

Since there is only one nominal interest rate, the relative interest rate equations for nominal bond rates do not impose any additional constraints on the model. But we can combine Eq. (8) (rewritten in 'gap' terms) with Eq. (27) to obtain a separate relationship between inflation and the output gap implied by the single currency area. This is:

$$\pi_t^R = -2\left(\sigma_D \widetilde{y}_t^R - \frac{\zeta\phi}{\sigma_D + \phi}\varepsilon_t^R\right) + 2\left(\sigma_D \widetilde{y}_{t-1}^R - \frac{\zeta\phi}{\sigma_D + \phi}\varepsilon_{t-1}^R\right). \tag{28}$$

This replaces Eq. (21) as an equilibrium condition under the single currency area. It represents a relationship linking relative demand, represented by the effect of output gaps on the terms of trade, to relative inflation. Note that it doesn't contain any parameters relevant to the monetary rule, and also, it is a dynamic equation; movements in relative inflation and output gaps won't satisfy the same stationarity characteristics as those under the multiple currency regime. The first point is obvious — there is only an aggregate monetary policy in the single currency area, and relative inflation is independent of the area-wide policy rule (under the symmetry assumptions we've made so far). The second follows since in a single currency area, the terms of trade can change only due to movements in domestic prices indices that occur gradually. This becomes an important distinction between the single currency area and the multiple currency regime at the zero lower bound — the dynamic properties of a single currency area ensure that the impact of the shock does not dissipate immediately after the shock expires. As we will see below, this gives a 'proxy' commitment aspect to the single currency area that doesn't exist naturally under the multiple currency regime.

We may combine Eqs. (13) and (28) as the equations determining the dynamics of relative inflation and the relative output gap following a savings shock. But before that, it is worth noting that while Eq. (28) gives a negative relationship between relative inflation and the relative output gap much like Eq. (21), the slope of the relationship is larger in the case of Eq. (28), since $2 > \frac{1-\mu}{\gamma-\mu}$. This implies that in 'normal' times, when the zero bound is not binding, a fall in relative inflation in a multiple currency area has a greater stabilizing effect on the output gap than in a single currency area. Under flexible exchange rates, a fall in relative inflation precipitates a fall in the home relative interest rate, and an exchange rate depreciation for the home economy. This can't happen in a single currency area — the fall in inflation stabilizes the output gap only by reducing the home relative price directly.

Since Eq. (28) is a dynamic equation, it can't be represented in the simple graphs of Figs. 1 and 2. But combining Eqs. (13) and (28), we may solve for the dynamics of inflation. The solution is given by:

$$\pi_t^R = \lambda \pi_{t-1}^R - \chi 2(\varepsilon_t^R - \varepsilon_{t-1}^R) \tag{29}$$

where $0 < \lambda < 1$, $\chi = -\frac{k}{2}\zeta\frac{\phi}{\Delta_{D1}} < 0$, and $\Delta_{D1} \equiv \sigma_D(1 - \beta\lambda + \beta(1-\mu)) + \frac{k}{2}(\phi + \sigma_D)$. The immediate effect of a savings shock is to reduce relative home country inflation. With a single currency area, this causes a terms of trade depreciation. The solution for the terms of trade is given as:

$$\tau_t = \lambda \tau_{t-1} + \chi 2 \varepsilon_t^R. \tag{30}$$

4.3. *Comparison*

We now compare the properties of the flexible exchange rate system with the single currency area. First focus on the case where the Taylor rule applies and the zero bound constraint is not binding. To compare the responses across the two regimes, we perform a simple numerical simulation. We make the following calibration assumptions; $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, and $v = 1.5$. In addition, we assume that there is a home country savings shock which persists with probability $\mu = 0.6$. Finally, assume a monetary rule $\gamma = 3$.[10]Fig. 3 illustrates the responses of π^R, y^R, and τ, and the home country trade balance, following the shock $\varepsilon = -0.5$.

With flexible exchange rates, relative inflation and relative output fall following the savings shock. The terms of trade depreciates sharply. By reducing the relative price of the home good, this acts to limit the fall in relative inflation and relative output. Under the single currency area, the terms of trade also depreciates, but this is muted and gradual. In order to achieve this depreciation, relative domestic inflation must fall much more sharply than under flexible exchange rates. At the same time, relative output falls by more under the single currency area. Note that the response of the trade balance is relatively unaffected by the exchange rate regime.[11]

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[9] The finding that the terms of trade (and the nominal exchange rate) must appreciate in face of the shock when the zero lower bound constraint is binding depends on the assumption that the shock expires at the same time that the constraint is relaxed. If the shock persists beyond the expiry of the zero bound constraint, and monetary policy follows a Taylor rule rather than directly offsetting the shock with an interest rate response, it is possible that the terms of trade will not appreciate. But even in this case, the same argument as above shows that the response of τ_t at the zero bound is always less than that under an operative Taylor rule (i.e. the zero bound constraint pushes relative prices in the 'wrong' direction). The assumption that the shock expires probabilistically is made so as to better illustrate the difference between the multiple exchange rate case and the single currency area, since it starkly reveals the commitment value of the latter regime. In a study of fiscal consolidation at the zero lower bound, Erceg and Linde (2010) show that a persistent fiscal contraction in a small economy may have greater or lesser effects on output in a single currency area relative to a multiple currency environment under the zero lower bound, depending on the degree of price flexibility and other parameters. In contrast to our paper, their results highlight the importance of shock persistence after the expiry of the zero lower bound. Our analysis emphasize the importance of the savings shocks that lead to the zero bound constraint on policy. If we assumed that rather than the Taylor rule, monetary policy was set optimally when the zero bound no longer binds (as in Section 5 below), then irrespective of the persistence of the shock, all gaps are zero after the expiry of the zero bound, and the terms of trade will always appreciate in the multiple currency case. These points are established in the online Appendix C.

[10] This value for γ is chosen to emphasize the stabilizing properties of the nominal exchange rate when normal monetary policy is in place. We follow this parametrization throughout the paper, except when otherwise stated.

[11] The response of the real trade balance may be approximated (around a zero initial trade balance) as

$$dTA = (v-2)\frac{\tau_t}{2} + \hat{y}_t^R - \hat{c}_t^R = \frac{2-v}{D}(\zeta\hat{y}_t^R - \varepsilon_t^r)$$

