Bank capital regulation, the lending channel and business cycles

Longmei Zhang
International Monetary Fund

Stefan Gerlach
EFG Bank

Bank Capital Regulation, the Lending Channel and Business Cycles

Longmei Zhang[1,2]
International Monetary Fund

April 2010

Abstract

This paper develops a Dynamic Stochastic General Equilibrium (DSGE) model to study how the instability of the banking sector can amplify and propagate business cycles. The model builds on Bernanke, Gertler and Gilchrist (BGG) (1999), who consider credit demand friction due to agency cost, but it deviates from BGG in that financial intermediaries have to share aggregate risk with entrepreneurs, and therefore bear uncertainty in their loan portfolios. Unexpected aggregate shocks will drive loan default rate away from expected, and have an impact on both firm and bank's balance sheets via the financial contract. In economic down turn, in addition to credit demand contraction induced by low firm net worth, low bank capital position can create strong credit supply contraction, and have a quantitatively significant effect on business cycle dynamics.

JEL classification: E32, E44, E52

Key words: Bank capital regulation; banking instability; financial friction; business cycles

[1] *Address for correspondence*: International Monetary Fund, 700 19th Street, N.W. Washington, D.C. 20431, U.S. *E-mail*: lzhang2@imf.org

[2] The author thanks the Deutsche Bundesbank for research support and seminar participants at Deutsche Bundesbank and Hong Kong Monetary Authority, the 7th workshop "Monetary and Financial Economics", MOOD Workshop, Joint BIS/ECB workshop on "Monetary Policy and Financial Stability" for discussions, thanks Stefan Gerlach, Hans Genberg, Simon Gilchrist, Heinz Herrmann, Frank Smets, Tommaso Monacelli, Eric Van Wincoop, Alexander L. Wolman, and Haibin Zhu for useful comments. The author is especially grateful to Thomas Laubach, Falko Fecht, and Alex Ho for detailed discussions.

1 Introduction

Financial frictions have long been ignored in the literature on business cycles. The main theoretical justification for this omission is the Modigliani-Miller proposition, which implies that financial structure is irrelevant for real economic outcomes. However, the ongoing global financial crisis has demonstrated that financial conditions play a central role in determining how real shocks are transmitted through the economy and has shown that financial disturbances can be a source of economic fluctuations. Moreover, there are many historical episodes in which distressed banking systems and adverse credit market conditions have triggered or contributed to serious macroeconomic contractions.[3] Yet in the canonical Dynamic Stochastic General Equilibrium (DSGE) models, there are no financial sector and consequently no financial shocks. Recently, a number of authors have sought to incorporate banking sector and related credit market frictions into DSGE models to study the interaction between the real economy and the financial sector. This paper is part of that effort.

Generally speaking, there are two aspects in integrating credit market frictions: one is credit frictions from the demand side, and the other is from the supply side. Earlier work by Bernanke and Gertler (1989) and Bernanke et al. (1999) (hereinafter BGG) investigated the role of credit demand friction, as a result of asymmetric information between the borrower and the lender. Their model established a link between firms' borrowing costs and net worth. In an economic downturn, firms' leverage ratio increases, causing them to face a higher external finance premium because information asymmetry is exacerbated, reducing capital demand. The drop in capital demand reinforces the initial decline of firms' net worth and the business cycle is propagated. This mechanism is known in the literature as the "financial accelerator". However, the financial friction coming from the credit supply side or the vulnerability of the financial intermediary itself has not been incorporated into DSGE models. Recent authors have tried to link the financial structure of banks to their lending rate to motivate the role of bank capital (e.g. Markovic (2006), Aguiar and Drumond (2007)) or have modeled the function of banks in a detailed manner (Gerali et al. (2009), Christiano et al. (2007)). However, by using financial contracts that insulate banks from aggregate shocks, previous models have avoided the key issue of linking systemic risk to banks' balance sheet and the related banking instability, which is then passed on to the macro economy through the credit market. This linkage is shown to be of critical importance in the current crises.

This paper focus on financial structure of banks and related credit supply frictions. The basic model is a closed economy DSGE model similar to BGG. Key deviations from the basic model is integrating a financial contract where borrowers and lenders share systemic risk. At the end of each period, a loan contract is signed based on two parties' expectation of future economic condition. Contractionary aggregate shocks in the next period

[3]Including past crisis in Scandinavia, Latin America, Japan, and other East Asian countries.

will lead to higher than expected loan default rate, and will therefore not only influence firm's balance sheet, i.e. net worth, but also bank's balance sheet, or capital position, as they face large write-offs from the unexpected loan losses. This is in contrast with the BGG model, in which banks issue state-contingent contract with entrepreneurs, therefore returns on their loan portfolio are indepedent of any aggregate shocks. Given its initial capital position, banks face the trade-off between increasing asset size and higher funding cost, as households perceive lower capital-asset ratio as more instable financial structure and charge a higher premium for holding banks' equity. Therefore, in economic downturn, in addition to the credit demand friction induced by low firm net worth, as captured in a financial accelerator, low bank capital position also gives rise to strong credit supply friction. Credit frictions from both sides will interact and reinforce each other, and drive the economy down further.

Model simulations show that instability of the banking sector alone can create strong credit supply frictions and can amplify and propagate short-run cycles significantly. Shocks that originate from the banking sector, e.g. a sudden decline in bank capital, can lead to strong contraction in the real economy. In the long run, instability of the banking sector implies a lower capital stock in the economy and therefore a lower level of investment and output.

This paper also compares the relative contribution of various frictions in shock transmission. Three cases are considered. In the first case, only nominal rigidities and capital adjustment cost are considered; in the second case, a financial accelerator effect is added; in the third case, the bank balance sheet channel is incorporated. Model simulations show that the bank capital channel is more important than the financial accelerator in amplifying policy shocks. This is consistent with previous findings in the literature that the financial accelerator contributes only marginally to monetary policy transmission.[4]However, the relative importance of the two channels is reversed when a positive technology shock hits the economy, when strong corporate balance sheets play an important role in driving up asset prices and increasing aggregate investment.

The model can also explain the long-established puzzle that aggregate lending does not decline immediately following a contractionary monetary policy shock but increases for four to six quarters and then falls. (Christiano et al. (1996)). The mechanism behind this phenomenon is that firm net worth contracts faster than asset prices in the initial period following a negative policy shock, and that therefore firms have to rely more on external financing. In the following period, contraction of firm net worth slows down, while asset price is declining faster, so that firm's external borrowing declines.

