

Before casting the small-scale DSGE model used as example in the textbook of Herbst & Schorfheide note that there are two minor mistakes.

First, the exogenous innovation $\varepsilon_{R,t}$ is not a state variable and hence does not belong to the vector x_t . Second, in eq. (2.5) y_t, π_t should be replaced by $\hat{y}_t, \hat{\pi}_t$.

Now casting the model into the canonical form

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi \varepsilon_t + \Pi \eta_t$$

yields

$$\begin{pmatrix} 1 & \frac{1}{\tau} & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ -(1-\rho_R)\psi_2 & -(1-\rho_R)\psi_1 & 1 & (1-\rho_R)\psi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ \hat{g}_t \\ \hat{z}_t \\ E_t \hat{y}_{t+1} \\ E_t \hat{\pi}_{t+1} \end{pmatrix} \\ = \begin{pmatrix} 1 & 0 & \frac{1}{\tau} & -(1-\rho_g) & -\frac{\rho_z}{\tau} & 0 & 0 \\ -\kappa & 1 & 0 & \kappa & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{R}_{t-1} \\ \hat{g}_{t-1} \\ \hat{z}_{t-1} \\ E_{t-1} \hat{y}_t \\ E_{t-1} \hat{\pi}_t \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{g,t} \\ \varepsilon_{R,t} \end{pmatrix} + \begin{pmatrix} 1 & \frac{1}{\tau} \\ 0 & \beta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_{y,t} \\ \eta_{\pi,t} \end{pmatrix}$$

with

$$\begin{aligned} \eta_{y,t} &= \hat{y}_t - E_{t-1} \hat{y}_t \\ \eta_{\pi,t} &= \hat{\pi}_t - E_{t-1} \hat{\pi}_t. \end{aligned}$$

The first two equations of this system are the NK-IS Curve and the NK-Phillips Curve shifted backward by one period and replacing $E_{t-1} \hat{y}_t$ and $E_{t-1} \hat{\pi}_t$ with $\hat{y}_t - \eta_{y,t}$ and $\hat{\pi}_t - \eta_{\pi,t}$, respectively. The third equation is just the interest rate rule. The fourth and fifth equations are the exogenous shock processes (1.25) and (1.26). The last two equations are the definitions of the one-step ahead rational forecast errors $\eta_{y,t}$ and $\eta_{\pi,t}$.