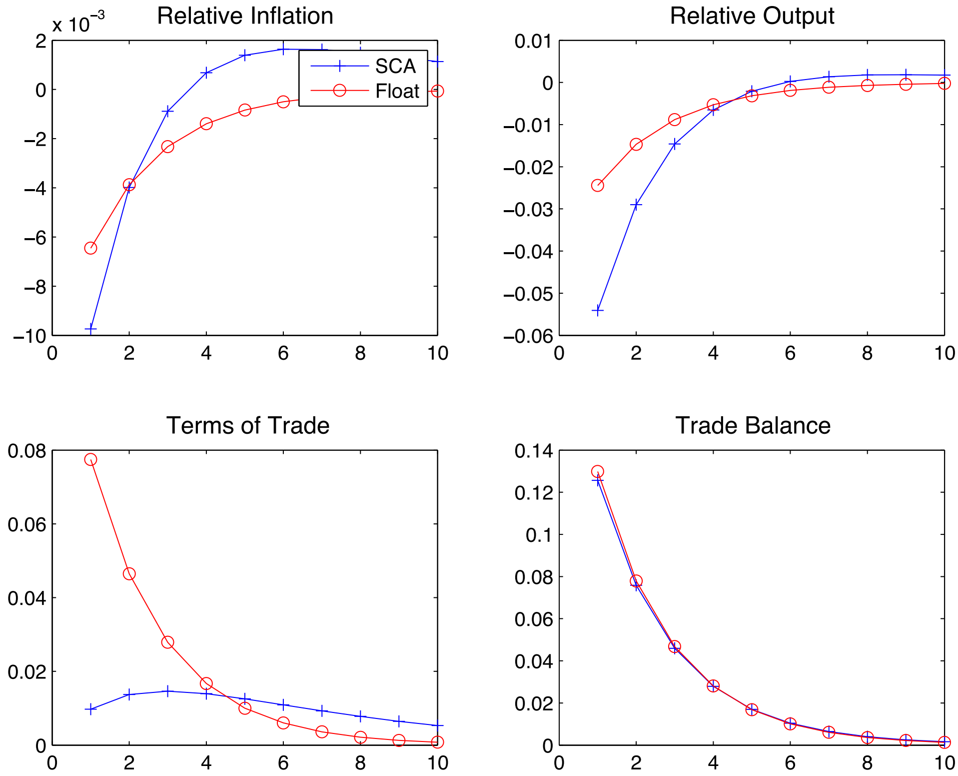

Fig. 3. Response to a saving shock under a Taylor rule versus the single currency area response. Parameter settings are as follows: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $v = 1.5$, $\mu = 0.6$, $\gamma = 3$ and $\varepsilon = -0.5$.

So far these results support the traditional merits of a flexible exchange rate in comparison with a single currency area. A country-specific demand shock requires an adjustment in relative prices. It is better to facilitate this adjustment with changes in the nominal exchange rate. A single currency area, by definition, can't achieve any nominal exchange rate adjustment. While the single currency has no consequences for overall world aggregates, it leads to excessive volatility in relative output, and insufficient flexibility in relative prices.

Fig. 4 shows the effect of a negative demand shock, but now assuming that both countries are constrained by the zero lower bound. The single currency area responses are the same as Fig. 3, since responses of relative inflation and output gaps in the single currency area don't depend on the monetary rule. But under flexible exchange rates, the terms of trade appreciates sharply, and there is a large fall in relative inflation, much larger for the flexible exchange rate than under the single currency area. The trade balance improvement under the flexible exchange rate is slightly less than that under a single currency area. We therefore see a dramatic reversal in the comparison between the multiple currency flexible exchange rate case and a single currency when the zero bound constraint is binding. Outside the zero bound, the flexible exchange rate stabilizes output. But at the zero bound, the exchange rate moves in the 'wrong direction', and relative output falls by substantially *more* than in the single currency area.

We conclude that when the zero lower bound is binding, flexible exchange rates do not act so as to stabilize the response to country specific shocks, and in fact impart greater relative macro instability than would exist under a single currency area. With flexible exchange rates, the exchange rate response compounds the original shock. In a single currency area, relative inflation rates move slowly, but move in a stabilizing fashion.[12]

As we show below, a critical feature of this comparison is that the monetary policy rule contains no commitment to future actions. The interest rate rule in Eq. (16) implies that when the shock expires, both countries will return to a zero inflation steady state. Then monetary policy cannot be relatively more expansionary in the home country, either at the time of the shock, or after the expiry of the shock. Thus, without the ability to adjust current interest rates, there is simply no policy response that can ensure the exchange rate moves in a stabilizing direction. By contrast, paradoxically, the single currency area has an inbuilt commitment, because relative prices can only change through domestic inflation, and even after the shock expires, the relative price of the home good will continue to be lower, as the terms of trade adjusts back to the steady state. This point is clarified in Section 6 below.

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using Eqs. (8) and (9). The results make clear that the benefits of flexible exchange rates are not tied to trade flows. Even in the case where movements in the terms of trade achieve full risk-sharing and balanced trade, as in the classic Cole and Obstfeld (1991) model (which is not the case here, since there is home bias in consumption), the exchange rate regime will still matter for the volatility of relative prices, consumption and output, unless prices are fully flexible. We note of course that the model assumes complete markets, so that the measured trade balance is not an integral part of cross country risk sharing.

[12] Although this is simply an impulse response to a one-time unanticipated shock, the contrast between the two regimes with and without the zero lower bound carries over to welfare comparisons. Using the welfare loss expressions from Section 5 we compute that the welfare loss of the single currency area is 0.3% higher than the flexible exchange rate regime under the Taylor rule, while it is 7% lower than the flexible exchange rate regime under the zero bound.

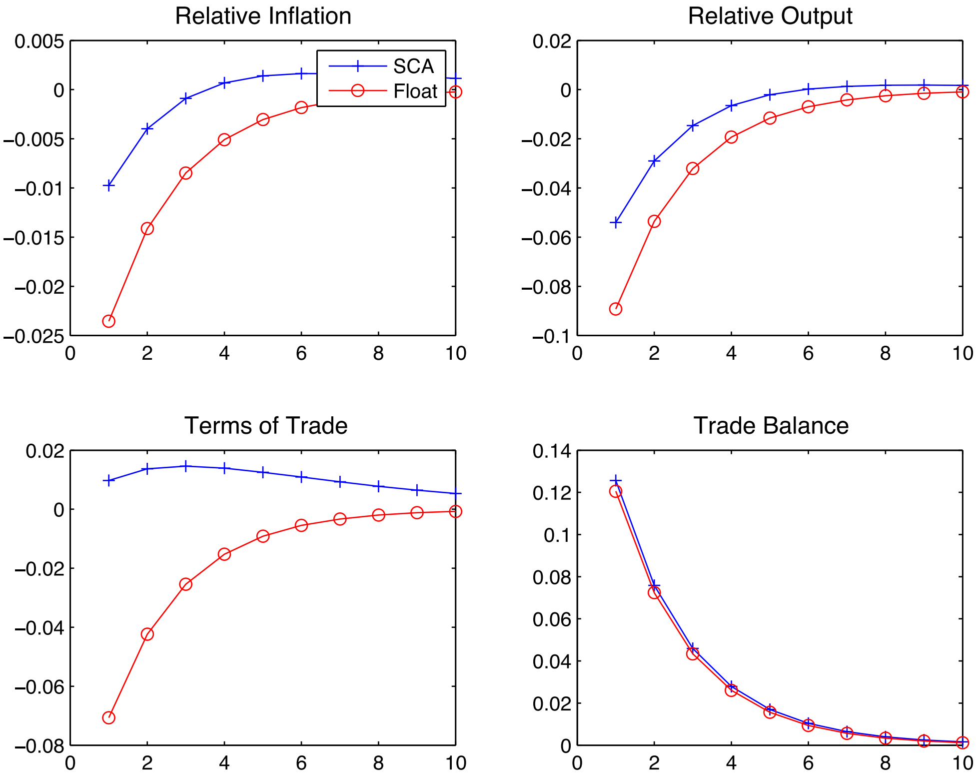

Fig. 4. Response to a saving shock at the zero bound versus the single currency area response. Parameter settings are as follows: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $v = 1.5$, $\mu = 0.6$, and $\varepsilon = -0.5$.