This paper is also related to the banking literature which focus on fragility of financial intermediaries. Representative work from Diamond and Dybvig (1983) shows the inherent

[4]See Meier and Mueller (2006), Christensen and Dib (2008).

instability coming from the bank liabilities side. Given the explicit or implicit government guarantee on bank deposit, this problem has been mitigated to a large extent. This paper shows the banking instability arising from the asset side. i.e., although banks can diversify away idiosyncratic shocks by holding a large loan portfolio, they are still vulnerable to any systemic risk. Another key difference is that in this model, financial instability is driven by fundamentals rather than pure self-fulfilling expectations.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 describes the calibration strategy. Section 4 discusses the effect of the bank capital channel on long-run steady states and short-run dynamics. Section 5 concludes.

2 Model

The economy is inhabited by four types of agents: households, entrepreneurs, retailers and bankers. The structure of the basic model is the following: Bankers raise equity and deposit from the households, and then intermediate these funds to the entrepreneurs. Entrepreneurs combine their own net worth and the money they borrowed from banks to purchase physical capital, which will be used in aggregate production together with labor supplied by households. The product will then be differentiated in the retail sector to become final goods, which is either invested or consumed by the agents. Nominal Return on risk-free assets (i.e.,deposit) is set by the central bank, who conducts monetary policy following a Taylor rule. Banks are subject to regulatory requirement on minimum capital ratio.

Next, we will present a financial contract where borrowers and lenders share aggregate risk, and then integrate it into a general equilibrium.

2.1 The Financial Contract

In this part, we discuss the design of an optimal financial contract between entrepreneurs and banks, which is the key deviation of our model to the original BGG model. The contract is derived in a partial equilibrium setting, taking the price of capital goods, entrepreneurs' net worth, the cost of deposits and bank capital as given. We then imbed the optimal contract in the general equilibrium setting.

There are two parties to the contract: an entrepreneur with net worth and a financial intermediary, which we call "bank". Bank takes deposit from and issue equity to households to finance the loan demanded by entrepreneurs. We will discuss the detail of the banking sector later. Both parties are assumed to be risk-neutral. At the end of period t, a continuum of entrepreneurs (indexed by $i \in (0, 1)$) need to purchase capital for production at $t+1$. The quantity of capital purchased by entrepreneur i is denoted K^i_{t+1}. The price of capital in period t is q_t (in real term). The return on capital is subject to both idiosyncratic and aggregate risk. The *ex-post* gross return to entrepreneur i is $\omega^i_{t+1} R^k_{t+1}$, where ω^i_{t+1} is an idiosyncratic productivity shock to entrepreneur i, and R^k_{t+1} is the *ex-post* aggregate rate of return on capital. ω^i_{t+1} is identically and independently distributed (across time and entrepreneurs) with log-normal distribution and unit mean.

To finance the purchase of capital, entrepreneurs use internal funds (net worth) and borrow the rest from a bank. Let N^i_{t+1} denote the net worth of entrepreneur i at the end of period of t; then borrows from the bank is the following:

$$L^i_{t+1} = q_t K^i_{t+1} - N^i_{t+1} \tag{1}$$

ω^i_{t+1} is private information to entrepreneur i and the bank has to pay a monitoring cost to observe it. Entreprenuers observe the realization of ω^i_{t+1} and decide whether to repay the debt or default. If they repay the debt, they pay $R^L_{t+1} L_{t+1}$. R^L_{t+1} is the gross loan rate specified in the contract that the entrepreneru need to pay to the bank. It can be fixed or state-contingent. If they default, the bank seizes the entrepreneur's remaining assets after paying the monitoring cost.[5] For a particular value of R^K_{t+1}, there is a corresponding cut-off value of idiosyncratic productivity $\overline{\omega}^i_{t+1}$, such that, if the realization of the idiosyncratic productivity falls below it, the entrepreneur defaults. That is:

$$\overline{\omega}^i_{t+1} R^k_{t+1} q_t K^i_{t+1} = R^L_{t+1} L^i_{t+1} \tag{2}$$

The monitoring cost is assumed to equal a proportion μ of the realized gross capital return $\omega^i_{t+1} R^k_{t+1} q_t K^i_{t+1}$. Parameter μ captures the degree of monitoring cost or information asymmetry. [6]

In BGG, entrepreneurs are assumed to bear all the aggregate risk. By issuing state-contingent loan contract, banks are insulated from aggregate shocks and always obtain risk-free rate of return on loan portfolios . The optimal contract, as a result, maximizes the expected return to entrepreneurs as following:

$$\max E_t \left\{ \int_{\overline{\omega}^i_{t+1}}^{\infty} \omega^i_{t+1} R^k_{t+1} q_t K^i_{t+1} f(\omega^i_{t+1}) d\omega - (1 - F(\overline{\omega}^i_{t+1})) \overline{\omega}^i_{t+1} R^k_{t+1} q_t K^i_{t+1} \right\} \tag{3}$$

where expectations are taken with respect to the random variable R^k_{t+1}, and $\overline{\omega}^i_{t+1}$ is a function of realization of R^k_{t+1} (and therefore, function of the states). f(.) and F(.) are respectively the density function and the cumulative distribution function of the random variable ω. The optimal contract must observe the participation constraints of the bank

[5] see Townsend (1979) and Gale and Hellwig (1985) .

[6] The existence of the banking sector in this paper is taken as given. It could also be motivated by assuming that banks have information advantage compared to households in monitoring the project outcome, i.e. $\mu_{bank} < \mu_{households}$.

as well, such that, for each possible realization of states of nature (and therefore, R^k_{t+1} and $\overline{\omega}^i_{t+1}$) the contract satisfies:

$$(1 - F(\overline{\omega}^i_{t+1}))R^L_{t+1}L^i_{t+1} + (1-\mu)\int_0^{\overline{\omega}^i_{t+1}} \omega E_t R^k_{t+1} q_t K^i_{t+1} f(\omega) d\omega = R^f_{t+1} L^i_{t+1} \tag{4}$$

In equation (4), the left hand side shows that banks' return on the loan portfolio has two components: the loan amount that is paid back by the entrepreneurs, and, in the default case, the acquisition of the firm' remaining assets after paying off the monitoring cost. R^f_{t+1} is the funding cost of the bank, which will be determined in the general equilibrium. Since the participation constraints hold for each realization of R^k_{t+1}, banks face no uncertainty in the return on loan portfolio, which equals to the risk-free rate.

The risk-sharing rule among entrepreneurs and banks is a bit stylized, nonetheless.[7] In reality, banks face great uncertainty in their loan portfolio. The major source of uncertainty is shocks to default risk. To account for this, *we assume that aggregate risk is shared between banks and entrepreneurs. The financial contract cannot be therefore contingent on the realized capital return but has to be written based on the two parties' expectation of capital return in the next period.*[8] Under this risk sharing rule, we have to make a distinction between the ex-post loan default threshold $\overline{\omega}^{i,b}_{t+1}$ and the ex-ante $\overline{\omega}^{i,a}_{t+1}$.

