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A small-scale DSGE-VAR model for the Romanian economy

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A R T I C L E I N F O

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This paper describes the theoretical structure and the estimation results for a DSGE-VAR model for the Romanian economy, an inflation targeting country since 2005. Having as benchmark the New-Keynesian model of Rabanal and Rubio-Ramirez (2005), the main additional feature introduced refers to the extension to a small open economy setting in order to account for this specific aspect of the Romanian economy.

Within the inflation targeting monetary policy regime, forecasts of central macro variables, inflation in particular, play an important part. Because inflation reacts to monetary measures with a considerable lag, the central bank's policy has to be forward-looking. Based on univariate measures of forecast performance, it is shown that the VAR with DSGE model prior produces forecasts that improve on those obtained using an unrestricted VAR model and the popular Minnesota prior in case of inflation, real exchange rate and nominal interest rate. Moreover, the DSGE-VAR model is informative about the structure of the economy and can help the "story-telling" in the central banks.

1. Introduction

In the last decades, inflation targeting has been adopted by an increasing number of central banks as their monetary policy framework. Due to the delays in the monetary policy transmission mechanism, central banks with quantitative inflation targets, Romania included, must have adequate tools to form views on future macroeconomic performance, especially on inflation prospects.

The vector autoregression models (VARs), introduced by Sims (1980), have long proven to be an effective method for modelling the dynamics of macroeconomic variables as well as forecasting. The VAR is an econometric model used to capture the linear interdependencies among multiple time series, the only prior knowledge required being a list of variables which can be hypothesized to affect each other intertemporally. In theory, the idea is to let the data guide the views regarding the true data generating process. In practice, however, the parameters in the VAR models are often not very precisely estimated using classical econometrics procedures due to the dimensionality problem: high number of parameters to be estimated using a limited number of observations. Therefore, alternative methods for estimating the coefficients in a VAR model have been developed, the most successful being the Bayesian approach, originally advocated by Litterman (1979). The Bayesian estimation method provides a logical and formally consistent way of introducing shrinkage by treating the parameters of the model as random variables with probability distributions which are used to summarize the status of the knowledge about each parameter (prior information). By combining the prior information with the information contained in the data (the likelihood function), an updated distribution for the parameters is obtained, known as the posterior distribution, which is used to carry inference about the value of the parameters. Thus, to the extent that the prior is based on non-sample information, the Bayesian approach offers a good framework for containing different sources of information when performing macroeconomic analysis. Karlsson (2013) provides a coherent survey on Bayesian approaches to inference in VAR models. Del Negro and Schorfheide (2011) and Koop and Korobilis (2009) present complementary reviews of Bayesian VAR models.

Even though it is proved that the Bayesian VAR model is a reliable forecasting tool (see, for example, Kinal and Ratner, 1986; Litterman, 1980 etc.), the specific functions of a central bank imply the usage of models that are based on much more economic theory than a VAR model and are thus useful as a "story-telling" device. The large scale-models that were used by central banks in the 1950s to 1970s were criticized because of the lack of microeconomic foundations, which made them subject to the Lucas critique (Lucas, 1976), as well as ad-hoc econometric restrictions. As a result, a new class of models have emerged, i.e. the dynamic stochastic general equilibrium (DSGE) models, built in recent years along the lines of New-Keynesian Economics. The DSGE models are microfounded, having a consistent behavioural structure which helps interpretation. Moreover, the structural parameters that govern the relations between the variables in a DSGE model are invariant to changes in economic policy, so, in

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principle, not subject to the Lucas critique. However, according to the empirical evidence, the DSGE models forecasts are usually dominated by univariate or multivariate time series models (see, among others, Del Negro and Schorfheide, 2013) and, therefore, many central banks are still reticent in adopting a DSGE model as the main tool for supporting the policy making.

In their seminal works, DeJong et al. (1993) and Ingram and Whiteman (1994) present an estimation methodology that unifies the two approaches mentioned above. DeJong et al. (1993) examine the impulse response functions generated by a VAR model estimated subject to the restrictions imposed by a monetary general equilibrium model, while Ingram and Whiteman (1994) demonstrate that prior information from a real business cycle model helps improve the forecasting performance in the case of movements in consumption, output, hours and investment for the US economy. Del Negro and Schorfheide (2004) significantly extend the earlier work: first, by showing how posterior inference for the VAR parameters can be translated into posterior inference for the DSGE model parameters, secondly by constructing a VAR identification scheme for the structural shocks based on a comparison of the contemporaneous VAR responses to shocks with the DSGE model responses and, finally, by illustrating how a VAR with DSGE model prior can be used to predict the effects of a permanent change in the policy rule. Lees et al. (2007) complement the analysis of a DSGE-VAR forecasting performance for the economy of New Zeeland along policy dimension: they use the estimated DSGE-VAR structure to identify optimal policy rules that are consistent with the Reserve Bank's Policy Targets Agreement. Other empirical applications of the DSGE-VAR methodology include Warne et al. (2013) for euro area, Bache et al. (2010) for Norway, Watanabe (2009) for Japan, Liu et al. (2007) for South Africa, etc.

This paper describes the theoretical structure and the estimation results for a DSGE-VAR model for the Romanian economy. The New-Keynesian model of Rabanal and Rubio-Ramirez (2005) is adopted, which serves as a minimal set of theory for modelling an inflation targeting economy. The model is extended to a small open economy setting in order to account for this specific feature of the Romanian economy. The forecasting performance of the DSGE-VAR model is evaluated against other VAR alternatives, i.e. an unrestricted VAR model and a VAR model with a Minnesota prior.

The rest of the paper is structured as follows. Section 2 briefly discusses the DSGE-VAR methodology. Section 3 presents the DSGE model used to construct prior beliefs about the VAR parameters, the data and the estimation results. Section 4 compares forecasts of the DSGE-VAR model to those obtained using the other VAR alternatives. Finally, Section 5 concludes.