5. Extension to optimal monetary policy

In the previous section we employed an arbitrary interest rate rule, and assumed that the zero lower bound constraint was binding in both countries or not at all. We now extend the argument to allow for the possibility that one country may not be constrained by the zero bound, and instead sets monetary policy optimally, assuming that central banks cooperate to maximize welfare. It might be thought that in this case, where one country freely adjusts interest rates, the conventional dominance of flexible exchange rates will prevail. We show that this is not the case. We emphasize two remarkable implications of the comparison under optimal monetary policy:

- The single currency area area may outperform the multiple-currency, flexible exchange rate system in welfare terms even when the zero bound is only partially binding.
- The fall of world output and inflation in the face of a demand shock may be less severe in a single currency area than in

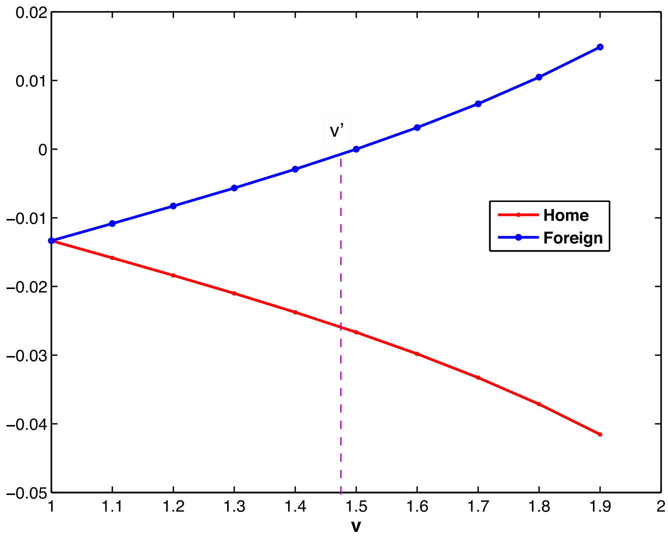

Fig. 5. Natural interest rates as a function of v. Parameter settings are as follows $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $\mu = 0.6$, $\gamma = 3$, and $\varepsilon = -0.5$.

a multiple-currency, flexible exchange rate system under a partially binding zero interest rate constraint.

The latter result arises from the fact that unconstrained countries will generally raise their interest rate to avoid perverse exchange rate movements. Thus, under optimal monetary policy, the world interest rate will generally be higher in the multiple currency case than in the single currency area, when the zero lower bound constraint does not constrain every country.

For now, we continue to assume that there can be no policy commitment; we characterize optimal policy under discretion.

Again, assume that the savings shock comes exclusively from the home country, so that $\varepsilon_t < 0$ and $\varepsilon_t^* = 0$, which obviously implies that $\varepsilon_t^W < 0$ and $\varepsilon_t^R < 0$. In addition, assume that the shock from the home country is large enough so that the world average natural interest rate is negative; i.e. $\bar{r}_t^W = \rho + \frac{(1-\mu)\phi}{\sigma+\phi}\varepsilon_t^W < 0$. From Eq. (12), then, the home natural interest rate, $\bar{r}_t = \bar{r}_t^W + \bar{r}_t^R$, must be negative, but the foreign natural interest rate $\bar{r}_t^* = \bar{r}_t^W - \bar{r}_t^R$, will depend on the degree of home bias v. For $v = 1$, the natural interest rates are identical, while for $v = 2$, the foreign natural interest rate is equal to ρ, and unaffected by the home country shock. Fig. 5 shows that for a negative home country shock $\varepsilon_t < 0$, there is a critical value of v, denoted v', such that for $v < v'$, $(v \geq v')$, the foreign natural interest rate is negative (positive).

First, we discuss the optimal policy with multiple currencies. A second order approximation to global welfare, in each period, for the model set out in Section 2 may be written as[13]

$$V_t = -(\hat{y}_t^R)^2 \cdot \frac{\sigma_D + \phi}{2} - (\hat{y}_t^W)^2 \cdot \frac{\sigma + \phi}{2} - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2. \tag{31}$$

Thus, the social welfare function faced by the policy maker depends upon average and relative world output gaps and inflation rates.

Under multiple currencies, the cooperative optimal monetary policy under discretion involves maximizing Eq. (31) subject to Eqs. (10)–(11) and (13)–(14), taking as given the expected future values of all variables, as well as the non-negativity constraints on nominal interest rates. Online Appendix A derives the full solution. Here we give an intuitive description of the solution for home and foreign interest rates, following Cook and Devereux (2013). Define Ψ and Ψ^* respectively as the multiplier on the non-negativity constraint for the home and foreign interest rate. Thus, the optimal solution must have the property that $\Psi \geq 0$, $r_t \geq 0$, and $\Psi \cdot r_t = 0$, and similarly for the foreign multiplier and foreign interest rate. Then, in the Appendix, we show that the optimal cooperative policy under discretion and multiple currencies requires that the following two conditions be satisfied:

$$\Omega_D(r_t^R - \bar{r}_t^R) = \Psi_t - \Psi_t^* \tag{32}$$

$$\Omega(r_t^W - \bar{r}_t^W) = \Psi_t + \Psi_t^* \tag{33}$$

where Ω_D and Ω are composite terms which have the following property; $\Omega_D < \Omega$ for $1 < v \leq 2$, $\Omega_D = \Omega$ for $v = 2$[14]. From Eqs. (32) and (33) we can show that, under the assumptions $\varepsilon_t < 0$, $\varepsilon_t^* = 0$ and $\bar{r}_t^W < 0$, the solution has the following properties; a) the home country is always constrained by the zero bound, so that $r_t = 0$, b) there exists a critical value $\hat{v} < v'$, such that for $1 \leq v \leq \hat{v}$, $r_t^* = 0$, while for $\hat{v} < v < 1$, the foreign interest rate solution is:

$$r_t^* = \bar{r}_t^* + \frac{\Omega_D - \Omega}{\Omega_D + \Omega}\bar{r}_t. \tag{34}$$

Thus, for v sufficiently greater than unity, the foreign country will choose to set its interest rate above zero, even if the world natural interest rate is negative. Moreover, because the second expression on the right hand side of Eq. (34) is positive, the foreign country may set a positive interest rate, even if its own natural interest rate is below zero.

The intuition behind this result is that in a cooperative optimal monetary policy outcome, under multiple currencies and the zero bound constraint applying to the home country, it may be optimal for the foreign country to raise policy rates in order to limit the appreciation of the home exchange rate. This reduces the extent of deflation and fall in output in the home country. At the same time, for v sufficiently high, the home terms of trade appreciation may generate inflation and a positive foreign output gap, so an increase in the foreign policy rate may also be warranted on that account.

This solution may be expressed in a different way. Note that the average world interest rate is $r_t^W = \frac{r_t + r_t^*}{2}$, so we may characterize the behavior of the average world interest rate under multiple currencies and flexible exchange rates as:

$$r_t^{W,mc} = max(0, \bar{r}_t^W - \frac{\Omega}{\Omega_D + \Omega}\bar{r}_t). \tag{35}$$

Under the single currency area, the cooperative optimal monetary policy chooses only a single world interest rate subject to Eqs. (10) and (11), and the non-negativity constraint. This policy differs from the multiple currency area case only in that it ignores the path of relative world output and relative world inflation, focusing only on the optimal path of world averages. It is immediate that the solution will be:

$$r_t^{W,sca} = max(0, \bar{r}_t^W). \tag{36}$$

Comparing Eqs. (35) and (36) we see that under optimal monetary policy, average policy interest rates will be higher in a multiple currency area than in a single currency area. This is because under the multiple currency area it may be optimal to raise foreign policy rates, even though the world natural rate is negative, in order to reduce the appreciation of the home country terms of trade.

Fig. 6 illustrates the response of average and relative world output, inflation, and the terms of trade under optimal monetary policy, in the multiple currency case, and the single currency area.[15] We choose the same parameter and shocks as before, assuming that the

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[13] See Cook and Devereux (2013). We make the simplifying assumption of a common welfare function under the single currency area and the multiple currency area under the zero lower bound. This is justified on the basis that we are approximating around a first-best undistorted steady state. In general, the welfare functions under the two regimes may differ, and approximation would require the use of use of the linear quadratic approach developed by Benigno and Woodford (2003). In addition, it is not in general the case that the loss functions under discretion and commitment will coincide.