Let $E_t R^k_{t+1}$ denote the expected capital return at the end of period t. *We assume that the entrepreneur can only offer the contract based on $E_t R^k_{t+1}$ instead of all possible realizations of R^k_{t+1}.* [9]The contract maximizes the expected return of the entrepreneur as following:

$$\int_{\overline{\omega}^{i,a}_{t+1}}^{\infty} \omega^i_{t+1} E_t R^k_{t+1} q_t K^i_{t+1} f(\omega^i_{t+1}) d\omega - (1 - F(\overline{\omega}^{i,a}_{t+1}))\overline{\omega}^{i,a}_{t+1} E_t R^k_{t+1} q_t K^i_{t+1} \tag{5}$$

where $\overline{\omega}^{i,a}_{t+1}$ is the cut-off idiosyncratic productivity that the entrepreneur is *expected* to default in period t+1 based on information up to period t. Correspondingly, the participation constraint of banks is also based on $E_t R^k_{t+1}$:

$$(1 - F(\overline{\omega}^{i,a}_{t+1}))R^L_{t+1}L^i_{t+1} + (1-\mu)\int_0^{\overline{\omega}^{i,a}_{t+1}} \omega^i_{t+1} E_t R^k_{t+1} q_t K^i_{t+1} f(\omega^i_{t+1}) d\omega = R^f_{t+1} L^i_{t+1} \tag{6}$$

By solving the contract we obtain the credit demand equation (see Appendix A):

$$E_t R^k_{t+1} = S(\frac{q_t K^i_{t+1}}{N^i_{t+1}}) R^f_{t+1} \tag{7}$$

[7]See footnote 10 in their paper.

[8]A state-contingent contract could be prevented by assuming that the state of the economy is not observed by the enforcement of the contract, but only observed at the very end of the period when people form expectations for the next period.

[9]Our assumption actually simplifies the characterization of the financial contract, as it corresponds to the problem of solving one case of no aggregate risk in the original BGG.

The property and interpretation of S(.) is identical to BGG, where S denotes the external finance premium, which captures the wedge (driven by the existence of monitoring cost) between the cost of finance from the firm's side and the cost of funds from the bank's side. $S' > 0$, implying that the higher is the leverage ratio of firms, the higher is the external finance premium.

After solving the optimal contract, the contractual lending rate could be derived as

$$R_{t+1}^{L} = \frac{\overline{\omega}_{t+1}^{i,a} E_t R_{t+1}^{k} q_t K_{t+1}^{i}}{L_{t+1}^{i}} \tag{8}$$

Note that in this model the contractual lending rate is *fixed* and independent to the realizations of the return on capital in t+1, whereas in BGG the lending rate is *state-contingent*:

$$R_{t+1}^{L} = \frac{\overline{\omega}_{t+1}^{i} R_{t+1}^{k} q_t K_{t+1}^{i}}{L_{t+1}^{i}} \tag{9}$$

.

In period t+1, given the specified loan rate R_{t+1}^{L} and the realized return on capital, the ex-post default threshold $\overline{\omega}^{i,b}$ is now determined by:

$$\overline{\omega}_{t+1}^{i,b} = \frac{R_{t+1}^{L} L_{t+1}^{i}}{R_{t+1}^{k} q_t K_{t+1}^{i}} \tag{10}$$

Recall that the expected default threshold is defined by:

$$\overline{\omega}_{t+1}^{i,a} E_t R_{t+1}^{k} q_t K_{t+1}^{i} = R_{t+1}^{L} L_{t+1}^{i} \tag{11}$$

This implies:

$$\overline{\omega}_{t+1}^{i,b} = \frac{\overline{\omega}_{t+1}^{i,a} E_t R_{t+1}^{k}}{R_{t+1}^{k}} \tag{12}$$

From this expression, we see that any deviation of the realized capital return from expected one will drive a wedge between ex-post loan default rate and ex-ante. We will discuss its impact on banking sector and aggregate economy later.

2.2 General Equilibrium

In this section, we analyze how can aggregate shocks influence firm and bank's balance sheet via the financial contract in a general equilibrium. In addition to a firm's credit demand curve which is contingent on its net worth (capturing the traditional financial accelerator effect), this model also derives an implicit credit supply curve, which is contingent on bank's capital position.

2.2.1 Households

There is a continuum of households in the economy, each indexed by $i \in (0,1)$. They consume the final good, c_t, invest in risk free bank deposits, d_{t+1}, and bank equity, e_{t+1}, supply labor h_t and own shares in a monopolistically competitive sector that produces differentiated varieties of goods. The households maximize the utility function:[10]

$$\max E_t \sum_{k=0}^{\infty} \beta^k [\ln(c_{t+k}) + \frac{d_{t+k+1}^{1+\varphi}}{1+\varphi} + \rho \ln(1 - h_{t+k})] \tag{13}$$

subject to the sequence of budget constraints:

$$d_{t+1} + e_{t+1} + c_t = w_t h_t + R_t^d d_t + R_t^e (1 - \phi_t) e_t + \Pi_t \tag{14}$$

d_{t+1} and e_{t+1} are deposits and bank equity(in real terms) held by the household from t to t+1. R_t^d and R_t^e reflect the gross real return on holding deposit and bank equity, and ϕ_t is the default rate on bank capital. h_t is household labor supply, w_t is the real wage for household labor, Π_t is dividends received from ownership of retail firms. Following Van den Heuvel (2008), the liquidity services of bank deposits are modeled by assuming that the household has a derived utility function that is increasing in the amount of deposits. The households'optimization problem yields following first-order conditions:

$$U_c(c_t) = \beta E_t R_{t+1}^e (1 - \phi_{t+1}) U_c(c_{t+1}) \tag{15}$$

$$U_c(c_t) - U_d(d_{t+1}) = \beta E_t R_{t+1}^d U_c(c_{t+1}) \tag{16}$$

$$-U_{c,t}/U_{h,t} = w_t \tag{17}$$

Equation (15) shows that households' intertemporal consumption decisions are determined by the default-adjusted return on holding bank equity. Equation (16) shows the optimality condition on bank deposit. Equation (17) describes the usual trade-off between consumption and leisure. In the model set up, bank equity has to offer higher return than deposit for two reasons: the first is the liquidity premium, since deposits can provide households extra utility in addition to carry a monetary reward; the second is to compensate for the default risk. As will be discussed later, banks will be shut down and default on capital return when their capital ratios fall below the regulatory threshold.[11]

2.2.2 Entrepreneurs

Other than difference in the financial contract, the entrepreneur sector at the aggregate level is identical to the BGG. We describe the entrepreneur sector for completeness purpose

[10] Inserting deposits into the utility function is just a modeling device to capture the bank' liquidity creation function. Model dynamics are robust if we consider a standard utility function with only consumption and leisure.