2. The DSGE-VAR methodology

This section briefly presents the DSGE-VAR methodology and outlines the Del Negro-Schorfheide algorithm used in the estimation procedure.

As mentioned in the Introduction section, the idea of the DSGE-VAR approach is to use the DSGE model to construct prior distributions for the VAR parameters.

The starting point for the estimation is an unrestricted VAR of order l:

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_l y_{t-l} + u_t, \tag{1}$$

where $t=1,2,\ldots,T$. $y_t=(y_{1t},y_{2t},\ldots,y_{nt})$ is a $n\times 1$ vector of observable variables, A_0 is a $n\times 1$ vector of constant terms, $A_1,A_2,\ldots,A_n$ are $n\times n$ matrices of autoregressive parameters and $u_t=(u_{1t},u_{2t},\ldots,u_{nt})$ is a vector of residuals following a multivariate normal distribution, i.e. $u_t \sim N(0,\Sigma_u)$. T is the size of the sample used for estimation. The model for the whole data set can be reformulated as:

$$Y = XA + U \tag{2}$$

where $Y=\begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_{T-1}' \\ y_T' \end{bmatrix}$, $X=\begin{bmatrix} 1 & y_0' & \cdots & y_{2-l}' & y_{1-l}' \\ 1 & y_1' & \cdots & y_{3-l}' & y_{2-l}' \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & y_{T-2}' & \cdots & y_{T-l}' & y_{T-1-l}' \\ 1 & y_{T-1}' & & y_{T+1-l}' & y_{T-l}' \end{bmatrix}$, $A=\begin{bmatrix} A_0' \\ A_1' \\ \vdots \\ A_{l-1}' \\ A_l' \end{bmatrix}$

and $U=\begin{bmatrix} u_1' \\ u_2' \\ \vdots \\ u_{T-1}' \\ u_T' \end{bmatrix}$. The system described by Eq. (2) is characterized by the following likelihood function:

$$p(Y/A,\Sigma_u) = |\Sigma_u|^{-T/2} exp\left\{-\frac{1}{2}\mathrm{tr}\left[\Sigma_u^{-1}(Y'Y - A'X'Y - Y'XA - A'X'XA)\right]\right\} \tag{3}$$

The prior distribution for the VAR parameters is based on the DSGE model representation as a reduced-form VAR,[1] characterized by a likelihood function similar with the one presented in Eq. (3), $p(Y(\xi)/A,\Sigma_u)$, where ξ represents the vector of structural parameters from the DSGE model.

Loosely speaking, imposing the prior from the DSGE model implies the augmentation of the dataset by a number of $T^*=\lambda T$ "artificial" observations, (Y^*,X^*), generated using the DSGE model, where λ is a hyper-parameter representing the ratio of "artificial" data relative to the size of the actual sample of data. The likelihood function for the combined sample of "artificial" and actual observations is obtained by pre-multiplying $p(Y/A,\Sigma_u)$ with $p(Y^*(\xi)/A,\Sigma_u)$, where the term $p(Y^*(\xi)/A,\Sigma_u)$ can be interpreted as a prior density for A and Σ_u of the Inverted Wishart (IW) – Normal (N) form, conditional on the vector of structural parameters ξ:

$$\Sigma_u/\xi \sim IW\left(\lambda T\Sigma_u^*(\xi), \lambda T - k, n\right) \tag{4}$$

$$A/\Sigma_u,\ \xi \sim N\left(A^*(\xi), \Sigma_u \otimes \left(\lambda T\Gamma_{XX}^*(\xi)\right)^{-1}\right)$$

$A^*(\xi)$ and $\Sigma_u^*(\xi)$ are maximum likelihood estimators based on the sample of "artificial" data generated using the DSGE model.

As Del Negro and Schorfheide (2004) demonstrated, the posterior distribution of the VAR parameters is also of Inverted Wishart-Normal form:

$$\Sigma_u/Y,\ \xi \sim IW\left((\lambda+1)T\widetilde{\Sigma}_u(\xi), (\lambda+1)T - k, n\right) \tag{5}$$

$$A/Y,\ \Sigma_u,\ \xi \sim N\left(\widetilde{A}(\xi), \Sigma_u \otimes \left(\lambda T\Gamma_{XX}^*(\xi) + X'X\right)^{-1}\right)$$

where

$$\widetilde{A}(\xi) = \left(\lambda T\Gamma_{XX}^*(\xi) + X'X\right)^{-1}\left(\lambda T\Gamma_{XY}^*(\xi) + X'Y\right) \tag{6}$$

$$\widetilde{\Sigma}_u(\xi) = \frac{1}{(\lambda+1)T}\Big[\left(\lambda T\Gamma_{YY}^*(\xi) + Y'Y\right) - \left(\lambda T\Gamma_{YX}^*(\xi) + Y'X\right) \left(\lambda T\Gamma_{XX}^*(\xi) + X'X\right)^{-1}\left(\lambda T\Gamma_{XY}^*(\xi) + X'Y\right)\Big]$$

represent maximum likelihood estimates of A and Σ_u, based on the combined sample of actual observations and "artificial" observations generated using the DSGE model. In the estimation, in order

[1] Giacomini (2013) presents a literature review on the econometric relationship between DSGE and VAR models from the point of view of estimation and model validation.

to avoid sampling variation, instead of moments from simulated data the expected moments of the DSGE model are used, i.e. $\Gamma^*_{XX}(\xi)$, $\Gamma^*_{XY}(\xi)$, $\Gamma^*_{YX}(\xi)$ and $\Gamma^*_{YY}(\xi)$, where for instance, $\Gamma^*_{XX}(\xi) = E_t[X^{*'}X^*]$, are replaced with the population moments.