[14] The exact expressions are

$$\Omega = \frac{\sigma + \phi}{\sigma}\frac{(1-\beta\mu) + k\theta(\sigma+\phi)}{\sigma(1-\beta\mu)(1-\mu) - k\mu(\sigma+\mu)},$$

$$\Omega_D = \frac{\sigma_D + \phi}{\sigma_D}\frac{(1-\beta\mu) + k\theta(\sigma_D+\phi)}{\sigma_D(1-\beta\mu)(1-\mu) - k\mu(\sigma_D+\mu)}.$$

[15] As before, Fig. 6 illustrates the impact effect, and the *expected* response of each variable following the shock.

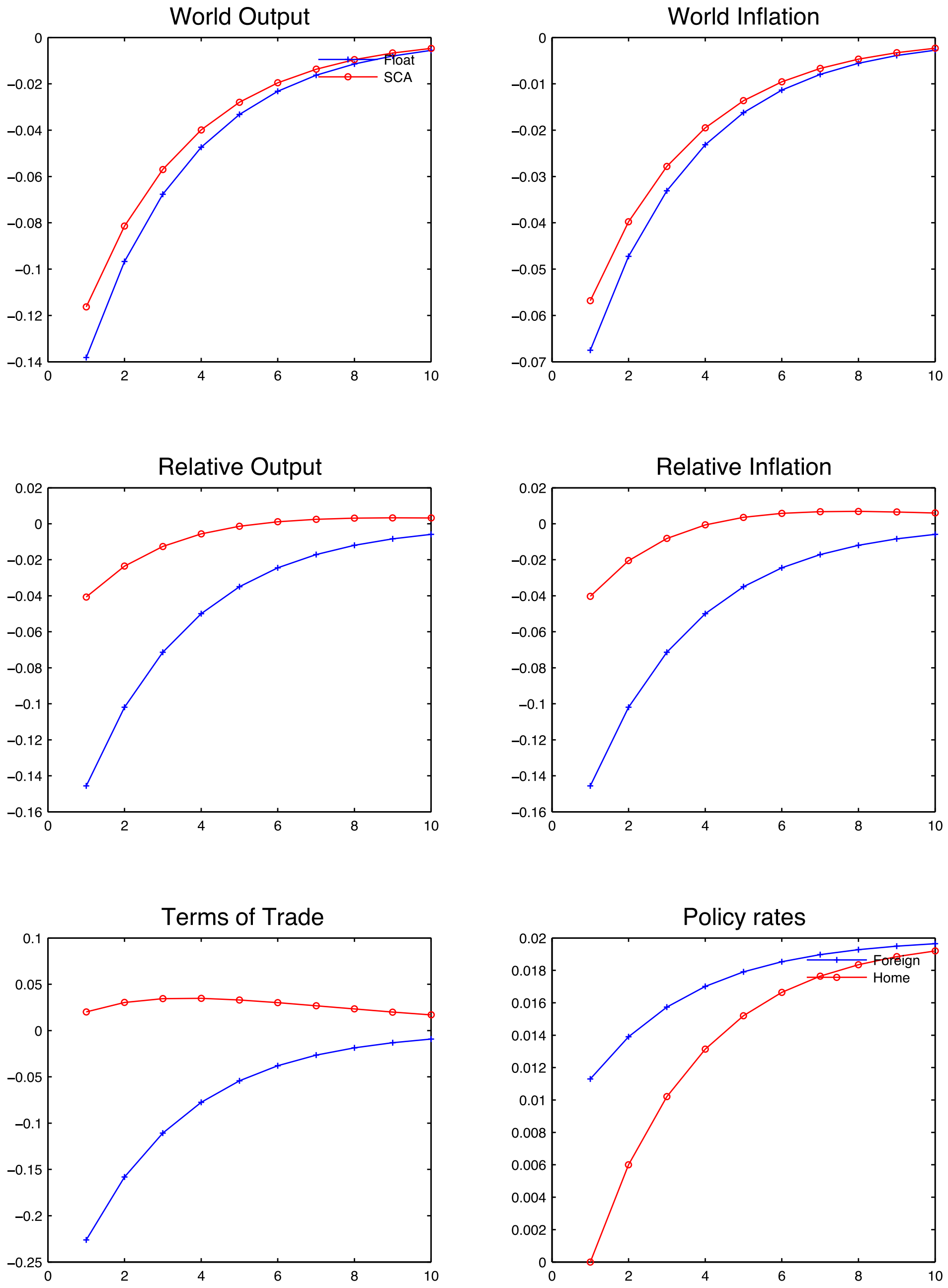

Fig. 6. Optimal policy when the foreign country adjusts interest rates, with home country at the zero lower bound. Parameter settings are as follows: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $v = 1.5$, $\mu = 0.6$, and $\varepsilon = -0.5$.

world natural interest rate falls below zero. The home bias parameter v is set so that $v > \hat{v}$, and the optimal response of the foreign country is to set a positive policy interest rate. Then, since average world interest rates are higher under multiple currencies than in the single currency area (zero), average world output and inflation falls by more in the response to the savings shock. Thus, due to the perverse response of the exchange rate under a multiple currency regime, at the zero lower bound, *overall world output* will fall by more with multiple currencies and flexible exchange rates than under a single currency area, *even when only the home country is constrained by the zero lower bound*. Fig. 6 likewise shows that relative output and inflation also falls by more in the multiple currency case than in the single currency area. Finally, even under the optimal policy rule, the terms of trade still appreciates under the multiple currency case, while as before, the terms of trade in the single currency area depreciates.[16]

These results indicate that, even when policy is set optimally and only one country in a multiple currency area is constrained by the zero lower bound, there is a perverse response of relative prices and output gaps under flexible exchange rates. How does this translate

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[16] This is not necessarily the case. In Section 6 below, we show that in an optimal discretionary equilibrium with relatively transitory shocks, the home terms of trade under multiple currencies may depreciate, but does so at a rate less than would take place in the single currency area.

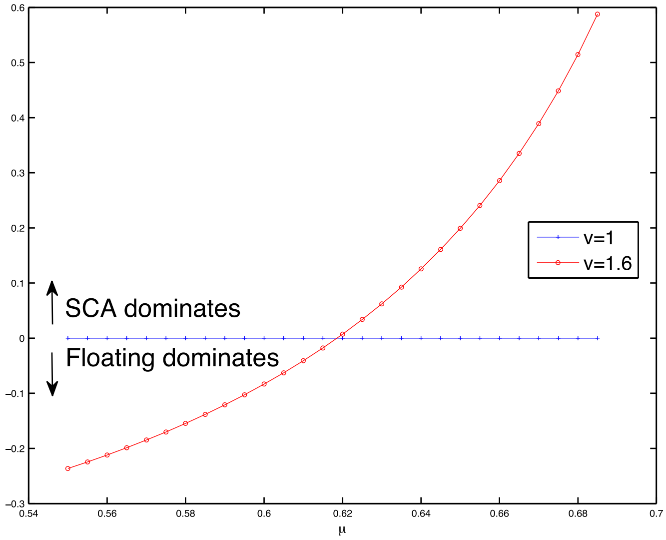

Fig. 7. Welfare comparison with optimal policy when foreign country adjusts interest rates with home country at the zero lower bound. Parameter settings are as follows: $\beta = 0.99, k = 0.05, \sigma = 2, \phi = 1, v = 1.5, \mu = 0.6$, and $\varepsilon = -0.5$.

into welfare terms? We can construct a welfare comparison by computing the loss associated with a savings shock using the welfare function in Eq. (31). To do this, we take the expected loss following a shock which follows the persistence properties described above. In particular we assume again that the shock such that the world natural interest rate is negative, and the shock persists with probability μ in each period in the future. Welfare in each regime is constructed by computing the discounted expected value of losses starting from the period of the shock. In the multiple currency case, expected welfare is constant in each future period, since the shock is either the same, with probability μ, or zero, with probability $1-\mu$, and if it is zero, all future gaps are closed. In the single currency area, welfare evolves over time, as the terms of trade gradually adjusts to the shock, and welfare doesn't go to zero after the shock expires, since the terms of trade continues to be away from its steady state level, and only gradually converges back to steady state.

