Households

$$max_{C_t, L_t, b_{t+1}} E_t \sum_{s=0}^{\infty} \beta^s \left\{ \log(C_{t+s} - \Phi C_{t+s-1}) - \chi L_{t+s}^{1+\sigma}/(1+\sigma) \right\},$$

subject to:

$$C_t + \frac{b_{t+1}}{1+r_t} = w_t L_t + \frac{b_t}{\pi_t} + \Pi_t + T_t$$

and $\lambda_{1,t}$ is the Lagrange multipliers of the budget constraints, respectively.
FOCs:

$$\frac{\partial}{\partial C_t} = U'(C_t) = \frac{1}{C_t - \Phi C_{t-1}} - \frac{\beta \Phi}{C_{t+1} - \Phi C_t} - \lambda_{1,t} = 0$$

$$\Rightarrow U'(C_t) = \lambda_{1,t}$$

$$\frac{\partial}{\partial L_t} = -\chi L_t^{\sigma} + \lambda_{1,t} w_t = 0$$

$$\Rightarrow L_t = (\frac{U'(C_t) w_t}{\chi})^{\frac{1}{\sigma}} \tag{1}$$

$$\frac{\partial}{\partial b_{t+1}} = -\frac{\lambda_{1,t}}{1+r_t} + \frac{\lambda_{1,t+1}}{\pi_{t+1}} = 0$$

$$\Rightarrow \frac{\pi_{t+1}}{1+r_t} = \frac{\lambda_{1,t+1}}{\lambda_{1,t}} = \frac{\beta U'(C_{t+1})}{U'(C_t)} \tag{2}$$

Intermediate good producers:

$$\max_{P_{t,j},L_{t,j},I_{t,j},S_{t,j},Z_{t,j},Y_{t,j},K_{t+1,j},N_{t+1,j},b_{t+1,j}} E_t \sum_{j=0}^{\infty} M_{t,t+j}\Pi_{t+j}(i)$$

where

$$\Pi_t(j) = \frac{P_t(j)}{P_t}Y_t(j) - w_t L_t(j) - I_t(j) - S_t(j) - \frac{\phi_P}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)^2 Y_t + \frac{b_{t+1}}{R_t} - \frac{b_t}{\pi_t}$$

subject to the following constraints:

$$Y_t(j) = K_t(j)^\alpha \left(Z_t(j)L_t(j)\right)^{1-\alpha} \tag{3}$$

$$Z_t(j) = A_t N_t(j)^\eta N_t^{1-\eta} \tag{4}$$

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$$

$$\frac{P_t(j)}{P_t}Y_t(j) \leq \zeta(K_{t+1}(j) - \frac{b_{t+1}}{1+r_t}) \tag{5}$$

$$K_{t+1} = (1-\delta_K)K_t + \Lambda_K\left(\frac{I_t}{K_t}\right)K_t \tag{6}$$

$$N_{t+1} = (1-\delta_N)N_t + \Lambda_N\left(\frac{S_t}{N_t}\right)N_t \tag{7}$$

with $\gamma_{i,t}, i = 1,2,3,4,5,6$ being their corresponding Lagrange multipliers.

FOCs:

$$\frac{\partial}{\partial P_{t,j}} = \frac{Y_t(j)}{P_t} - \phi_P \frac{1}{\pi P_{t-1}(j)}\left(\frac{P_t(i)}{\pi P_{t-1}(j)} - 1\right)Y_t + M_{t,t+1}\phi_P \frac{P_{t+1}(j)}{\pi P_t^2(j)}\left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1\right)Y_{t+1} -$$

$\varepsilon\gamma_{3,t}\frac{1}{P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon-1}Y_t - \gamma_{4,t}\frac{1}{P_t}Y_t(j) = 0$
or

$$1 - \phi_P\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right) + M_{t,t+1}\phi_P\frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{Y_{t+1}}{Y_t} - \varepsilon - (1-\varepsilon)\gamma_{4,t} + \varepsilon\gamma_{1,t} = 0 \tag{8}$$

$$\frac{\partial}{\partial L_{t,j}} = -w_t + \gamma_{1,t}(1-\alpha)K_t(j)^\alpha Z_t(j)^{1-\alpha}L_t(j)^{-\alpha} = 0$$

$$w_t = \gamma_{1,t}(1-\alpha)\frac{Y_{j,t}}{L_{j,t}} \tag{9}$$

$$\frac{\partial}{\partial K_{t+1,j}} = \gamma_{1,t+1}\alpha\frac{Y_{j,t+1}}{K_{j,t+1}} + \gamma_{4,t}\zeta - \gamma_{5,t} + \gamma_{5,t+1}\left(1 - \delta_K + \Lambda_K\left(\frac{I_{t+1}}{K_{t+1}}\right) - \frac{I_{t+1}}{K_{t+1}}\Lambda_K'\left(\frac{I_{t+1}}{K_{t+1}}\right)\right) = 0$$