The hyper-parameter λ governs the tightness of the prior distribution generated by the DSGE model for the parameters in the VAR model. In particular, setting $\lambda = 0$ delivers OLS-estimated VAR, i.e. DSGE prior is not important, while large λ values shrink coefficients towards the DSGE solution, i.e. data is not important.

In the empirical application, the hyper-parameter λ is estimated jointly with the deep parameters ξ using the algorithm proposed by Del Negro and Schorfheide (2004). They construct a hierarchical prior consisting of a marginal distribution for ξ and a conditional distribution for the VAR parameters given ξ. Bayes theorem then leads to a joint posterior distribution for the DSGE model and VAR model parameters which is estimated using a Markov-Chain-Monte-Carlo algorithm. The optimal λ value is the one that maximizes the marginal data density and is determined by performing a grid search.

3. The model

The econometric procedure presented in Section 2 is applied to a four variables VAR model consisting of real GDP, real exchange rate, inflation and nominal interest rate. The prior distribution for the VAR is derived from a New-Keynesian model adopted from Rabanal and Rubio-Ramirez (2005) and extended to a small open economy setting in order to reflect this specific feature of the Romanian economy.[2]

The model economy consists of a continuum of small open economies represented by the unit interval. Each small open economy is composed of: (i) a continuum of intermediate good producers, each producing a specific good that is an imperfect substitute for the other goods; (ii) a continuum of competitive final good producers; (iii) a continuum of infinitively lived households, each of them selling a type of labour that is an imperfect substitute for the other types; (iv) a monetary authority, without international policy coordination.

Firms operate in a monopolistically competitive environment, setting the prices according to Calvo staggered pricing rule (Calvo, 1983). The production function is linear in labour and abstracts from capital accumulation.[3] Technology is assumed to follow a unit root process and is common to both the domestic and world economies.

The representative household derives utility from consumption (subject to a preference-shifter shock), leisure and real balances. Wages are sticky in an analogous way to goods price.

Monetary policy is specified by a flexible Taylor rule, allowing for interest rate smoothing and penalizing the deviation of price inflation from the target, as well as the output gap. Also, as a measure of a central bank's non-systematic behaviour, a monetary policy shock is considered.

International financial markets are assumed to be perfect. The households discriminate between domestic and foreign goods, even though all goods can be traded internationally. The exchange rate is introduced into the model via purchasing power parity. The foreign demand for home produced goods is modelled as an exogenous AR(1) process.

The linearized equations of the model are provided below:

$$c_t = E_t[c_{t+1}] - \sigma\left(R_t - E_t[\pi_{t+1}]\right) + \sigma\left(1 - \rho_g\right)g_t \tag{7}$$

$$y_t = a_t + (1 - \alpha)n_t \tag{8}$$

[2] An overview of the structure of the model is provided in Appendix A. We refer to Rabanal and Rubio-Ramirez (2005) and Gali (2008) for a more through discussion of the model and literature references.

[3] Following McCallum and Nelson (1999), the capital stock is treated as fixed and investment is set to zero in the short run.

$$mrs_t = \frac{1}{\sigma}c_t + \gamma n_t - g_t \tag{9}$$

$$R_t = \rho_r R_{t-1} + \left(1 - \rho_r\right)\left(\Phi_\pi \pi_t + \Phi_y y_t\right) + z_t \tag{10}$$

$$wr_t = wr_{t-1} + \left(\Delta w_t - \pi_t\right) \tag{11}$$

$$\pi_{H,t} = \gamma_b \pi_{H,t-1} + \gamma_f E_t[\pi_{H,t+1}] + \frac{\kappa_p}{1 + \beta\chi_p}\left(wr_t + n_t - y_t\right)$$

$$\text{where } \kappa_p = \frac{(1-\alpha)(1-\theta)(1-\beta\theta)}{\theta\left(1 + \alpha\left(\varepsilon_p - 1\right)\right)},$$

$$\gamma_b = \frac{\chi_p}{1 + \beta\chi_p} \text{ and } \gamma_f = \frac{\beta}{1 + \beta\chi_p} \tag{12}$$

$$\Delta w_t - \chi_w \pi_{t-1} = \beta E_t\left[\Delta w_{t+1}\right] - \beta\chi_w \pi_t + \kappa_w(mrs_t \text{-} wr_t)$$

$$\text{where } \kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1 + \varepsilon_w \gamma)} \tag{13}$$

$$y_t = (1 - \varepsilon_B)c_t + \varepsilon_B y_t^* + \left(\varepsilon_B \varepsilon_H + \varepsilon_F \frac{\varepsilon_B}{1 - \varepsilon_B}\right) rer_t \tag{14}$$

$$\pi_t = \pi_{H,t} + \frac{\varepsilon_B}{1 - \varepsilon_B}(rer_t - rer_{t-1}) \tag{15}$$

$$c_t = y_t^* + \sigma rer_t + \sigma g_t \tag{16}$$

where the small letter variables represent the log of large letter variables. Output is denoted Y, consumption C, wage W, number of hours worked N. mrs represents the desired marginal rate of substitution between consumption and hours worked. R is the nominal interest rate, RER is the real effective exchange rate and π denotes the CPI inflation. g_t, a_t, z_t and y_t^* are a preference shifter shock, a technology shock, a monetary policy shock and a foreign demand shock; the evolution of these shocks is specified as follows:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{19}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{20}$$

$$z_t = \varepsilon_t^z \tag{21}$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \tag{22}$$

where each innovation ε_t^q follows a normal $(0, \sigma_q^2)$ distribution, $q = \{a, g, z, y^*\}$.

The description of the parameters is provided in Tables 1 and 2 below and also in Appendix A.

3.1. Data

The model is set at quarterly frequency, the sample covering the period from 2000 Q1 to 2015 Q4. The observed variables are real GDP, real exchange rate, CPI inflation net of the first round effects of the VAT rate changes and nominal interest rate, expressed as deviations from their trends. The exchange rate used in the estimation is an effective import-weighted exchange rate based on the bilateral exchange rates of the Romanian leu versus Eurozone and the United States of America respectively. The rest of the variables are considered not to be observed directly and are inferred using the Kalman filter.