Fig. 7 illustrates the welfare comparison for different values of the persistence parameter μ, and for two different values of v. The figure illustrates the value of

$$\frac{|V(mc)|}{|V(sca)|} - 1$$

where $V(mc)$, $(V\,L(sca))$, is the welfare function under multiple currencies (single currency area). For $v = 1$, there is no difference between the two measures. For $v > 1$, the loss from the multiple currency area increases, relative to that under the single currency area, as μ rises. The welfare comparison between the multiple currency case and the single currency area involves a trade-off. On the one hand, under multiple currencies, the savings shock leads the home terms of trade to appreciate, and relative output is destabilized during the period of the shock. But under a single currency area, the shock causes a persistent movement in the terms of trade, relative output, and inflation rates, even after the shock ends. For low values of μ, the welfare effect of the second factor dominates, and the multiple currency area, under the optimal choice of monetary policy, is preferable. But for high μ, the first factor becomes more important, and the multiple currency area is worse in welfare terms, *even if monetary policy is chosen optimally*. This is also clear from Fig. 6 which shows the expected values of the home and foreign interest rate[17] , where the calibration for this figure involves $v = 1.5$ and $\mu = 0.6$. Hence, for this case, the foreign country is actively adjusting its interest rate. Despite this, welfare is higher in the single currency area.

6. The need for forward guidance

A key aspect of the comparison so far has been that monetary policy is myopic; we have allowed no possibility for policy-makers to commit to future actions. In general, the literature on the zero lower bound in closed economy settings has stressed the benefits of *forward guidance*. As shown by Eggertsson and Woodford (2003), and Jung et al. (2005), with full commitment, an optimal monetary policy can significantly alleviate the consequences of the zero lower bound constraint. This is done by promising to follow, in the future, after the conditions leading to the zero bound have elapsed, a more expansionary policy than would otherwise be appropriate for the economy's conditions at that time. In Eggertsson and Woodford (2003) and Jung et al. (2005), this involves keeping a zero interest rate policy for a period of time *after* the shock which drives the natural interest rate below zero has disappeared.

We now extend the analysis to allow for commitment in monetary policy.[18] Again, we focus on cooperative optimal monetary policy, but assume that policymakers can choose a path of interest rates for each country (in the multiple currency case) or a world interest rate (in the single currency case) that will hold for current and future periods, subject to the zero bound constraint. To simplify the analysis, we focus on a special case of perfect foresight, where the preference shock is known to last a fixed number of periods. Specifically, assume that at time $t = 1$, there is a shock to home preferences $\varepsilon < 0$, which drives world natural interest rates below zero, and further, it is known at time 1 that the shock will last exactly T periods. Thus, the shock leads to a fall in the world natural interest rate below zero for T periods. Then, in both the single currency case and the case of multiple currencies, an optimal monetary policy at time $t = 1$ is characterized by a path of interest rates for all $t \geq 1$.

We focus on a case where $v \geq \hat{v}$, which implies that in the equilibrium of the optimal discretionary monetary policy, the foreign country would set a positive interest rate. In this case, we find that it is also optimal for the foreign country to pursue a positive interest rate in the optimal policy problem with commitment. So again, we are focusing on a case where only the home country is currently constrained by the zero bound.

In the multiple currency case, an optimal policy with commitment involves a path of interest rates r_t^W and r_t^R for $t \geq 1$ to minimize the discounted sum of losses given by:

$$V_0 = -\sum_{t=0}^{\infty} \beta^t \left[(\hat{y}_t^R)^2 \cdot \frac{\sigma_D + \phi}{2} + (\hat{y}_t^W)^2 \cdot \frac{\sigma + \phi}{2} \right] - \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta}{4k} (\pi_t^W + \pi_t^R)^2 + \frac{\theta}{4k} (\pi_t^W - \pi_t^R)^2 \right] \tag{37}$$

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[17] Note that the *actual* value of the home policy rate is zero so long as the savings shock continues.

[18] In this discussion we assume that commitment is synonymous with forward guidance in monetary policy. In general, the two policy stances may differ. See Woodford (2013) for a discussion of forward guidance as monetary policy. In addition, we take the loss functions for policymakers under commitment and discretion to be identical. This depends on the assumption that the initial steady state is efficient. In addition, we follow the approach of Eggertsson and Woodford (2003) and Jung et al. (2005), by assuming that the loss function at the zero bound is equivalent to the loss function approximated for monetary policy during 'normal' times.

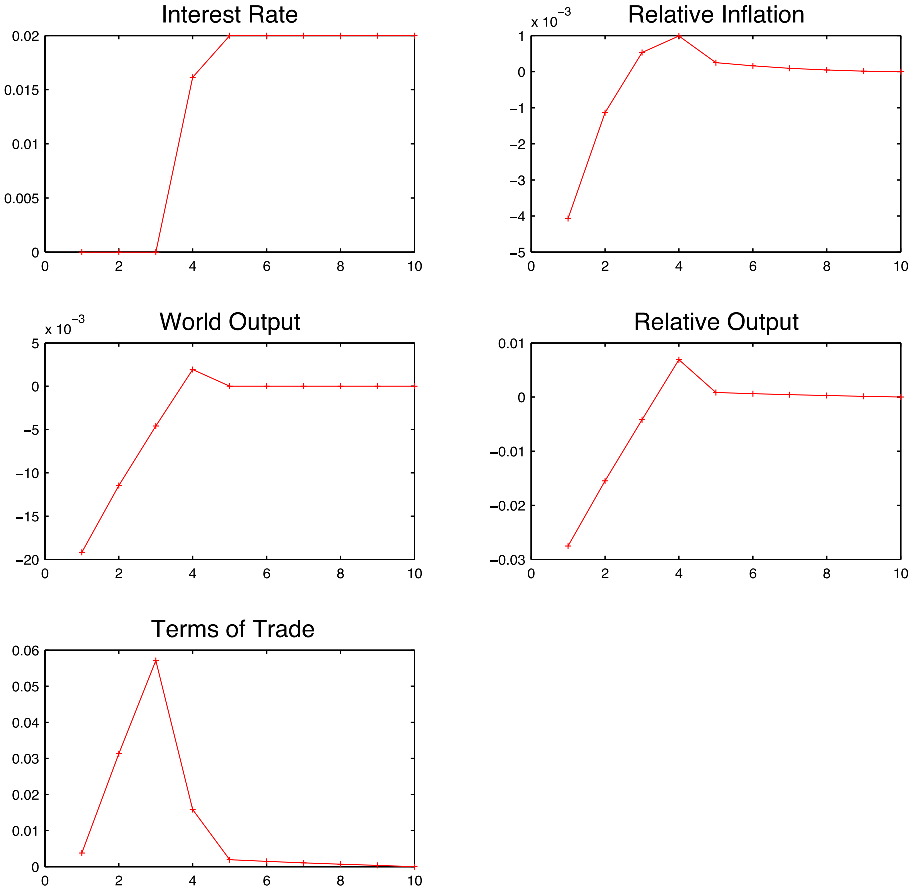

Fig. 8. Optimal policy under discretion in a single currency area with a 3 period liquidity trap. Parameter settings are as follows: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, and $v = 1.5$, and $\varepsilon = -0.5$.

subject to Eqs. (10)–(11) and (13)–(14) for each period t, as well as the non-negativity constraints on national nominal interest rates. In the single currency area case, the optimal policy involves a choice of the path of r_t^W for $t \geq 1$ to minimize Eq. (37) subject to Eqs. (10)–(11) and the non-negativity constraint on r_t^W.