[11] This paper assumes a relationship between households and bankers as delegated monitoring. Therefore, households do not care about the capital structure of banks in their decision.

below. After signing the financial contract, entrepreneurs combine loans acquired from the bank and their own net worth to purchase capital. They use capital and labor to produce wholesale goods and sell them on a perfect competitive market at a price equal to their nominal marginal cost. The aggregate production function is given by :

$$Y_t = A_t K_t^{\alpha_k} (h_t)^{\alpha_h} (h_t^e)^{\alpha_e} (h_t^b)^{\alpha_b} \tag{18}$$

Following BGG, We assumed entrepreneurs and bankers supply one unit of labor services inelastically to the general labor market: $h_t^e = h_t^b = 1$. As will be see later, α_e and α_b are calibrated so that these two additional labor forces have a negligible effect on the output level and model dynamics.[12]

The optimization problem of production remains standard:

$$z_t = \alpha_k mc_t \frac{Y_t}{K_t} \tag{19}$$

$$w_t = \alpha_h mc_t \frac{Y_t}{h_t} \tag{20}$$

$$w_t^e = \alpha_e mc_t \frac{Y_t}{h_t^e} \tag{21}$$

$$w_t^b = \alpha_b mc_t \frac{Y_t}{h_t^b} \tag{22}$$

where z_t is the real rental rate of capital and w_t,w_t^e and w_t^b are, respectively, the real wage of households, entrepreneurs and bankers. mc_t denotes real marginal cost. The expected return on capital is then:

$$E_t R_{t+1}^k = E_t \left(\frac{z_{t+1} + (1-\delta) q_{t+1}}{q_t} \right) \tag{23}$$

The accumulation of entrepreneurs' net worth consists of two parts: profits from operating the firms and labor income. It is assumed that, in every period, entrepreneur will die with the probability $1 - \gamma$. This assumption ensures that entrepreneurs never accumulate enough net worth to finance a project without external financing. Those entrepreneurs who die at time t will consume $(1-\gamma)V_t$. The evolution of aggregate net worth is therefore given by:

$$N_{t+1} = \gamma V_t + w_t^e \tag{24}$$

where V_t represents gross return on operating business. It is the difference between gross capital return and loan payment.

$$V_t = \int_{\overline{\omega}^b}^{\infty} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - (1 - F(\overline{\omega}^b)) R_{t+1}^L L_{t+1}^i \tag{25}$$

[12]The salary that bankers earn from labor supply could be understood as fee income collected from transaction services, a function of financial intermediaries that is not modeled in the paper.

2.2.3 Capital Producers

Capital producers purchase a fraction of final goods from the retailer as investment goods i_t and combine this with the existing capital stock to obtain capital stock in the next period. A quadratic capital adjustment cost is included to motivate a variable price of capital, which contributes to the volatility of firm net worth and bank capital. Capital producers will choose the quantity of investment goods to maximize profit subject to the adjustment cost:

$$\max E_t \left[q_t i_t - i_t - \frac{\chi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t \right] \tag{26}$$

where q_t is the real price of capital. The optimization problem yields the following capital supply curve:

$$q_t = 1 + \chi(\frac{i_t}{k_t} - \delta) \tag{27}$$

where χ captures the sensitivity of capital price to investment fluctuation. The higher χ is, the more volatile the price of capital. The aggregate capital stock evolves according to:

$$k_{t+1} = i_t + (1-\delta)k_t \tag{28}$$

where δ is the depreciation rate.

2.2.4 Banking Sector

The banks' equity value is accumulated through retained earnings:

$$\begin{aligned} e_{t+1} = (1-\phi_t)e_t + [R^L_{t+1} L_{t+1}(1 - F(\overline{\omega}^b)) \\ +(1-\mu)\int_0^{\overline{\omega}^b} \omega R^k_{t+1} q_t K_{t+1} f(\omega) d\omega - R^f_{t+1} L_{t+1}] + w^b_t \end{aligned}$$

where ϕ_t is the bank default rate, which will be explained in the bank regulation section. Aggregate bank equity at time t+1 consists of three parts: $(1-\phi_t)e_t$ is equity from those banks who did not default at time t; the term inside the square bracket is unexpected gains or losses in the loan portfolio; w^b_t is bankers' wages.
Substituting equation (6) into the above equation, we get:

$$\begin{aligned} e_{t+1} = (1-\phi_t)e_t + R^L_{t+1} L_{t+1}(F(\overline{\omega}^a) - F(\overline{\omega}^b)) \\ +(1-\mu)\int_0^{\overline{\omega}^b} \omega R^k_{t+1} q_t K_{t+1} f(\omega) d\omega \\ -(1-\mu)\int_0^{\overline{\omega}^a} \omega E_t R^k_{t+1} q_t K_{t+1} f(\omega) d\omega + w^b_t \end{aligned}$$

Notice from the financial contract, we have derived following relationship between loan default threshold and aggregate capital return:

$$\overline{\omega}^b_{t+1} = \frac{\overline{\omega}^a_{t+1} E_t R^k_{t+1}}{R^k_{t+1}} \tag{29}$$

Consider the case when a contractionary shock hits the economy, which reduces realized capital return R^k_{t+1} below the expected value $E_t R^k_{t+1}$. This will lead to higher ex-post loan default threshold $\overline{\omega}^b_{t+1}$ than expected $\overline{\omega}^a_{t+1}$, correspondingly higher loan default rate $F(\overline{\omega}^b)$ than anticipated $F(\overline{\omega}^a)$, and creat unexpected losses $R^L_{t+1}L_{t+1}(F(\overline{\omega}^a) - F(\overline{\omega}^b))$ that write down bank's capital position. This is the key difference of our model from the original BGG model in terms of shocks transmission. In the BGG setting, all aggregate shocks are absorbed by firms' balance sheets; while in our model, aggregate shocks are absorbed partly by firms' balance sheets and partly by banks' balance sheets via the financial contract.

Given the aggregate loan size, L_t, and bank equity, we obtain the aggregate capital ratio:

$$\Delta_t = \frac{e_t}{L_t} \tag{30}$$

The rest of bank funding

$$d_t = L_t - e_t \tag{31}$$

will be collected from the households in the form of deposits. Therefore, from an aggregate level, the opportunity cost of bank funding is a linear combination of cost of bank equity and cost of deposits, where the proportion of each type of funding varies according to the bank capital ratio.

$$R^f_{t+1} = \Delta_t R^e_{t+1} + (1 - \Delta_t) R^d_{t+1} \tag{32}$$

The respective costs of deposits R^d_{t+1} and equity R^e_{t+1} are derived endogenously from households' optimization problem.