The data is collected from the Romanian National Institute of Statistics database and National Bank of Romania database.

The trends are computed in the following way: for the output and the real effective exchange rate the Hodrick-Prescott filter is used, the smoothing parameter being set equal to 1600; in case of the inflation rate, the trend is set equal to the inflation target; for the nominal interest rate the trend is determined as the sum between the natural interest rate and the inflation target, where the natural interest rate is approximated by the average of the real interest rate taken on 2 sub-samples of data, before and after the economic crisis manifestation in the Romanian economy, i.e. the end of 2008.

An important modelling choice for the DSGE-VAR model is the lag length. In the estimation the lag length is set to four quarters, the usual approach in the existing DSGE-VAR literature when working with quarterly data (see, for example, Del Negro and Schorfheide, 2004).

3.2. Full-sample estimation results

Three parameters are calibrated, being kept fixes throughout the estimation. First the discount factor β is calibrated to match in steady state the sample average real interest rate. Secondly, the labour income share ratio, i.e. $1-\alpha$, is set to the average share of labour for the chosen sample period. The degree of openness in the economy, ε_B, is calibrated to the sample average of the ratio of total trade to gross domestic product. The values for these parameters are displayed in Table 1.

The rest of the parameters are estimated using the Bayesian approach. Structural parameters' prior distributions are presented in Table 2. In general, the priors are tight given the relatively small data sample.

The prior for price stickiness parameter, θ, is set to 0.667, implying a price duration of three quarters, slightly above the value resulting from micro-evidence as presented by Iordache and Pandioniu (2015). Wages are assumed to be renegotiated with annual frequency, the wage stickiness parameter, θ_w, being set to 0.75. The prior uncertainty is assumed to be relatively low, namely 0.075. The priors for the indexation parameters to past inflation are centered at 0.5, with an associated standard deviation of 0.1.

The priors for the Taylor rule parameters are centered around values similar to those in Copaciu et al. (2016) who estimated an extended version of Christiano et al. (2011) model for the Romanian economy. Thus, the prior for the persistence parameter in the reaction function is centered at 0.8 (standard deviation of 0.05), the parameter governing the response of interest rate to inflation to 1.7 (standard deviation of 0.1) and the parameter governing the response of interest rate to the deviation of output from the trend to 0.15 (standard deviation of 0.01).

Following a wide literature, the priors for the elasticities of substitution are set at 3, with associated standard deviations of 0.1, with one exception: elasticity of substitution among labour varieties, ε_w, for which the prior value is set to 11 (standard deviation of 0.1), implying a wage mark-up of 10%.

The prior for the inverse Frisch elasticity is set to 2, with a standard deviation of 0.5, i.e. a labour supply elasticity of 0.5, in line with empirical estimates (Lee, 2001; Ziliak and Kniesner, 2005 etc.).

The structural shocks are assumed to follow AR(1) processes, with the exception of the monetary policy shock, with the prior value for the mean being set at 0.7 (standard deviations of 0.1). The prior for the standard deviations of the shocks is fixed at values similar to those in Lees et al. (2007), who estimated the model of Lubik and Schorfheide (2007) for New Zeeland, which, as Romania, is a small open economy country with an inflation targeting monetary policy regime.

The posterior parameters and standard deviations values are reported in Table 2 below. The estimation results are based on 400,000 Metropolis Hastings simulations from the parameters' posterior distributions (out of the 600,000 draws one-third was discarded).

For most of the parameters the estimated posterior mean is not far from the prior mean as the priors, as mentioned, are relatively tight. Moreover, the results present a relatively high degree of uncertainty surrounding the posterior mean values, as measured by the 10th and 90th percentiles. This is due to short data sample available for the estimation, a specific feature of the emerging economies.

Table 1
Calibrated parameters.

Parameter	Description	Value
β	Discount factor	0.999
α	Capital share in production	0.45
ε_B	Degree of openness in the economy	0.7

However, the estimated Calvo parameter for prices points towards a lower degree of price stickiness as the one implied by the prior value. Also the estimated weight of price indexation to past inflation is lower than the one assumed a priori. These facts are most probably associated with the highly volatile observed inflation series as depicted in Fig. 1. Copaciu et al. (2016) find a slightly higher value (0.4) for the Calvo parameter associated with domestic prices using a shorter data sample in estimation, i.e. 2005 Q3–2014 Q3; the result obtained for the parameter reflecting the weight of price indexation to past inflation is similar with the one reported here.

Following Del Negro and Schorfheide (2004), the hyper-parameter λ is chosen to maximize the marginal data density by searching over a grid of λ values. The estimated value is 0.4, which corresponds to 25 artificial observations from the DSGE model, i.e. a weight of approximately 25% of the DSGE model prior relative to the weight of the actual sample in the estimation of the DSGE-VAR model.

The impulse response functions for the estimated model are reported in Appendix B, while the variance decomposition of output and inflation[4] over the 8-quarter horizon is displayed in Table 3 below. The technology shock explains over 50% of the variation in output, while it has a limited influence (below 10%) in case of inflation. The high contribution of foreign demand shock in determining the evolution of inflation highlights the importance of the open economy dimension of the model. The innovations to the Taylor rule are important for both output and inflation (about 25% of the variance explained). Regarding the preference shifter shock, it contributes little to the variance decomposition of inflation, while the effects on output are more pronounced (it explains around 17% of the variance decomposition of output).

Table 4 presents model moments, namely means and standard deviations, versus data counterparts. The model matches the data mean for real GDP and real exchange rate, while it understates the ones for CPI inflation and nominal interest rate. However, from Fig. 1 it can be observed that the positive data mean associated with CPI inflation and nominal interest rate is related with the disinflationary trend that characterized the first part of the analysed period, for the rest of the sample the data mean for both variables being close to the model's mean. The analysis of the standard deviations reveals the model overestimates the volatility of GDP while underestimating the standard deviation of the real exchange rate and CPI inflation. Also the model fails to generate the high volatility of the nominal interest rate, which is also associated mostly with the first part of the sample.

