Utility is

$$U(c_t(i),\ldots) = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\frac{\epsilon^B_{t+\tau}(c_{t+\tau}(i) - hc_{t+\tau-1})^{1-\sigma_c}}{1-\sigma_c} \right) \ldots \tag{1}$$

So, the marginal utility is given by

$$\lambda_t = \epsilon^B_t (c_t - hc_{t-1})^{-\sigma_c} - \beta h \mathbb{E}_t \epsilon^B_{t+1}(c_{t+1} - hc_t)^{-\sigma_c} \tag{2}$$

At the steady state,

$$\lambda = (c - hc)^{-\sigma_c}(1 - \beta h) \tag{3}$$

with the resulting log linear form

$$\widehat{\lambda}_t = \frac{1}{1-\beta h}\left(\hat{\epsilon}^B_t - \beta h \mathbb{E}_t \hat{\epsilon}^B_{t+1}\right) - \frac{\sigma_c}{(1-h)(1-\beta h)}\left[\hat{c}_t - h\hat{c}_{t-1} - \beta h \mathbb{E}_t(\hat{c}_{t+1} - h\hat{c}_t)\right] \tag{4}$$

Euler equation in SW is

$$\beta \mathbb{E}_t \lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}} = \lambda_t \tag{5}$$

at the ss

$$\beta \lambda \frac{R}{\pi} = \lambda \tag{6}$$

with log linear approximation

$$\widehat{\lambda}_{t+1} + \hat{R}_{t+1} - \hat{\pi}_{t+1} = \hat{\lambda}_t \tag{7}$$

Now I exploit $\hat{\lambda}_t$ to obtain the following consumption eqaution

$$\hat{c}_t = \frac{1}{1+h+\beta h^2}\left[(1+\beta+\beta h^2)\hat{c}_{t+1} + h\hat{c}_{t-1} - \beta h \hat{c}_{t-2}\right] - \frac{1-h}{\sigma_c(1+h+\beta h^2)}\left[\hat{\epsilon}^B_{t+1} - \beta h \hat{\epsilon}^B_{t+2} - \hat{\epsilon}^B_t - \beta h \hat{\epsilon}^B_{t+1}\right]$$
$$- \frac{(1-h)(1-\beta h)}{\sigma_c(1+h+\beta h^2)}(\hat{R}_{t+1} - \hat{\pi}_{t+1})$$