Bank regulation In modern banking regulation, capital requirement has become the focal point.[13] Given the implicit or explicit government guarantee on bank deposit, bank capital regulation is imposed to curb banks' excessive risk-taking. In 1987, the Basel Commitee of Banking Supervision established the Basel I Accord, which provided a uniform capital standard for all banks in the member countries. Basel I required the ratio of banks' capital to risk-weighted assets to amount to a minimum of 8 percent, with at least 50 percent of it being tier 1 capital. By 1993, nearly all of the world's big banks satisfied the Basel capital requirement. Many of them have been increasing their capital ratio. Figure 1 presents a histogram of the risk-based total capital ratios of U.S. commercial banks in the fourth quarter of 2000. As we can see from the figure, capital ratios vary across banks, with most of them between 10 and 11 percent, and very few below 10 percent.

[13] In this paper, bank capital regulation is taken as given, instead of being motivated from a micro perspective. It could be understood to mean that the threshold requirement is set to keep the government or the central bank from having to shoulder the burden of massive bank failures.

Motivated by this empirical observation, the capital ratio across banks in the model is assumed to have *log-normal* distribution. The mode of the distribution is given by the aggregate capital ratio derived above. $\Delta_{i,t}$ log-normal (Δ_t, σ).[14] The health of the banking sector as a whole will depend largely on the variation of aggregate capital ratio. With a higher aggregate ratio, the distribution moves to the right, and fewer banks will fall short of the 8 percent threshold and thus default, and vice versa. The default probability is given by the cumulative distribution function up to the regulatory threshold:[15]

$$\phi_t = cdf(\Delta_t, \sigma) \tag{33}$$

The higher the default probability, the more it costs banks to raise equity. Therefore, a low capital position today will lead to higher equity costs in the next period. This increase in funding costs will dampen banks' incentive to supply credit, and reduce aggregate investment.

By contrast, in the BGG model, banks' funding costs are independent of banks' capital structure, and is always equal to the risk free rate. In economic downturns, even though large loan losses lead to a weak capital position, funding costs remain the same, as households do not charge a risk premium for the increased banking instability; therefore, there is no amplification effect of business cycles from banks.

2.2.5 Retail Sector

The retail sector is introduced into the model to motivate sticky prices. We assume monopolistic competition and Calvo pricing. Retailers purchase the wholesale good from entrepreneurs at a price equal to its nominal marginal cost and differentiate them at no cost. They then sell these differentiated retail goods in a monopolistically competitive market. Let $Y_t(i)$ be the quantity of output sold by retailer i, measured in units of wholesale goods, and let $P_t(i)$ be the nominal price. Total final usable goods Y_t are the following composite of retail goods:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)} \tag{34}$$

with $\epsilon \geq 1$ representing the degree of monopolistic competition. The corresponding price index is given by

$$P_t = \left[\int_0^1 P_t(i)^{(1-\epsilon)} di \right]^{1/(1-\epsilon)} \tag{35}$$

[14]The conditional distribution of bank capital ratio could be derived endogenously from the bank equity accumulation equation. For simplicity, in the simulation only the mean of the distribution is used, while the variance is assumed constant. As Krusell and Smith (1998) has shown, the behavior of the macroeconomic aggregates can be described almost perfectly using only the mean of the wealth distribution.

[15]Since banks that fall below the regulatory threshold cannot make new loans, they exit from the industry. Note that the default case in this model is benign, i.e. banks default because of bad fundamentals. Irrational bank runs caused purely by shifts in people's expectations are not considered here.

Following Calvo (1983), in a given period the retailer receives the signal to adjust the price with probability $1-\theta$ and otherwise has to maintain the previous price. Let $P_t^*(i)$ denote the price set by retailers who are able to change price at t, and $Y_t^*(i)$ the demand given this price. The retailer will thus choose this price to maximize future expected discounted real profits, given by:

$$\max E_t \sum_{k=0}^{\infty} \left[\theta^k \Lambda_{t,k} \Omega_{t+k}(i)/P_{t+k}\right] \tag{36}$$

subject to the demand function

$$Y_{t+k}^*(i) = \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \tag{37}$$

where the discount rate $\Lambda_{t,k} = \beta^k C_t/C_{t+k}$ (given assumed log utility in consumption) is the household intertemporal marginal rate of substitution, which the retailer takes as given. Ω_{t+k} is nominal profits given by $(P_t^*(i) - MC_{t+k})Y_{t+k}^*(i)$. The optimization problem yields the following condition:

$$P_t^*(i) = \frac{\theta}{\theta-1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} MC_{t+k}(i) Y_{t+k}^*(i)/P_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} Y_{t+k}(i)/P_{t+k}} \tag{38}$$

Given that the share θ of retailers do not change their price in period t, the aggregate price evolves according to:

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{39}$$

Combining the optimal pricing and the evolution of aggregate price and then log-linearizing, we obtain a standard Phillips curve where $\hat{mc}_t$ represents the real marginal cost gap.

$$\beta E_t \pi_{t+1} = \pi_t - (1-\beta\theta)\frac{1-\theta}{\theta}\hat{mc}_t \tag{40}$$

2.2.6 Monetary Policy

To facilitate comparison with previous models, we assume a simple rule according to which the central bank adjusts the current nominal interest rate in response to the lagged inflation rate and the lagged interest rate.

$$r_t^n = \rho_r r_{t-1}^n + \rho_\pi \pi_{t-1} + \epsilon_t \tag{41}$$

3 Calibration

In the household utility function, ρ is chosen so that steady-state labor is 0.3. φ is calibrated so that the steady-state liquidity premium is 380 bp on an annual basis. β is calibrated at 0.983..

In the aggregate production function, the capital share is 0.33, the share of household labor is 0.66, the share of entrepreneur labor is 0.00956 and the share of banking labor is 0.00044. Capital depreciates at 2.5 percent quarterly. Capital adjustment parameter χ is calibrated at 2 based on the estimates in Chirinko (1993).

In the retail sector, the degree of monopolistic competition ϵ is calibrated at 6, which implies a steady-state mark-up of 20 percent. The Calvo probability that a firm does not change price in a given period θ is set to 0.75, which implies that prices in the economy are adjusted every four quarters on average. In monetary policy, the autoregressive coefficient is set to 0.65 and the coefficient of lagged inflation 1.2. These calibrations are standard in the literature.

In the financial contract, the monitoring cost parameter μ is set to 0.12, following BGG 1999. The probability that entrepreneurs die in a given period $1-\gamma$ is set to 0.019. The variance of idiosyncratic productivity is set to 0.265. These parameterizations lead to a capital-to-net worth ratio of 2 (leverage ratio of 0.5), an annual loan default rate of 2.56 percent and an annual external finance premium of 180 bp. In the distribution of the bank capital ratio, the steady-state ratio is calibrated at 10 percent and the variance of the distribution is set to match a steady-state annual bank default rate of 1 percent. Based on Dimson et al. (2002), Annualised return on equity is calibrated at 5.8 percent and return on deposit 1 percent.