The first order conditions for the optimal policy with commitment are quite familiar from previous literature. The conditions are described fully in the online Appendix. Here we illustrate the results and comparisons in Fig. 8–11. The figures are based on the assumption that at time $t = 1$, there is an unanticipated savings shock in the home country equal to $\varepsilon = -0.5$, which is known to persist for three periods (so that $T = 3$). After that $\varepsilon = 0$. The figures use the calibration used in Section 5, and assume that $v = 1.5$.

For comparison, Figs. 8 and 9 illustrate the case of discretionary monetary policy, under multiple currencies and the single currency area. The qualitative features are the same as in Section 5. Under multiple currencies, there is no persistence beyond $t = 3$. In response to the savings shock, the home country policy rate is stuck at the zero bound for 3 periods only, while as shown in the previous section for relatively high values of v, the foreign policy rate is positive. Relative inflation is negative for three periods. In the single currency area, the world interest rate is at the zero bound for three periods, but does not converge immediately to the steady state natural rate. This is because the home terms of trade ends the third period above its steady state, and the home country must experience some relative inflation in order to converge back to steady state. This is achieved by having a world interest rate lower than the steady state, which, in conjunction with a home country terms of trade above its steady state, facilitates more inflation in the home country. This mechanism illustrates the in-built commitment dynamic of the single currency area — producing relative home country inflation after the expiry of the shock, even in a discretionary equilibrium.

Figs. 10 and 11 now focus on the commitment equilibrium under the multiple currency case and the single currency area. Under commitment, the policymaker can choose the whole future path of interest rates so as to produce the desired outcome, announcing interest rates to hold even after the shock elapses. We see that under the multiple currency area, there is a dramatic difference from the discretionary outcome. In particular, the home country keeps its policy rate at zero for an additional two periods, even after the shock expires. Moreover, the foreign country's interest rate, while still rising immediately following the shock as in the case of discretion, converges to steady state only gradually. This conjunction of policy announcements sharply reduces the deflation experienced in the home country, and as a result, the home terms of trade experiences an immediate depreciation, as would occur under the outcome outside of the zero bound. The fall in world average output and world relative output is now much smaller than under discretion. More

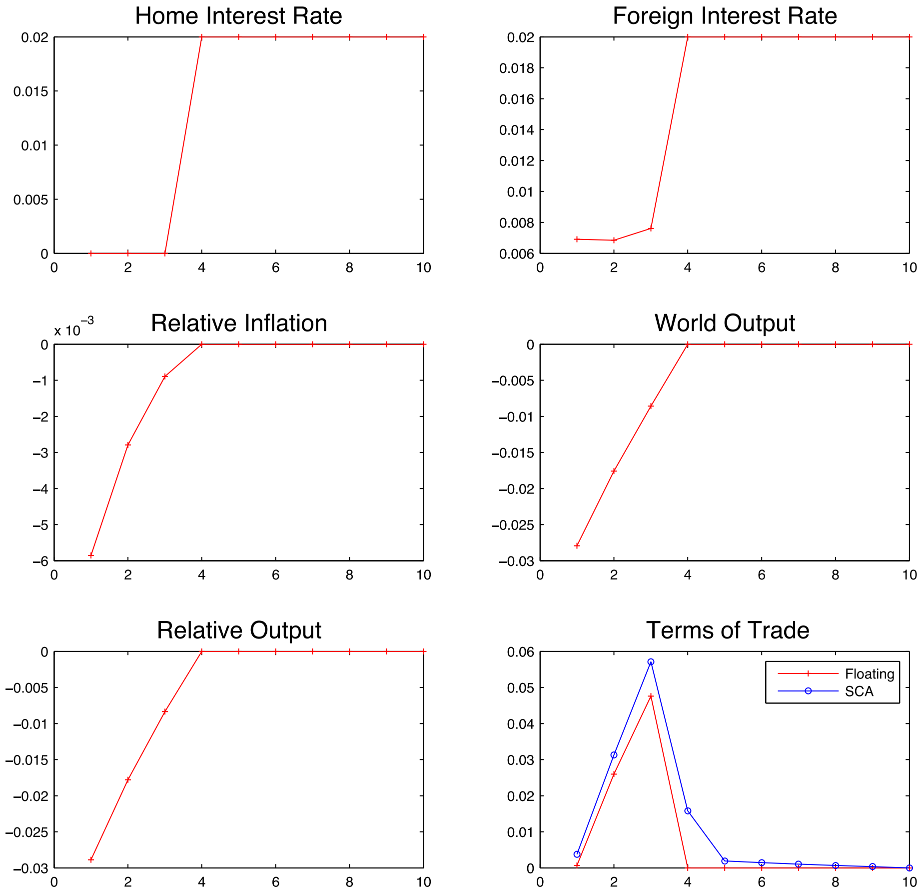

Fig. 9. Optimal policy under discretion with multiple currencies with a 3 period liquidity trap. Parameter settings are as follows: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $v = 1.5$, and $\varepsilon = -0.5$.

importantly, the outcomes under multiple currencies with full commitment are better than those under the single currency area with commitment, which is illustrated in Fig. 11. This figure shows that with full commitment, the single currency area policy rate remains at zero for one period after the shock expires. The home terms of trade depreciates, but by less than that under multiple currencies with full commitment. The movement in the average world interest rate means that average world output falls by about the same amount as under multiple currency areas, but the inability to fully adjust the home terms of trade leads to a greater fall in relative world output. Thus, with full commitment, the macro outcomes under the multiple currency area, even constrained by the zero lower bound, seem to dominate those of the single currency area.

The substance of these results indicate that in exploring the benefits of multiple currencies and flexible exchange rates in an environment constrained by the zero bound, it is critical to have well functioning forward guidance as part of the policy toolkit. Table1 1 makes this clear in terms of welfare evaluation.

The table reports the discounted sum of losses under discretion and commitment, using an optimal policy in each case, for the multiple currency area and the single currency case, for the shock process and the calibration example described in the previous paragraphs. For the discretionary case, the single currency area still dominates, as implied by the previous section. But with full policy commitment, the traditional result applies — when policy can effectively employ forward guidance, the welfare benefits of multiple currencies and flexible exchange rates re-emerge.

7. Discussion

We view this paper essentially as a theoretical contribution, cautioning against applying some standard open economy policy prescriptions that rely on the presence of activist monetary policy to an environment constrained by the zero bound. Nevertheless, because the essential mechanism driving the perverse response of the exchange rate at the zero bound is very straightforward, we might ask whether there is any evidence for this mechanism. In this section, we provide a very brief discussion of the experience of Japan in the late 1990's and Switzerland following the financial crisis of 2008. In both cases, these countries moved to ultra-low interest rate policies relative to their major trading partners, and in both cases there is some unconditional evidence that their exchange rates appreciated.

