Semistructural model form Brazilian Central Bank

Link to the paper:
https://www.bcb.gov.br/content/ri/relatorioinflacao/202406/ri202406b12p.pdf

The model has the following equations

Phillips Curve

$$(1)\ \pi_t^{L,sa} = \alpha_{1\mathrm{L}}\pi_{t-1}^{L,sa} + \alpha_{1\mathrm{I}}\frac{\sum_{i=1}^{4}\pi_{t-\mathrm{i}}^{IPCA,sa}}{4} + (1-\alpha_{1\mathrm{L}}-\alpha_{1\mathrm{I}})\frac{\pi_{t,t+4|t}^{e}}{4} + \alpha_2\hat{\pi}_t^{*} + \alpha_3\widehat{\Delta e}_{t-1} + \alpha_4 h_t + \frac{\sum_{i=0}^{2}(\alpha_5 d_{t-i}^{el} + \alpha_6 d_{t-i}^{la})Clima_{t-i}^{2}}{3} - \frac{\sum_{i=3}^{5}(\alpha_5 d_{t-i}^{el} + \alpha_6 d_{t-i}^{la})Clima_{t-i}^{2}}{3} + \epsilon_t^{\pi^L}$$

Where

$\pi_t^{L,sa}$	Represents the quarterly inflation of free prices in the Broad National Consumer Price Index (IPCA) with seasonal adjustment.
$\pi_t^{IPCA,sa}$	Represents the quarterly inflation of the IPCA with seasonal adjustment.
$\pi_{t,t+4\|t}^{e}$	It is the expectation at time t, determined by the Focus survey, regarding the expected inflation for the next four quarters (the average over the quarter of the expectations for the next twelve months). It is NOT an "rational", "adaptative" etc expectation
$\hat{\pi}_t^{*}$	imported from commodities, measured by the deviation of the variation of the Commodity Index – Brazil (IC-Br) in BRL (brazilian reais) in relation to the domestic inflation target.
$\widehat{\Delta e}_t$	deviation of the quarterly variation of the nominal exchange rate (BRL/USD) from its long-term variation as predicted by the purchasing power parity theory (ppp was set to 2% according to Central Bank report).
h_t	GDP gap (unobservable)
$Clima_t^2$	Represents the variable that captures supply shocks from climate anomalies (temperature of pacific ocean – ONI form NOAA).
d_{t-i}^{el}	It is a dummy variable that takes the value 1 when the climate anomaly is positive (El Niño events)
d_{t-i}^{la}	t is a dummy variable that takes the value 1 when the climate anomaly is negative (La Niña events).
$\epsilon_t^{\pi^L}$	Error term

IS curve

$$(2)\quad h_t = \beta_1 h_{t-1} - \beta_2\frac{\hat{r}_{t-1}}{4} - \beta_3\,\widehat{rp}_t + \beta_4 h_t^{*} + s_t^{h}$$

(2.1) $\hat{r}_t = i^e_{t,t+4|t} - \pi^e_{t,t+4|t} - \overline{rr}^{IS}_t$

(2.2) $s^h_t = \beta_5 s^h_{t-1} + \epsilon^h_t$

(2.3) $\overline{rr}^{IS}_t = \overline{rr}^{trend}_t + \widehat{rr}^{IS}_t$

(2.4) $\widehat{rr}^{IS}_t = \widehat{rr}^{IS}_{t-1} + \epsilon^{rr,IS}_t$

$\hat{r}_t$	Real interest gap
$i^e_{t,t+4\|t}$	Expectation of the Selic rate (nominal interest rate) determined by the Focus survey over a twelve-month horizon. It is NOT an "rational", "adaptative" etc expectation
$\overline{rr}^{IS}_t$	Unobservable equilibrium real interest rate
$\widehat{rp}$	It is a measure of the central government's primary balance corrected for outliers and adjusted for the economic cycle, expressed as a deviation from its trend.
h^*_t	Measure of the global output gap relevant to the Brazilian economy
s^h_t	Autoregressive shock
ϵ^h_t	Error term
$\widehat{rr}^{IS}_t$	Component of high persistence
$\epsilon^{rr,IS}_t$	Error term
$\overline{rr}^{trend}_t$	Trend of HP filtered series $i^e_{t,t+4\|t} - \pi^e_{t,t+4\|t}$

Taylor Curve

(3) $i_t = \theta_1 i_{t-1} + \theta_2 i_{t-2} + (1-\theta_1-\theta_2)\left[\overline{rr}^{taylor}_t + \pi^{meta}_t + \theta_3\left(\pi^e_{t,t+4|t} - \pi^{meta}_t\right)\right] + \epsilon^i_t$

(3.1) $\overline{rr}^{taylor}_t = \overline{rr}^{trend}_t + \widehat{rr}^{taylor}_t$

(3.2) $\widehat{rr}^{taylor}_t = \widehat{rr}^{taylor}_{t-1} + \epsilon^{rr,taylor}_t$

i_t	Represents nominal Selic rate annualized
π^{meta}_t	Represents inflation target annualized
ϵ^i_t	Error term
$\overline{rr}^{taylor}_t$	This term is unobservable and is analogous to equilibrium real interest rate.
$\widehat{rr}^{taylor}_t$	Random walk
$\epsilon^{rr,taylor}_t$	Error term

Uncovered Interest Parity

(4) $\Delta e_t = \Delta e^{ppc}_t - \delta\left(i^{dif}_t - i^{dif}_{t-1}\right) + \epsilon^e_t$

(4.1) $i^{dif}_t = i_t - (i^*_t + CDS_t)$

(4.2) $\Delta e^{ppc}_t = (\pi^{meta}_t - \pi^{*ss})/4$

Δe_t^{ppc}	Expected variation in the exchange rate in the long term follows Purchasing Power Parity.
π^{*ss}	External equilibrium inflation
i_t^{dif}	omestic and external interest rate differential
CDS	Risk premium, measured in this estimation by the Credit Default Swap
i_t^*	Fed funds rate

Inflation expectations

$$(5) \quad \pi^e_{t,t+4|t} = \varphi_1 \pi^e_{t-1,t+3|t-1} + \varphi_2 E_t \pi_{t,t+4} + \varphi_3 \sum_{i=1}^{4} \pi^{IPCA}_{t-i} + (1 - \varphi_1 - \varphi_2 - \varphi_3)\pi^{meta}_{t} + \epsilon^e_t$$

$E_t\pi_{t,t+4}$	Represents the expectation of inflation accumulated over four quarters ahead consistent with the model (model-consistent expectations).
π_t^{IPCA}	quarterly inflation of the IPCA
ϵ_t^e	Error term