Based on King and Rebelo (1999), the aggregate productivity shock follows an AR (1) process, with a coefficient of 0.9 and a standard deviation of 0.0056.

4 Simulation

Technology shocks, monetary policy shocks and financial shocks are considered in the simulation. First, the impulse responses to shocks are analyzed; then the model is compared with a model where the only financial friction comes from the credit demand side and with a baseline model with no financial friction. The marginal contributions of the bank capital channel to the long-run steady state and short-run dynamics are studied.

4.1 Technology shocks

Figure 2 and 3 display impulse reponses to a positive technology shock with the size of one standard deviation. After a positive technology shock, the realized capital return is higher than expected, leading to lower than expected loan default rate. This generates unexpected gain on the loan portfolio, which strengthens banks' capital position. Given the improvement in banks' balance sheets, households expect a lower bank default rate in the next period and are therefore willing to hold bank capital at lower rates of return. The reduction in the cost of funding from the banks' side expands credit supply and drive up investment in equilibrium. On the other hand, after a positive technology shock, firms' net worth increases and leverage ratios decline, causing them to face lower agency costs in the credit market and enabling them to obtain loans at lower external finance premiums. The positive reaction from both the credit supply and credit demand side drive up aggregate lending to a large extent, which implies an investment boom. This raises output, consumption, and asset prices. The marginal cost of production falls after productivity increases; therefore, inflation falls.[16]

4.2 Monetary policy shocks

Figure 4 and 5 show impulse reponses to an unanticipated twenty-five basis point increase in the policy rate. After a monetary policy tightening, the cost of deposits rises, bank credit supply declines, ex-post loan default rate goes up. The unexpected loss in loan portfolio will write off bank's capital. The deterioration in banks' balance sheets will lead households to demand higher returns for holding bank capital in the next period. The difficulty in raising capital will further depress banks' credit supply and propagate the monetary policy shock. On the other hand, the net worth of entrepreneurs falls, the leverage ratio rises. This makes them look less attractive in the credit market and forces them to pay a higher external finance premium. *Note that, despite the contraction in both credit supply and credit demand, the aggregate lending rises for about four to six quarters and then falls.* This behavior has been well documented in empirical studies. Christiano et al. (1996) argue that "following a contractionary shock to monetary policy, net funds raised by the business sector increases for roughly a year, and then fall". Recall aggregate lending is determined by: $L_{t+1} = q_t K_{t+1} - N_{t+1}$. The reason for the temporary increase in the loan amount is that, after a monetary policy tightening, there is contraction in firm net worth, capital stock and asset prices. The adjustment speed of capital is low; therefore, the change in aggregate lending depends on the adjustment speed of net worth and asset prices. Since at the beginning net worth decreases much faster than the asset price, the firm has to borrow more external funds to finance a reduced amount of investment. In the following period, contraction of firm net worth slows down, while asset price is declining faster, firm's external borrowing therefore goes down. The rest of dynamics are standard: after interest rates are increased, inflation and consumption fall. Contraction of investment and consumption reduces the output level.

4.3 Financial shock

Figure 6 depicts the model dynamics after a negative shock to bank capital. Assume that there is an exogenous deterioration of bank's balance sheet and therefore a sudden drop of bank capital, possibly due to the burst of an asset price bubble, which leads to larger write-offs of bank equity compared to the case where asset swing is only driven by fundamental

[16]In all the graphs in the simulation part, the X-axis represents the number of quarters after shocks hit the economy, the Y-axis represents the percentage point deviation from the steady state value.

as modeled in this paper. From the simulation we can see that, a sudden drop of bank's capital position leads to strong contraction in bank's credit supply. We observe a decrease in aggregate lending and an increase in credit spreads. Tightening of credit market leads to dampened aggregate investment, which further deteriorates firm's balance sheet, loan default rate goes up. Weak aggregate demand leads to both low output and inflation.

4.4 Model Comparison: Marginal Effect of Banking Instability

Next, we compare this model with a model where only the BGG type of financial friction exists as well as with a standard model with no financial friction. The results show that banking instability can lead to lower capital stock and investment in the long run and have an acceleration effect on the short-run dynamics of the model.

4.4.1 Long-run effect

Table 1 displays steady states of model economy with different frictions. In the long run, instability of the banking sector implies higher bank funding cost, compared to the risk-free rate in BGG model. Given the increased funding cost, banks are only willing to finance project with higher return. Since the marginal return on capital is decreasing at the aggregate level, this implies a lower capital stock in the equilibrium, and therefore lower investment, output and consumption level.

4.4.2 Short-run effect

Figure 5 and 6 compare the relative importance of various frictions in shock transmission. The dashdot line describes impulse responses in a standard DSGE model, where only nominal rigidity and capital adjustment costs are considered. The dashed line incorporates the additional friction coming from the credit demand side, or the financial accelerator effect. The solid line captures the model dynamics where the bank capital channel is added to the previous frictions.

As we can see from the figures, the bank capital channel has a strong acceleration and propagation effect on both the impulse responses to the technology shock and the monetary policy shock. Compared to previous literature (e.g. Markovic (2006)), where the bank capital channel can generate the acceleration effect, but very little propagation effect, as the marginal contribution of credit supply friction vanishes after around 8 quarters following a policy shock. In this model, by introducing bank capital as a state variable, low capital position not only amplifies the cycle, but also creates more persistence of cycles. This corresponds to the real world senario, where after a one-time deteriation of banks' balance sheets, it takes time to repair the balance sheets and restore credit supply.

The most significant effect of bank capital is on investment, asset prices, and credit spreads. The instability in the banking sector introduces extra volatility to these variables, while its impact on output is relatively minor. This is because consumption, which accounts for 80 percent of output in the model calibration, is not strongly subject to the influence of banking instability. If consumer loan is incoporated, bank capital channel will have a much larger effect on the consumption level, and therefore a more significant impact on output.

Another observation from figure 5 is that the bank capital channel is more important than the financial accelerator in amplifying policy shocks. This is consistent with previous findings in the literature that the financial accelerator contributes only marginally to monetary policy transmission. However, the relative importance of the two channels is reversed when a positive technology shock hits the economy, where strong corporate balance sheets play an important role in driving up asset prices and aggregate investment.

5 Conclusion

This paper extends a general equilibrium model with a BGG-type financial accelerator to a model in which financial friction coming from both the credit supply and credit demand sides are considered. By integrating a financial contract, in which entrepreneurs and banks share aggregate risk, aggregate shocks will have impact not only on firms' balance sheet, but also banks' balance sheet. In economic downturn, in addition to credit demand friction induced by low firm net worth, this paper shows that low bank capital position also creates strong credit supply friction, and leads the economy to contract further. This bank balance sheet to credit market linkage has been shown to be critical importance in the current crisis.