7.1. Japan

Japan introduced a zero interest rate policy in 1999. This is illustrated in the panels of Fig. 12. From 1996, Japan's GDP deflator began falling, stabilizing at a level about negative one percent annually or below. By comparison, US inflation ranged between 1% to 3% over this period. After a long cycle of policy easing, the Bank of Japan's policy instrument (the uncollateralized overnight call money rate) reached the zero lower bound during the first quarter of 1999. This can be compared with a Fed Funds rate which was near 5 or 6%. Falling policy rates were associated with weakening in the yen to a level near 140 per US dollar. Once the policy rate hit zero however,

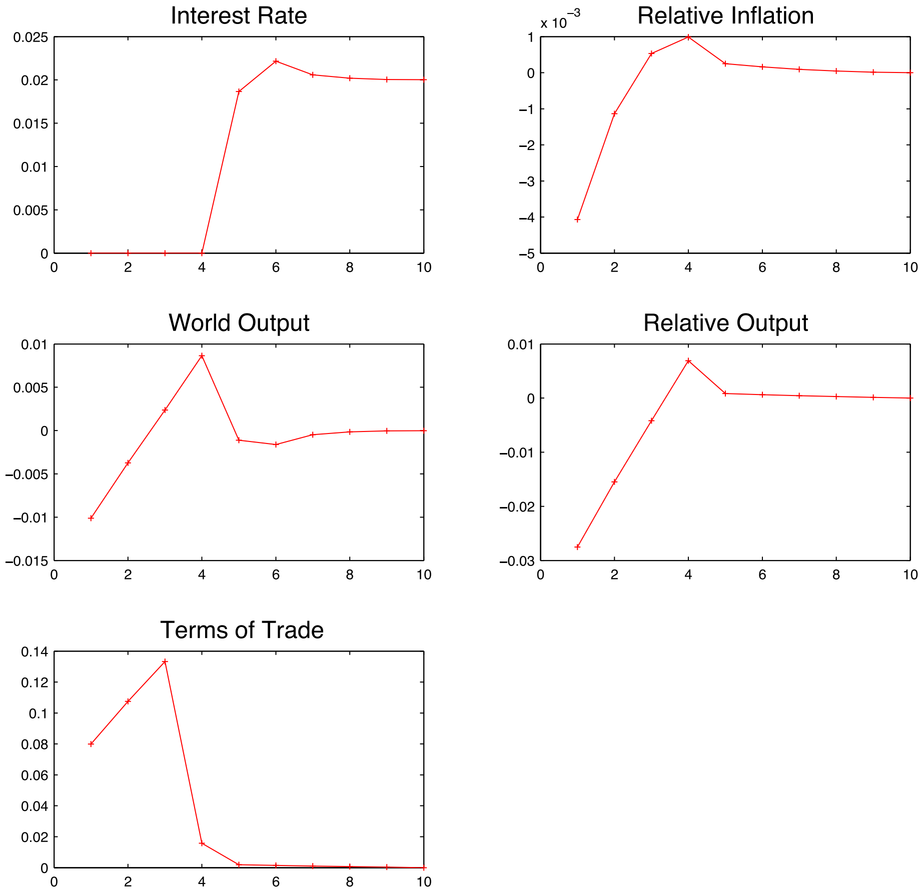

Fig. 10. Optimal policy with commitment in a single currency area with a 3 period liquidity trap. Parameter settings are as follows: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $v = 1.5$, and $\varepsilon = -0.5$.

the continuing Japanese deflation was associated with a rising real interest rate. The Yen then experienced an appreciation to a level near 104.

7.2. Switzerland

The case of Switzerland is illustrated in Fig. 13. In 2008, both the Swiss National Bank and the ECB sharply reduced policy rates. The Swiss rate moved to the zero lower bound by the end of 2008, staying there subsequently. This was more drastic than the drop in the Euro rate which stayed above zero until 2012. Despite the more aggressive monetary easing by the Swiss central bank, the Swiss Franc strengthened continuously against the Euro through 2011, when policymakers committed to an exchange rate peg against the Euro to supplement the zero lower bound. Panel (C) of Fig. 13 shows the year on year growth rate in the GDP deflator in the Eurozone and Switzerland. In the immediate aftermath of the crisis, Switzerland experienced deflation. After 2010, the GDP deflator was basically flat. Though inflation in the Eurozone was below the 2% target, it was generally higher than in Switzerland. The combination of low inflation in Switzerland after the crisis and the constraint on reducing the nominal interest rate imposed by the zero lower bound led to a relatively high real interest rate. Panel (D) shows the difference between the policy rate and the 4 quarter ahead year on year inflation rate. By this measure, the real interest rate in Switzerland exceeded that in the Eurozone. It can be argued that flight-to-quality style capital flows exacerbated the appreciation of the Swiss franc. However, it seems from Fig. 13 that the high real interest rates induced by the liquidity trap may have reinforced disinflation and exchange rate appreciation.

Finally, again we caution that these examples are taken simply as illustrative. They suggest that instances where interest rates hit the zero bound may not be associated with currency depreciation due to the fact that the zero bound can lead to relatively high real interest rates. It is important to emphasize that this is very far from being a test or a validation of the model, since the model relies on an exogenous savings shock which precipitates the zero bound episode, and obviously we cannot identify this shock in the data without far more extensive empirical work than we do here.

8. Conclusions

A growing recent literature has demonstrated that conventional responses to macroeconomic shocks can be substantially different when monetary policy is constrained by the zero bound on nominal interest rates (see e.g. Eggertsson 2011, Cook and Devereux, 2013). This present paper extends that literature by showing that the conventional reasoning on the benefits of flexible exchange rates and the costs of a single currency area can be reversed in a situation of a liquidity trap. When monetary policy is ineffective, the conventional response of the exchange rate to aggregate demand shocks may be reversed, and the exchange rate exacerbates rather than ameliorates economic instability.

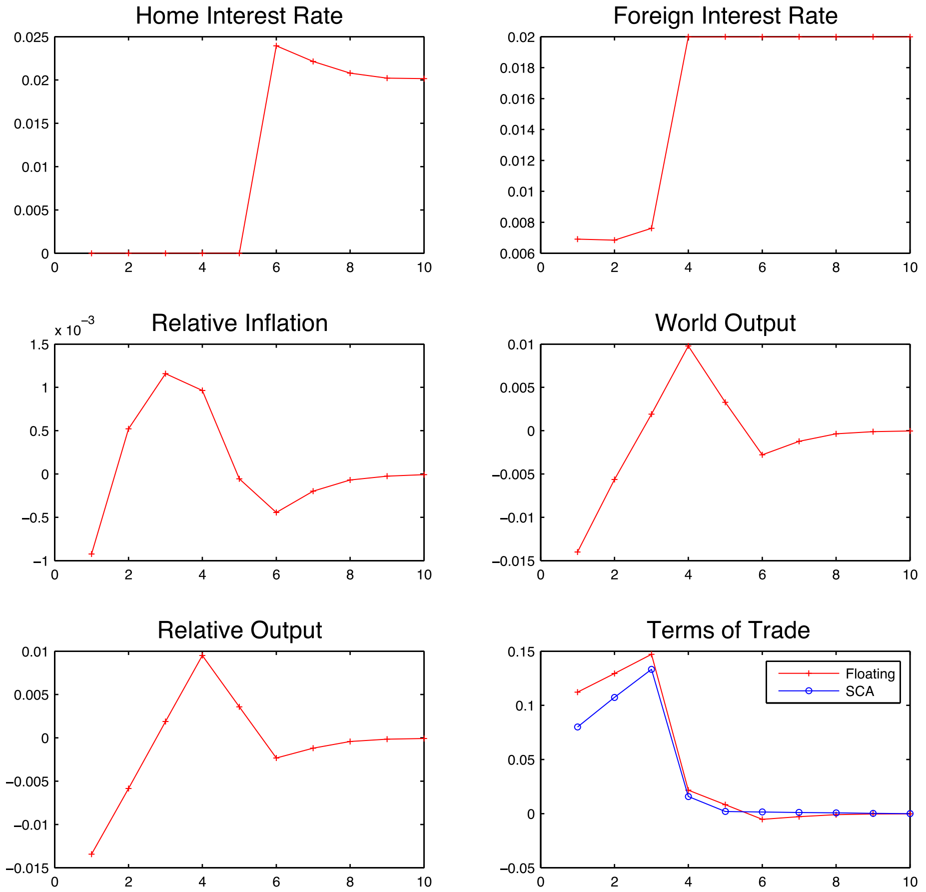

Fig. 11. Optimal policy with commitment under multiple currencies with a 3 period liquidity trap. Parameter settings are as follows: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $v = 1.5$, and $\varepsilon = -0.5$.