4. Evaluating forecasting performance

The effectiveness of inflation targeting depends upon the ability of the central bank to forecast accurately, but also to provide a credible “story” in order to explain and justify current policy actions.

Following Del Negro and Schorfheide (2004), the DSGE-VAR model's forecasting performance is assessed against an unrestricted VAR and a VAR with a Minnesota prior (which treats all variables symmetrically and shrinks the VAR coefficients towards a random walk process).

[4] We report here the results for the two variables that make the object of the monetary policy reaction function. For the rest of the variables the results are available upon request from the author.

Table 2
Estimated structural parameters.

	Description	Prior			Posterior			
		Distr.	Mean	s.d.	Mean	s.d.	10%	90%
ε_p	Elasticity of substitution between varieties of goods produced in any country	Γ	3	0.1	2.964	0.099	2.803	3.127
ε_H	Elasticity of substitution between domestic and foreign good in the domestic aggregate demand	Γ	3	0.1	2.999	0.100	2.832	3.162
ε_F	Elasticity of substitution between importing countries	Γ	3	0.1	2.986	0.099	2.823	3.148
ε_w	Elasticity of substitution among labour varieties	Γ	11	0.1	10.997	0.100	10.834	11.162
θ	Calvo parameter of price rigidity	β	0.667	0.075	0.245	0.045	0.195	0.305
θ_w	Calvo parameter of wage rigidity	β	0.75	0.075	0.717	0.083	0.587	0.858
χ_p	Weight of price indexation to past inflation	β	0.5	0.1	0.336	0.089	0.192	0.480
χ_w	Weight of wage indexation to past inflation	β	0.5	0.1	0.521	0.099	0.358	0.685
Φ_π	Reaction coefficient to the deviation of price inflation from the target in the Taylor-type rule	N	1.7	0.1	1.672	0.102	1.503	1.840
Φ_y	Reaction coefficient to the output gap in the Taylor-type rule	N	0.15	0.01	0.154	0.010	0.137	0.170
ρ_r	Inertia in the Taylor-type rule	β	0.8	0.05	0.680	0.043	0.609	0.751
σ	Elasticity of intertemporal substitution	Γ	3	0.1	2.942	0.097	2.784	3.104
γ	Inverse of the elasticity of labour supply with respect to real wages	Γ	2.0	0.5	2.016	0.501	1.203	2.809
ρ_g	Persistence of preference shifter shocks	β	0.7	0.1	0.808	0.074	0.695	0.926
ρ_a	Persistence of productivity shocks	β	0.7	0.1	0.675	0.097	0.516	0.830
ρ_{y^*}	Persistence of foreign demand shocks	β	0.7	0.1	0.639	0.079	0.514	0.773

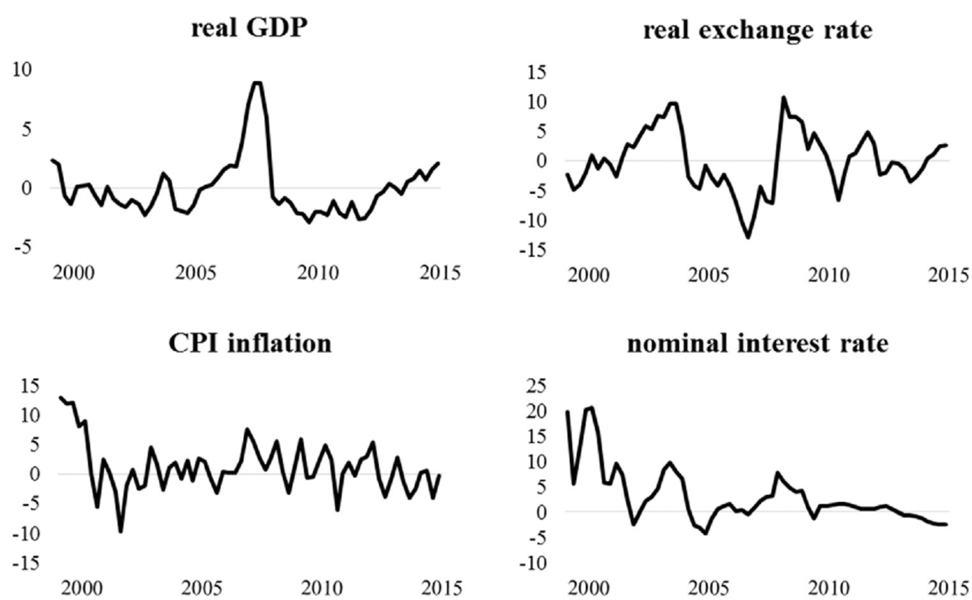

Fig. 1. Observable series as used in estimation, 2000 Q1–2015 Q4
Note: The variables are expressed as deviations from their trends and in the case of CPI inflation the values are annualized.
Source: Author's calculations based on NBR and NIS data.

To simulate the forecasting performance, all the three models are estimated recursively for 40 quarters from 2000 Q1 to 2013 Q4, i.e. the first estimation is performed using the sub-sample from 2000 Q1 to 2009 Q4, while the last one using the sub-sample from to 2004Q1 to 2013 Q4. The out-of-sample forecasting performance of the models is then evaluated at horizons *h* of 1 to 8 quarters ahead using the sub-sample from 2010 Q1 to 2015 Q4. The forecast error associated with the *h*-step-ahead forecast made at time *t* is determined as the forecasted value minus the actual value. Notice that all the parameters in the unrestricted VAR model, the Bayesian VAR model and the DSGE-VAR model, including the hyper-parameter λ, are re-estimated in each recursion. The forecasting performance is evaluated using the root square mean forecast error (RMSFE) indicator.