The extended model enables us to study how real shocks, e.g, technology shocks, monetary shocks, affect the financial sector and how shocks originate in the banking sector can influence the real economy. The model also facilitate us to understand the role of different frictions in shock transmissions.

In future research, this model could be extended to consumer loans. Since consumption is the major component of output, once the feedback from banking instability to consumption is incorporated, the effect on output will be much more significant compared to the corporate-loans-only case. The model could also be extended to an open economy and study how the instability of a financial intermediary in one country could influence the real sector in the other economy.

References

Aguiar, A. and Drumond, I. (2007). Business cycle and bank capital: Monetary policy transmission under the basel accords. FEP Working Paper Series 242.

Bernanke, B. and Gertler, M. (1989). Agency costs, net worth, and business fluctuations. *American Economic Review*, 79:14–31.

Bernanke, B., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. and Woodford, M., editors, *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. North-Holland, Amsterdam.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12:383–398.

Chirinko, R. (1993). Business fixed investment spending: a critical survey of modeling strategies, empirical results, and policy implications. *Journal of Economic Literature*, 31:1875–1911.

Christensen, I. and Dib, A. (2008). The financial accelerator in an estimated new keynesian model. *Review of Economic Dynamics*, 11:155–178.

Christiano, L., Motto, R., and Rostagno, M. (2007). Financial factors in business cycles. Manuscript.

Christiano, L. J., Eichenbaum, M., and Evans, C. (1996). The effects of monetary policy shocks: Evidence from the flow of funds. *Review of Economics & Statistics.*, 78:16–34.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91:401–19.

Dimson, E., Marsh, P., and Staunton, M. (2002). Long-run global capital market returns and risk premia. avaliable at http://papers.ssrn.com.

Gale, D. and Hellwig, M. (1985). Incentive-compatible debt contracts: the one-period problem. *Review of Economic Studies*, 52:647–664.

Gerali, A., Neri, S., Sessa, L., and Signoretti, F. M. (2009). Credit and banking in a dsge model of the euro area. Bank of Italy Economic Working Papers 740.

King, R. G. and Rebelo, S. T. (1999). Resuscitating real business cycles. In Taylor, J. and Woodford, M., editors, *Handbook of Macroeconomics*, chapter 14, pages 927–1007. North-Holland, Amsterdam.

Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106:867–896.

Markovic, B. (2006). Bank capital channels in the monetary transmission mechanism. Bank of England Working Paper No. 313.

Meier, A. and Mueller, G. (2006). Fleshing out the monetary transmission mechanism: Output composition and the role of financial frictions. *Journal of Money, Credit and Banking*, 38:1999–2133.

Townsend, R. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21:265–293.

Van den Heuvel, S. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics*, 55:298–320.

Appendix A: The Financial Contract

In the financial contract, the entrepreneurs maximize expected profit subject to the participation constraint of the bank,

$$\max \int_{\overline{\omega}^{i,a}}^{\infty} \omega E_t R^k_{t+1} q_t K^i_{t+1} f(\omega) d\omega - (1 - F(\overline{\omega}^{i,a}))\overline{\omega}^{i,a} E_t R^k_{t+1} q_t K^i_{t+1}$$

subject to

$$(1 - F(\overline{\omega}^{i,a}))R^L_{t+1} L^i_{t+1} + (1-\mu) \int_0^{\overline{\omega}^{i,a}} \omega E_t R^k_{t+1} q_t K^i_{t+1} f(\omega) d\omega = R^f_{t+1} L^i_{t+1}$$

Recall that

$$\overline{\omega}^{i,a}_t E_t R^k_{t+1} q_t K^i_{t+1} = R^L_{t+1} L^i_{t+1}$$

The key difference in solving the contract compared to BGG is that, in BGG, the expectation operator is outside the brackets, since ω itself is not fixed but instead contingent on R^k_{t+1}:

$$\max E_t \left\{ \int_{\overline{\omega}^{i,a}}^{\infty} \omega R^k_{t+1} q_t K^i_{t+1} f(\omega) d\omega - (1 - F(\overline{\omega}^{i,a}))\overline{\omega}^{i,a} R^k_{t+1} q_t K^i_{t+1} \right\}$$

subject to

$$(1 - F(\overline{\omega}^{i,a}))R^L_{t+1} L^i_{t+1} + (1-\mu) \int_0^{\overline{\omega}^{i,a}} \omega R^k_{t+1} q_t K^i_{t+1} f(\omega) d\omega = R^f_{t+1} L^i_{t+1}$$

This participation constraint has to hold for each realization of R^k_{t+1}; therefore, $\overline{\omega}^a$ is a function of R^k_{t+1}. By contrast, in our model the participation constraint only holds for $E_t R^k_{t+1}$ and will break down ex-post if the realization of R^k_{t+1} deviates from expectation. Define

$$\Gamma(\overline{\omega}^{i,a}) = \int_{\overline{\omega}^{i,a}}^{\infty} \omega f(\omega) d\omega - (1 - F(\overline{\omega}^{i,a}))\overline{\omega}^{i,a} \tag{A-1}$$

$$G(\overline{\omega}^{i,a}) = \int_0^{\overline{\omega}^{i,a}} \omega f(\omega) d\omega \tag{A-2}$$

The financial contract can then be transformed into

$$\max_{K^i_{t+1}, \overline{\omega}^{i,a}} (1 - \Gamma(\overline{\omega}^{i,a})) E_t R^k_{t+1} q_t K^i_{t+1}$$

subject to

$$(\Gamma(\overline{\omega}^{i,a}) - \mu G(\overline{\omega}^{i,a})) E_t R^k_{t+1} q_t K^i_{t+1} = R^f_{t+1}(q_t K^i_{t+1} - N^i_{t+1})$$

Define external finance premium $s^i = \frac{E_t R^k_{t+1}}{R^f_{t+1}}$ and firm leverage ratio $k^i = \frac{K^i_{t+1}}{N^i_{t+1}}$ and let λ be the Lagrange multiplier on the bank participation constraint. First-order conditions imply that:

$$\lambda = \frac{\Gamma'(\overline{\omega}^{i,a})}{\Gamma'(\overline{\omega}^{i,a}) - \mu G'(\overline{\omega}^{i,a})} \tag{A-3}$$

$$s^i = \frac{\lambda}{1-\Gamma(\overline{\omega}^{i,a}) + \lambda(\Gamma(\overline{\omega}^{i,a}) - \mu G(\overline{\omega}^{i,a}))} \tag{A-4}$$

