COVID Worker Value Functions

We extend on the pre-COVID design with the following value functions for employed and unemployed respectively:

$$U_t = b + \beta[p(\theta_{t+1})E_{t+1} + (1 - p(\theta_{t+1}))U_{t+1}]$$

$$E_t = w + \beta[\delta(1 - p(\theta_{t+1}))U_{t+1} + (1 - \delta(1 - p(\theta_{t+1})))E_{t+1}]$$

Adding SIR-model transitions ($\pi, \iota\pi, \gamma^R, \gamma^D$ stand for employed infection, unemployed infection, recovery, and death rates respectively):

$$U_t^S = b + \beta[\iota\pi_t U_{t+1}^I + (1 - \iota\pi_t)(p(\theta_{t+1})E_{t+1}^S + (1 - p(\theta_{t+1}))U_{t+1}^S)]$$

$$U_t^I = b + \beta[(1 - \gamma^R - \gamma^D)U_{t+1}^I + \gamma^R(p(\theta_{t+1})E_{t+1}^R + (1 - p(\theta_{t+1}))U_{t+1}^R)]$$

$$U_t^R = b + \beta[p(\theta_{t+1})E_{t+1}^R + (1 - p(\theta_{t+1}))U_{t+1}^R]$$

$$E_t^S = w_t + \beta[\pi_t(\delta U_{t+1}^I + (1 - \delta)E_{t+1}^I) + (1 - \pi_t)(\delta(1 - p(\theta_{t+1}))U_{t+1}^S + (1 - \delta(1 - p(\theta_{t+1})))E_{t+1}^S)]$$

$$E_t^I = w_t + \beta[(1 - \gamma^R - \gamma^D)(\delta U_{t+1}^I + (1 - \delta)E_{t+1}^I) + \gamma^R(\delta(1 - p(\theta_{t+1}))U_{t+1}^R + (1 - \delta(1 - p(\theta_{t+1})))E_{t+1}^R)]$$

$$E_t^R = w_t + \beta[\delta(1 - p(\theta_{t+1}))U_{t+1}^R + (1 - \delta(1 - p(\theta_{t+1})))E_{t+1}^R]$$

COVID Firm Behaviour

The pre-COVID firm decision was to choose $\{e_t, v_t\}_{t=0}^{\infty}$ in order to maximise

$$\sum_{t=0}^{\infty} \beta^t (y_t - w_t e_t - \kappa v_t)$$

$$s.t. \quad y_t = A_t n_t^s, \quad n_t = q(\theta_t)v_t + (1 - \delta)n_{t-1}.$$

The resulted in the F.O.C.

$$\frac{\kappa}{q(\theta_t)} = s\frac{y_t}{n_t} - w_t + \beta(1 - \delta)\frac{\kappa}{q(\theta_{t+1})}$$

Adding SIR-model transitions, firms choose $\{e_t, v_t\}_{t=0}^{\infty}$ in order to maximise

$$\sum_{t=0}^{\infty} \beta^t \{-w_t e_t - \kappa v_t + (1-\phi) y_t + \phi(1-\pi)^t y_t + \phi \eta y_t \sum_{j=0}^{t-1} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-1}] +$$
$$\phi y_t \sum_{j=0}^{t-1} \sum_{k=2}^{t-j} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-k} \gamma^R]\} \ s.t.$$
$$y_t = A_t n_t^s, \quad n_t = q(\theta_t) v_t + (1-\delta)(1-\phi \sum_{j=0}^{t-1} \sum_{k=2}^{t-j} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-k} \gamma^D]) n_{t-1}.$$

Equivalently, the firms choose $\{e_t\}_{t=0}^{\infty}$ in order to maximise

$$\sum_{t=0}^{\infty} \beta^t \{A_t n_t^s [(1-\phi) + \phi(1-\pi)^t + \phi \eta \sum_{j=0}^{t-1} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-1}] +$$
$$\phi \sum_{j=0}^{t-1} \sum_{k=2}^{t-j} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-k} \gamma^R]] - w_t n_t -$$
$$\frac{\kappa}{q(\theta_t)} [n_t - (1-\delta)(1-\phi \sum_{j=0}^{t-1} \sum_{k=2}^{t-j} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-k} \gamma^D]) n_{t-1}]\}.$$

The F.O.C. reads:

$$\frac{\kappa}{q(\theta_t)} = s \frac{y_t}{n_t} \{(1-\phi) + \phi(1-\pi)^t + \phi \eta \sum_{j=0}^{t-1} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-1}] +$$
$$\phi \sum_{j=0}^{t-1} \sum_{k=2}^{t-j} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-k} \gamma^R] - w_t +$$
$$\beta(1-\delta)(1-\phi \sum_{j=0}^{t} \sum_{k=2}^{t-j+1} [(1-\pi)^j \pi (1-\gamma^R-\gamma^D)^{t-j-k} \gamma^D]) \frac{\kappa}{q(\theta_{t+1})}\}$$

In order to be able to simulate the COVID equilibrium, we require either a recursive representation of the F.O.C. or to rewrite the system using value functions:

$$\frac{\kappa}{q(\theta_t)} = \phi J_t^S + (1-\phi) J_t^R$$
$$J_t^S = -w_t + (1-\pi_t)(s \frac{y_t}{n_t} + \beta(1-\delta) J_{t+1}^S) + \pi_t (s \frac{\eta y_t}{n_t} + \beta(1-\delta)(1-\gamma^D) J_{t+1}^I)$$
$$J_t^I = -w_t + (1-\gamma^R-\gamma^D)(s \frac{\eta y_t}{n_t} + \beta(1-\delta)(1-\gamma^D) J_{t+1}^I) + \gamma^R (s \frac{y_t}{n_t} + \beta(1-\delta) J_{t+1}^R)$$
$$J_t^R = -w_t + s \frac{y_t}{n_t} + \beta(1-\delta) J_{t+1}^R$$