$$\Rightarrow \gamma_{1,t+1}\alpha\frac{Y_{j,t+1}}{K_{j,t+1}} + \gamma_{4,t}\zeta = \frac{1}{\Lambda_K'(\frac{I_t}{K_t})} - \frac{1}{\Lambda_K'(\frac{I_{t+1}}{K_t 1})}\left(1 - \delta_K + \Lambda_K\left(\frac{I_{t+1}}{K_{t+1}}\right) - \frac{I_{t+1}}{K_{t+1}}\Lambda_K'\left(\frac{I_{t+1}}{K_{t+1}}\right)\right) \tag{10}$$

$$\frac{\partial}{\partial N_{t+1,j}} = \gamma_{2,t+1}\eta\frac{Z_{j,t+1}}{N_{j,t+1}} - \gamma_{6,t} + \gamma_{6,t+1}\left(1-\delta_N+\Lambda_N\left(\frac{S_{t+1}}{N_{t+1}}\right)-\frac{S_{t+1}}{N_{t+1}}\Lambda_N'\left(\frac{S_{t+1}}{N_{t+1}}\right)\right)=0$$

$$\Rightarrow \gamma_{1,t+1}(1-\alpha)\eta\frac{Y_{j,t+1}}{N_{j,t+1}} = \frac{1}{\Lambda_N'(\frac{S_t}{N_t})} - \frac{1}{\Lambda_N'(\frac{S_{t+1}}{N_{t+1}})}\left(1-\delta_N+\Lambda_N\left(\frac{S_{t+1}}{N_{t+1}}\right)-\frac{S_{t+1}}{N_{t+1}}\Lambda_N'\left(\frac{S_{t+1}}{N_{t+1}}\right)\right) \tag{11}$$

$$\frac{\partial}{\partial Z_t(j)} = \gamma_{1,t}(1-\alpha)\left(K_t(j)\right)^{\alpha} Z_t(j)^{-\alpha} L_t(j)^{1-\alpha} - \gamma_{2,t} = 0$$

$$\gamma_{2,t} = \gamma_{1,t}(1-\alpha)\frac{Y_{j,t}}{Z_{j,t}}$$

$$\frac{\partial}{\partial Y_t(j)} = \frac{P_t(j)}{P_t} - \gamma_{1,t} - \gamma_{3,t} - \frac{P_t(j)}{P_t}\gamma_{4,t} = 0$$

$$\gamma_{1,t} + \gamma_{3,t} + \gamma_{4,t} = 1$$

$$\frac{\partial}{\partial b_{t+1}} = \frac{1}{R_t} - \frac{M_{t,t+1}}{\pi_{t+1}} - \frac{\gamma_{4,t}\zeta}{1+r_t} = 0$$

$$\frac{\pi_{t+1}}{R_t} - \gamma_{4,t}\zeta\frac{\pi_{t+1}}{1+r_t} = M_{t,t+1} \tag{12}$$

$$\frac{\partial}{\partial I_t} = -1 + \gamma_{5,t}\Lambda_K'(\frac{I_t}{K_t}) = 0$$

$$\Rightarrow \gamma_{5,t} = \frac{1}{\Lambda_K'(\frac{I_t}{K_t})}$$

$$\frac{\partial}{\partial S_t} = -1 + \gamma_{6,t}\Lambda_N'(\frac{S_t}{N_t}) = 0$$

$$\Rightarrow \gamma_{6,t} = \frac{1}{\Lambda_N'(\frac{S_t}{N_t})}$$

Market clearing conditions

Goods market:

$$Y_t = C_t + I_t + S_t + \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t \tag{13}$$

definition:

$$R_t = 1 + (1-\tau)r_t \tag{14}$$

monetary policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\left(\frac{\Delta Y_t}{\Delta Y}\right)^{\rho_Y}\exp(\sigma_r \varepsilon_{r,t}), \tag{15}$$

aggregate productivity:

$$\text{Log}(A_t) = (1-\rho_a)\text{Log}(A) + \rho_a \text{Log}(A_{t-1}) + \sigma_a \varepsilon_{a,t} \tag{16}$$

government budget balance:

$$T_t = b_{t+1}\left(\frac{1}{R_t} - \frac{1}{1+r_t}\right) \tag{17}$$

ALL DE-TRENDED EQUATIONS

I define $\tilde{X}_t = \frac{X_t}{N_t}$ and also $g_{N,t} = \frac{N_t}{N_{t-1}}$.

Set of endogenous variables:

$$\left\{ \mathrm{L}_t, \tilde{C}_t, \tilde{I}_t, \tilde{S}_t, \tilde{K}_{t+1}, g_{N,t+1}, \tilde{Y}_t, \pi_t, \tilde{Z}_t, b_{t+1}, \tilde{w}_t, R_t, r_t, T_t, \gamma_{1,t}, \gamma_{4,t}, A_t \right\}$$

$$g_{N,t+1}\tilde{K_{t+1}} = (1-\delta_K)\,\tilde{K_t} + \Lambda_K\left(\frac{\tilde{I}_t}{\tilde{K_t}}\right)\tilde{K_t} \tag{1}$$