Table 5 reports the RMSFE improvements of the DSGE-VAR model relative to the unrestricted VAR and the Minnesota-VAR. The DSGE-VAR model clearly outperforms the unrestricted VAR and the Minnesota-VAR in terms of real exchange rate and nominal interest rate

Table 3
Variance decomposition (%) over the 8-quarter horizon of output and inflation at posterior mean.

Variables/ shock	Monetary policy shock	Foreign demand shock	Technology shock	Preference shifter shock
Real GDP	23.20	9.07	50.36	17.38
CPI inflation	31.63	58.18	9.70	0.49

Note: The variables are expressed as deviations from their trends and in the case of CPI inflation the values are annualized.

Table 4
Data and model moments.

Variable	Means		St. dev.	
	Data	Model	Data	Model
Real GDP	0.00	0.00	2.49	3.55
Real exchange rate	0.00	0.00	4.97	1.58
CPI inflation	1.14	0.00	4.18	1.58
Nominal interest rate	3.02	0.00	5.42	0.83

Note: The variables are expressed as deviations from their trends and in the case of CPI inflation the values are annualized.

Table 5
Percentage gain (loss) in RMSFEs: DSGE prior versus unrestricted VAR and Minnesota prior.

Horizon	Real GDP		Real exchange rate	
	Unrestricted VAR	Minnesota VAR	Unrestricted VAR	Minnesota VAR
1	0.16	0.12	1.52	1.16
2	0.31	0.23	1.47	1.18
3	0.53	0.40	1.41	1.19
4	0.68	0.54	1.41	1.27
5	0.80	0.67	1.22	1.31
6	0.83	0.73	1.13	1.39
7	0.84	0.79	1.28	1.48
8	0.88	0.82	1.35	1.43
Horizon	CPI inflation		Nominal interest rate	
	Unrestricted VAR	Minnesota VAR	Unrestricted VAR	Minnesota VAR
1	0.85	0.98	1.32	1.36
2	1.06	1.21	1.79	1.73
3	1.02	1.17	2.02	2.03
4	0.97	1.04	1.88	2.27
5	0.86	1.16	1.69	2.50
6	0.99	1.49	1.82	2.63
7	1.18	1.32	1.89	2.73
8	1.09	1.14	2.02	2.83

Note: The variables are expressed as deviations from their trends and in the case of CPI inflation the values are annualized. The rolling sample is 2000Q1–2013Q4 (56 periods). 40 observations are used to estimate the VAR models.

forecasts. The DSGE-VAR model also dominates the Minnesota-VAR model in terms of CPI inflation, having a relative similar forecasting performance as the unrestricted VAR model. For the real GDP, the DSGE-VAR model forecasts are worse than the ones obtained with the alternatives VAR models considered.

Overall, these results suggest that the DSGE-VAR model is competitive and, to some extent, improves upon the unrestricted VAR model and the Minnesota-VAR model. Moreover, the DSGE-VAR model is informative about the structure of the economy and can help the "story-telling" in the central banks. If for the VAR models either there is no parametric restrictions, i.e. the unrestricted VAR model, or the prior used incorporates no behavioural interpretations of parameters or equations, i.e. the Bayesian VAR model with Minnesota prior (which has only a statistical justification and not an economic one), in case of the DSGE-VAR model beliefs about the behavioural parameters in DSGE models are used to generate a prior distribution for the parameters of the VAR model. This specific feature of the DSGE-VAR model makes it possible to use the model for counterfactual policy simulations. As emphasized by Sims (2006), the DSGE-VAR model appears to be the most promising direction to follow in developing models that combine accurate probability modelling of the behaviour of economic series with insights from DSGE models.

5. Concluding remarks

It is generally recognized that central banks policies must be forward looking, as there are long lags between monetary policy actions and their impact on the economy. Therefore, macroeconomic forecasting has always been among the top priorities within central banks, the way in which forecasts are realized undergoing important changes over the past decades.

This paper describes the theoretical structure and the estimation results for a DSGE-VAR model for the Romanian economy and tests whether forecasts using this model are competitive with forecasts from an unrestricted VAR model and a VAR with a Minnesota prior. The relative forecast performance, as measured by the RMSFE indicator, indicates that the DSGE-VAR model is competitive and improves upon the alternative VAR models for three of the four variables considered in the analysis, i.e. CPI inflation, real exchange rate and nominal interest rate.

Moreover, the DSGE-VAR model is informative about the structure of the economy and can help the "story-telling" in the central banks.

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Appendix A. The DSGE model

A.1. Firms

It is assumed that different economies share identical preferences, technology and market structure. Firms are identical across countries and have the same Cobb-Douglas production function:

$$Y_{jt} = A_t N_{jt}^{1-\alpha} \tag{A.1}$$

where Y_j is the output produced by firm j, A is the economy-wide technology level and N_j is an index of labour input used by firm j and defined by a constant elasticity of substitution (CES) function that bundles the continuum of differentiated labour services provided by the households:

$$N_{jt} = \left[\int_0^1 \left(N_{jt}^i \right)^{\frac{\varepsilon_W - 1}{\varepsilon_W}} di \right]^{\frac{\varepsilon_W}{\varepsilon_W - 1}} \tag{A.2}$$

α represents the capital share of output, while ε_w denotes the elasticity of substitution among different labour types.

Price rigidity is introduced by using the staggered pricing rule of Calvo (1983). In any given period only a randomly chosen fraction of the firms $(1-\theta)$ are allowed to reoptimize their prices. The rest of the firms (Θ) adjust their prices by partial indexation to previous period inflation. χ_p measures the degree of price indexation to last period's inflation.

Firm j chooses its inputs and price in order to maximize the present value of its future profits.