Our paper focuses on the role of the exchange rate as an equilibrating mechanism in a situation with country-specific shocks. Some recent literature argues that in a single currency area, there will exist tax-subsidy policies that can substitute for the absent exchange rate variation. Fahri et al. (2013) describe how a mix of tax and subsidies can achieve *Fiscal Devaluation* in a small economy, exactly replicating the effects of a nominal exchange rate devaluation. Therefore, if fiscal policy is sufficiently flexible, it can completely eliminate the loss of monetary autonomy implied by a fixed exchange rate regime. More generally, it has been established by Correa et. al. (2013) that a combination of state-contingent taxes and subsidies can undo the effects of the zero bound and fully replicate the flexible price equilibrium in standard New Keynesian models. In online Appendix B, we see how these results extend to our setting. We show that a combination of VAT adjustment and payroll tax changes can be used to ensure price stability and zero output gaps, achieving the fully optimal flexible price equilibrium in face of shocks which would drive the world interest rate in a single currency area, or the home interest rate in a multiple currency area, to the zero bound. But the key finding is that when monetary policy is constrained by the zero bound, fiscal adjustment will be required even in a situation of flexible exchange rates. We find that a fall in average and relative VAT taxes combined with a rise in average and relative payroll taxes achieves the first best outcome, leaving all gaps and inflation rates equal to zero. But the main result we show is that the tax-subsidy policy is the same for the multiple currency case and the single currency area. Hence, the key need for the optimal fiscal response is not the inability of exchange rates to adjust, but the zero lower bound constraint itself.

It is important to emphasize that the paper does not argue unconditionally for the benefits of a single currency area. For instance, we have entirely ignored a whole set of factors that have been identified as important in the Eurozone crisis, such as sovereign debt constraints, moral hazard elements of a single currency/single market, potential for bubbles in financial markets, and financial and banking fragility. In addition, we have shown that effective forward guidance in monetary policy can restore the traditional advantages of multiple currencies and flexible exchange rates. Hence, the key message of the paper is that, when monetary policy is constrained by the zero lower bound, the support for traditional policy conclusions is acutely dependent on the ability for policy-makers to make credible future commitments.

Table 1
Welfare comparison

	SCA	Float
Discretion	−0.007	−0.017
Commitment	−0.0039	−0.002

Notes: Compares present value of welfare under optimal policy under discretionary and commitment policies under each regime

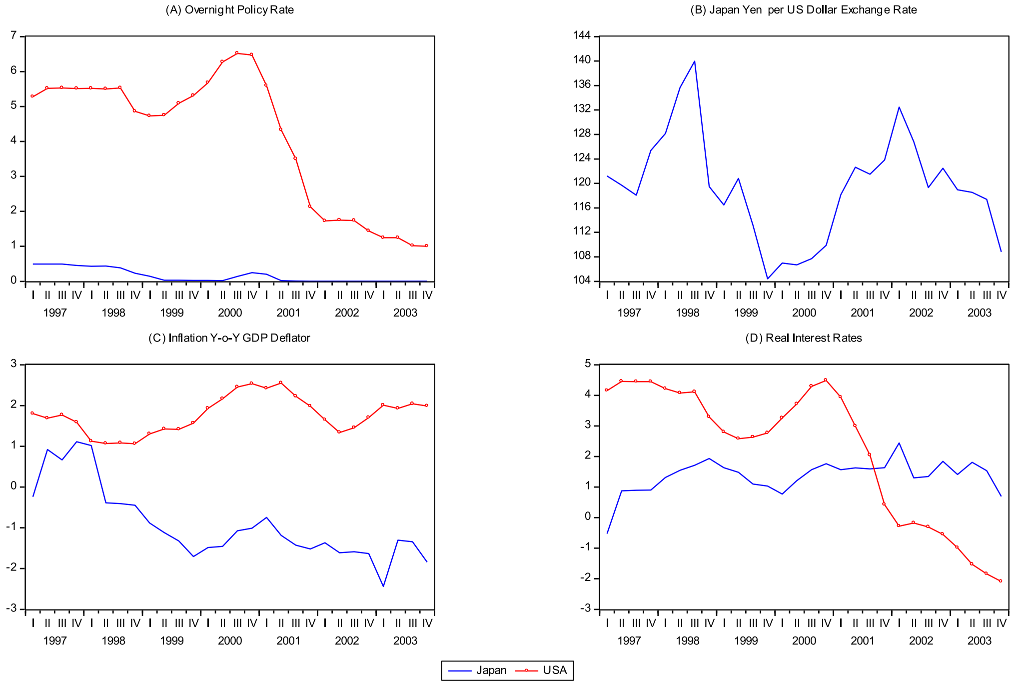

Fig. 12. Japan at the zero bound. Source: OECD Main Economic Indicators and Japanese and US Quarterly National Accounts.

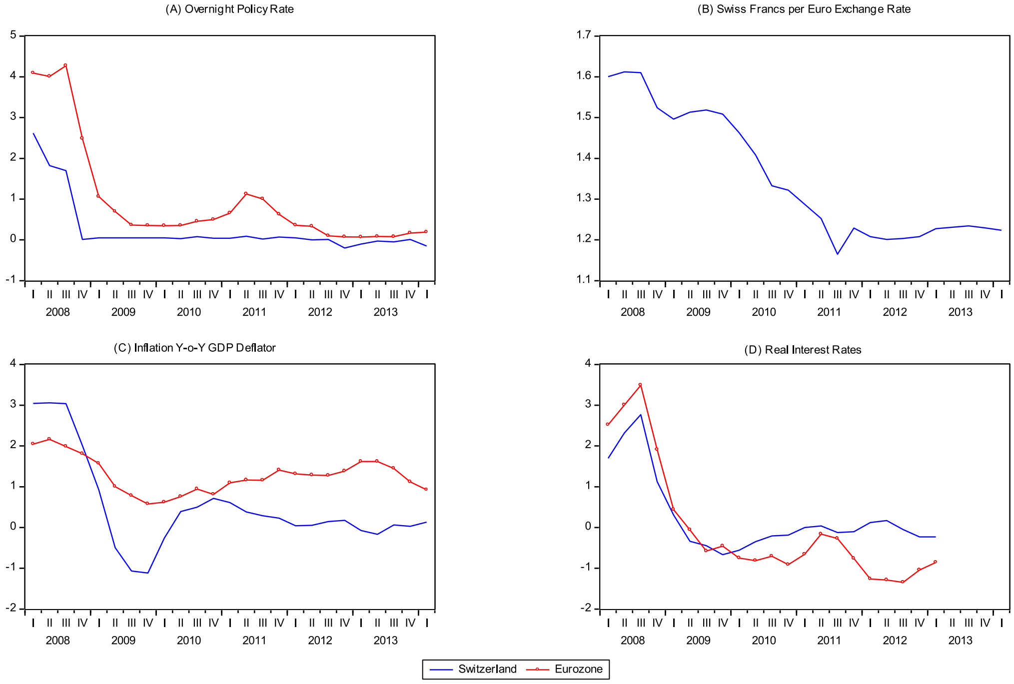

Fig. 13. Switzerland at the zero bound. Source: OECD Main Economic Indicators and Swiss and Euro Area Quarterly National Accounts.

Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx.10.1016/j.jinteco.2016.03.011.

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