$$g_{N,t+1} = (1-\delta_N) + \Lambda_N\left(\tilde{S}_t\right) \tag{2}$$

$$U'(\tilde{C}_t) = \frac{1}{\tilde{C}_t - \Phi\frac{\tilde{C_{t-1}}}{g_{N,t}}} - \frac{\beta\Phi}{g_{N,t+1}\tilde{C_{t+1}} - \Phi\tilde{C}_t}$$

$$U'(\tilde{C_{t+1}}) = \frac{1}{g_{N,t+1}\tilde{C_{t+1}} - \Phi\tilde{C}_t} - \frac{\beta\Phi}{(g_{N,t+2}\times g_{N,t+1})\tilde{C_{t+2}} - \Phi g_{N,t+1}\tilde{C_{t+1}}}$$

$$\tilde{M_{t,t+1}} = \frac{\beta U'(\tilde{C_{t+1}})}{U'(\tilde{C_t})}$$

$$\frac{\pi_{t+1}}{1+r_t} = \tilde{M_{t,t+1}} \tag{3}$$

$$L_t = \left(\frac{U'(\tilde{C}_t)\tilde{w}_t}{\chi}\right)^{\frac{1}{\sigma}} \tag{4}$$

$$\gamma_{1,t+1}\alpha\frac{\tilde{Y_{j,t+1}}}{\tilde{K_{j,t+1}}} + \gamma_{4,t}\zeta = \frac{1}{\Lambda'_K(\frac{\tilde{I}_t}{\tilde{K}_t})} - \frac{1}{\Lambda'_K(\frac{\tilde{I}_{t+1}}{\tilde{K}_t 1})}\left(1-\delta_K+\Lambda_K\left(\frac{\tilde{I}_{t+1}}{\tilde{K}_{t+1}}\right) - \frac{\tilde{I}_{t+1}}{\tilde{K}_{t+1}}\Lambda'_K\left(\frac{\tilde{I}_{t+1}}{\tilde{K}_{t+1}}\right)\right) \tag{5}$$

$$\gamma_{1,t+1}(1-\alpha)\eta\tilde{Y_{j,t+1}} = \frac{1}{\Lambda'_N(\tilde{S}_t)} - \frac{1}{\Lambda'_N(\tilde{S_{t+1}})}\left(1-\delta_N+\Lambda_N\left(\tilde{S_{t+1}}\right) - \tilde{S_{t+1}}\Lambda'_N\left(\tilde{S_{t+1}}\right)\right) \tag{6}$$

$$\Lambda_K\left(\frac{\tilde{I}_t}{\tilde{K}_t}\right) = a_{K,1} + \frac{a_{K,2}}{1-\frac{1}{\tau_K}}\left(\frac{\tilde{I}_t}{\tilde{K}_t}\right)^{1-\frac{1}{\tau_K}}$$

$$\Lambda'_{K,t} = a_{K,2}\left(\frac{\tilde{I}_t}{\tilde{K}_t}\right)^{-\frac{1}{\tau_K}}$$

$$\Lambda_N\left(\tilde{S}_t\right) = a_{N,1} + \frac{a_{N,2}}{1-\frac{1}{\tau_N}}\tilde{S}_t^{1-\frac{1}{\tau_N}}$$

$$\Lambda'_{N,t} = a_{N,2}\tilde{S}_t^{-\frac{1}{\tau_N}}$$

$$\tilde{w_t} = \gamma_{1,t}(1-\alpha)\frac{\tilde{Y}_t}{L_t} \tag{7}$$

$$\tilde{Y}_t = \tilde{K}_t^{\alpha}\left(\tilde{Z}_t L_t\right)^{1-\alpha} \tag{8}$$

$$\tilde{Z}_t = A_t \tag{9}$$

$$1-\phi_P\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi}-1\right)+\tilde{M}_{t,t+1}\phi_P\frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{g_{N,t+1}\tilde{Y}_{t+1}}{\tilde{Y}_t}-\varepsilon-(1-\varepsilon)\gamma_{4,t}+\varepsilon\gamma_{1,t}=0 \tag{10}$$

$$\tilde{Y}_t = \zeta\left(\tilde{K}_{t+1}-\frac{b_{t+1}}{1+r_t}\right) \tag{11}$$

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{S}_t + \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi}-1\right)^2\tilde{Y}_t \tag{12}$$

$$\mathrm{b}_t = 0$$

$$\frac{\pi_{t+1}}{R_t} - \gamma_{4,t}\zeta\frac{\pi_{t+1}}{1+r_t} = M_{t,t+1} \tag{13}$$

$$R_t = 1 + (1-\tau)r_t \tag{14}$$

$$T_t = b_{t+1}\left(\frac{1}{R_t} - \frac{1}{1+r_t}\right) \tag{15}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}\frac{\tilde{g}_t}{\bar{g}}\right)^{\rho_Y}\exp(\sigma_r\varepsilon_{r,t}), \tag{16}$$

$$\mathrm{Log}(A_t) = (1-\rho_a)\mathrm{Log}(A) + \rho_a\mathrm{Log}(A_{t-1}) + \sigma_a\varepsilon_{a,t} \tag{17}$$