A.2. Households

The lifetime utility function which a typical household i seeks to maximize is additively separable in consumption, leisure and real money holdings respectively:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[G_t^i \frac{\left(C_t^i\right)^{1-1/\sigma}}{1-1/\sigma} - \frac{\left(N_t^i\right)^{1+\gamma}}{1+\gamma} + \frac{\left(M_t^i/P_t\right)^{1-\nu}}{1-\nu} \right] \tag{A.3}$$

where β is the discount factor, σ is the elasticity of intertemporal substitution, γ is the inverse of the elasticity of labour supply with respect to real wages and ν is the elasticity of money holdings with respect to transactions. N^i denotes the labour services provided by the household i: each household specializes in one type of labour, which is supplied monopolistically. M^i/P represents real money holding of household i, while G^i is a preference-shifter shock that affects the marginal utility of consumption. The variable C is a composite consumption index determined by both home and foreign goods, aggregated together by the perfectly competitive final good producers:

$$C_t = \left[(1-\varepsilon_B)^{\frac{1}{\varepsilon_H}} (C_{Ht})^{\frac{\varepsilon_H - 1}{\varepsilon_H}} + \varepsilon_B^{\frac{1}{\varepsilon_H}} (C_{Ft})^{\frac{\varepsilon_H - 1}{\varepsilon_H}} \right]^{\frac{\varepsilon_H}{\varepsilon_H - 1}} \tag{A.4}$$

where ε_B measures the degree of openness in the economy and ε_H denotes the substitutability between domestic and foreign goods from the viewpoint of domestic consumers. C_H is an index of consumption of domestic goods, given by the CES-function:

$$C_{Ht} = \left[\int_0^1 (C_{Hjt})^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{A.5}$$

where ε_p represents the elasticity of substitution between varieties of goods produced in any country. C_F is an index of consumption of imported goods:

$$C_{Ft} = \left[\int_0^1 (C_{kt})^{\frac{\varepsilon_F - 1}{\varepsilon_F}} dk \right]^{\frac{\varepsilon_F}{\varepsilon_F - 1}} \tag{A.6}$$

where ε_F denotes the elasticity of substitution between importing countries. Finally, C_k is an index of the different goods imported from country k:

$$C_{kt} = \left[\int_0^1 (C_{kjt})^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{A.7}$$

The typical household i's maximization problem is subject to a one-period budget constraint:

$$\int_0^1 P_{Hjt} C_{Hjt}^i dj + \int_0^1 \int_0^1 (S_{kt} P_{kjt}) C_{kjt}^i dj\, dk + M_t^i + \frac{1}{1+ir_t} B_t^i \le M_{t-1}^i + B_{t-1}^i + W_t^i N_t^i + T_t^i \tag{A.8}$$

The domestic price on good j is denoted P_{Hj}, the price on good j imported from country k and expressed in country's k currency is denoted P_{kj}, while S_k represents the bilateral nominal exchange rate, i.e. the price of country k's currency in terms of domestic currency. B^i is the quantity of one-period nominally riskless bonds purchased each period by household i, which pay one unit of money at maturity and have a price of $\frac{1}{1+ir}$ units of money, ir being the nominal interest rate. W^i represents the nominal wage received for the type of labour provided by household i. Each period only a constant fraction of households $(1-\theta_w)$ can reoptimize the price of their labour services, while for the remaining fraction of the households (θ_w) the wage they had last period is adjusted by partial indexation to previous period inflation (χ_w measures the degree of wage indexation to last period's inflation). T^i denotes lump-sum additions or subtractions to household i's period income (taxes, dividends etc.).

A.3. Monetary authority

It is assumed that the central bank follows a Taylor-type rule which allows for interest rate smoothing and penalizes the deviation of price inflation from the target π, as well as the output gap (y).

Appendix B. Impulse response functions (IRFs)

To solve the model, optimality conditions are derived for the maximization problems. The dynamics of the model are obtained

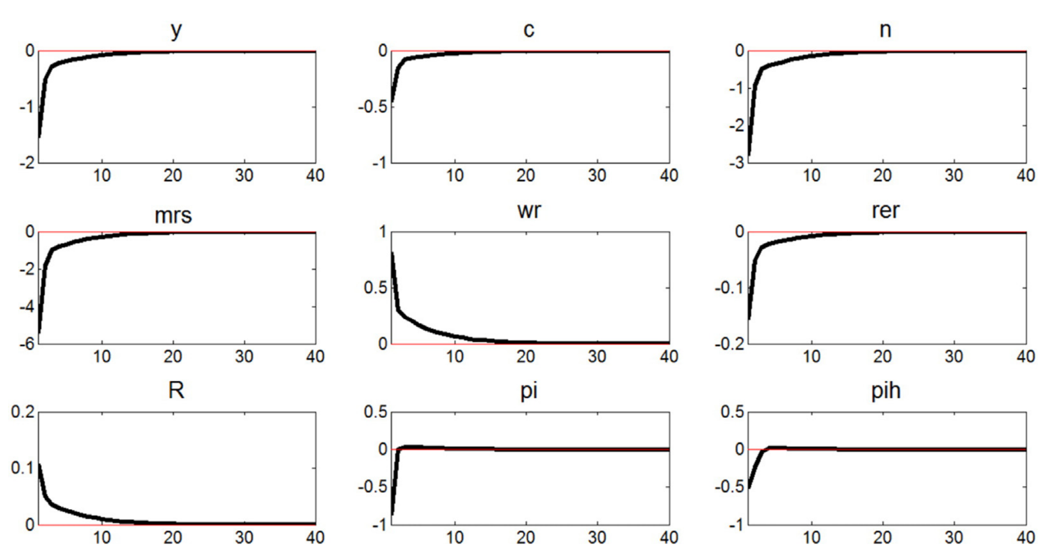

Fig. B.1. IRFs to a monetary policy shock.

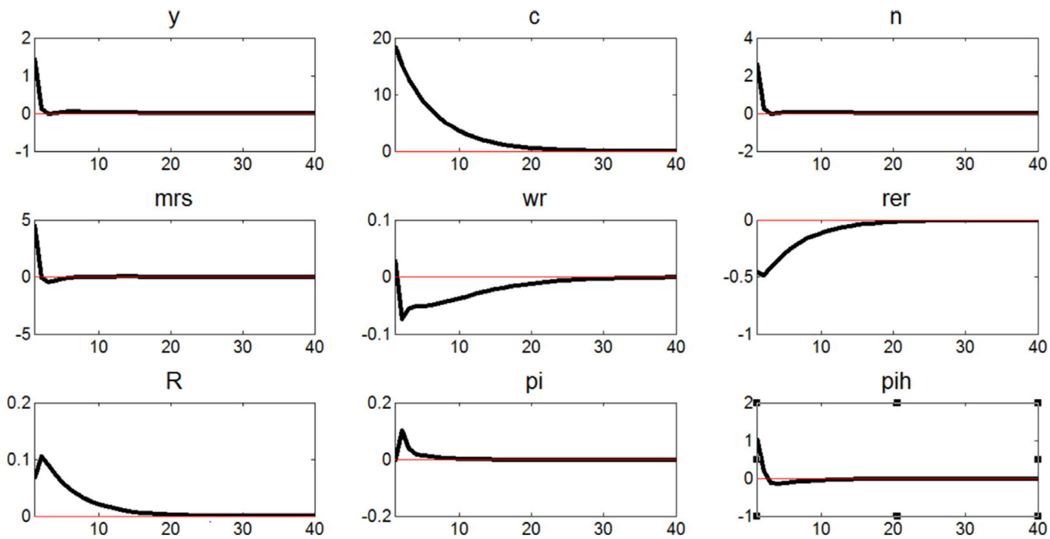

Fig. B.2. IRFs to a preference shifter shock.

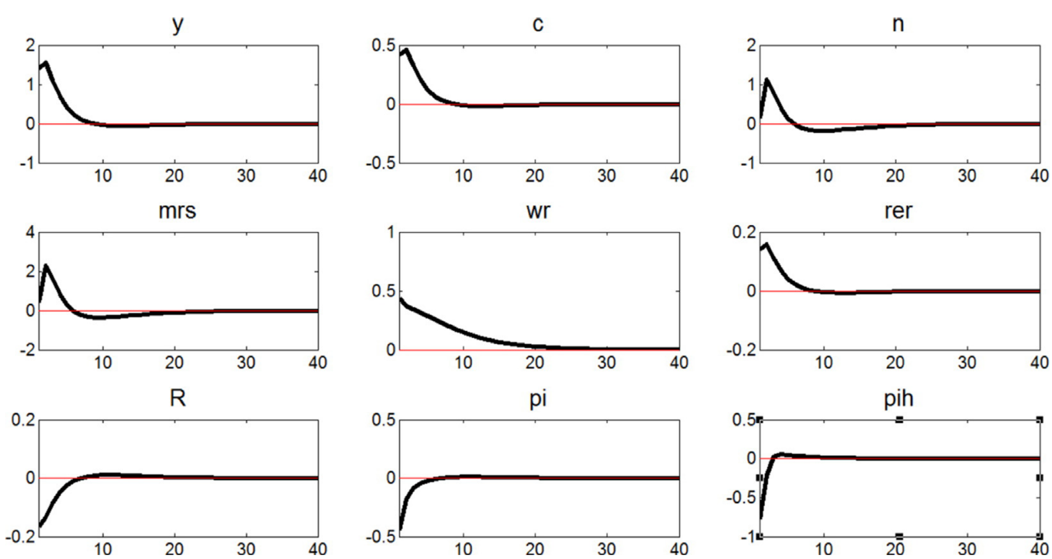

Fig. B.3. IRFs to a technology shock.

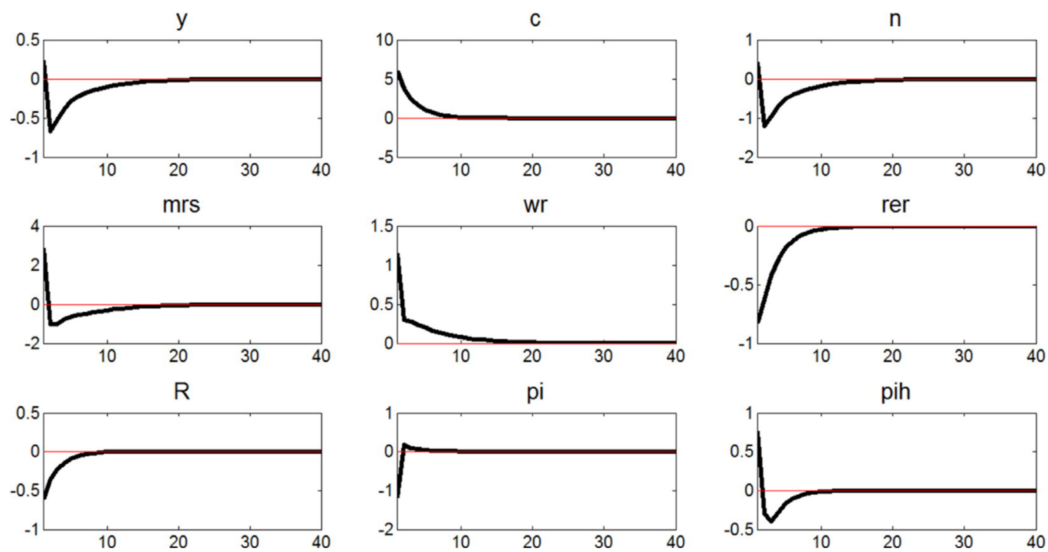

Fig. B.4. IRFs to a foreign demand shock.

by taking a log-linear approximation around the steady-state equilibrium.

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