Combining first-order conditions with the participation constraint enables us to derive a one-to-one relationship between the external finance premium and the cut-off threshold value, as well as a one-to-one relationship between the leverage ratio and the cut-off threshold value:

$$s^i = s(\overline{\omega}^{i,a}) = \frac{\lambda(\overline{\omega}^{i,a})}{1-\Gamma(\overline{\omega}^{i,a}) + \lambda(\overline{\omega}^{i,a})(\Gamma(\overline{\omega}^{i,a}) - \mu G(\overline{\omega}^{i,a}))} \tag{A-5}$$

$$k^i = k(\overline{\omega}^{i,a}) = 1 + \frac{\lambda(\Gamma(\overline{\omega}^{i,a}) - \mu G(\overline{\omega}^{i,a}))}{1-\Gamma(\overline{\omega}^{i,a})} \tag{A-6}$$

Therefore there exists a one-one relationship between the firm leverage ratio and the external finance premium:

$$k^i = \varphi(s^i) \tag{A-7}$$

or $q_t K^i_{t+1} = \varphi(s^i) N^i_{t+1}$. Since the leverage ratio is the same across firms, they pay the same external risk premium s. We can thus easily aggregate this equation, and derive the following:

$$q_t K_{t+1} = \varphi(\frac{E_t R^k_{t+1}}{R^f_{t+1}}) N_{t+1} \tag{A-8}$$

where K_{t+1} and N_{t+1} represent aggregate capital and firm net worth. We can also rewrite this equation into equation (6) in the paper:

$$E_t R^k_{t+1} = s(\frac{q_t K_{t+1}}{N_{t+1}}) R^f_{t+1} \tag{A-9}$$

Appendix B: First-Order Conditions

$$U_c(c_t) = \beta E_t R^e_{t+1}(1-\phi_{t+1})U_c(c_{t+1}) \tag{B-1}$$

$$U_c(c_t) - U_d(d_{t+1}) = \beta E_t R^d_{t+1} U_c(c_{t+1}) \tag{B-2}$$

$$-U_{c,t}/U_{h,t} = w_t \tag{B-3}$$

$$z_t = \alpha_k mc_t \frac{Y_t}{K_t} \tag{B-4}$$

$$w^h_t = \alpha_h mc_t \frac{Y_t}{h_t} \tag{B-5}$$

$$w^e_t = \alpha_e mc_t \frac{Y_t}{h^e_t} \tag{B-6}$$

$$w^b_t = \alpha_h mc_t \frac{Y_t}{h^b_t} \tag{B-7}$$

$$q_t = 1 + \chi(\frac{i_t}{k_t} - \delta) \tag{B-8}$$

$$k_{t+1} = i_t + (1-\delta)k_t \tag{B-9}$$

$$R^k_{t+1} = \frac{z_{t+1} + (1-\delta)q_{t+1}}{q_t} \tag{B-10}$$

$$E_t R^k_{t+1} = S(\frac{q_t K_{t+1}}{N_{t+1}})R^f_{t+1} \tag{B-11}$$

$$R^f_{t+1} = \Delta_t R^e_{t+1} + (1-\Delta_t)R^d_{t+1} \tag{B-12}$$

$$\overline{\omega}^a_{t+1} E_t R^k_{t+1} q_t K_{t+1} = R^L_{t+1} L_{t+1} \tag{B-13}$$

$$\frac{q_t K_{t+1}}{N_{t+1}} = 1 - s(\overline{\omega}^a_{t+1})(\Gamma(\overline{\omega}^a_{t+1}) - \mu G(\overline{\omega}^a_{t+1})) \tag{B-14}$$

$$\overline{\omega}^b_{t+1} = \frac{\overline{\omega}^a_{t+1} E_t R^k_{t+1}}{R^k_{t+1}} \tag{B-15}$$

$$N_{t+1} = \gamma V_t + w^e_t \tag{B-16}$$

$$V_t = \int_{\overline{\omega}^b}^{\infty} \omega R^k_{t+1} q_t K_{t+1} f(\omega) d\omega - (1 - F(\overline{\omega}^b))R^L_{t+1} L^i_{t+1} \tag{B-17}$$

$$\phi_t = cdf(\Delta_t, \sigma) \tag{B-18}$$

$$P^*_t(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} MC_{t+k}(i) Y^*_{t+k}(i)/P_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} Y_{t+k}(i)/P_{t+k}} \tag{B-19}$$

$$
\begin{aligned}
e_{t+1} = (1-\phi_t)e_t + R^L_{t+1}L^i_{t+1}(F(\overline{\omega}^a) - F(\overline{\omega}^b)) \\
+(1-\mu)\int_0^{\overline{\omega}^b} \omega R^k_{t+1} q_t K_{t+1} f(\omega) d\omega \\
-(1-\mu)\int_0^{\overline{\omega}^a} \omega E_t R^k_{t+1} q_t K_{t+1} f(\omega) d\omega + w^b_t
\end{aligned}
\tag{B-20}
$$

Appendix C: The Steady States

$$R^e_{ss}(1-\phi_{ss}) = 1/\beta \tag{C-1}$$

$$\frac{c^\sigma_{ss}}{d^\varphi_{ss}} = R^e_{ss}(1-\phi_{ss}) - R^d_{ss} \tag{C-2}$$

$$\frac{c^{-\sigma}_{ss}}{l^\phi_{ss}} = w^h_{ss} \tag{C-3}$$

$$mc_{ss} = \frac{\theta-1}{\theta} \tag{C-4}$$

$$z_{ss} = \alpha_k mc_{ss}\frac{y_{ss}}{k_{ss}} \tag{C-5}$$

$$w^h_{ss} = \alpha_h mc_{ss}\frac{y_{ss}}{h_{ss}} \tag{C-6}$$

$$w^e_{ss} = \alpha_e mc_{ss}\frac{y_{ss}}{h^e_{ss}} \tag{C-7}$$

$$w^b_{ss} = \alpha_b mc_{ss}\frac{y_{ss}}{h^b_{ss}} \tag{C-8}$$

$$q_{ss} = 1 \tag{C-9}$$

$$i_{ss} = \delta k_{ss} \tag{C-10}$$

$$R^k_{ss} = z_{ss} + 1 - \delta \tag{C-11}$$

$$lev_{ss} = \frac{q_{ss}K_{ss}}{N_{ss}} \tag{C-12}$$

$$R^k_{ss} = S(lev_{ss})R^f_{ss} \tag{C-13}$$

$$R^f_{ss} = \Delta_{ss}R^e_{ss} + (1-\Delta_{ss})R^d_{ss} \tag{C-14}$$

$$\overline{\omega}^a_{ss} = \frac{R^l_{ss}L_{ss}}{R^k_{ss}q_{ss}K_{ss}} \tag{C-15}$$

$$\overline{\omega}^b_{ss} = \overline{\omega}^a_{ss} \tag{C-16}$$

$$lev_{ss} = 1 - s(\overline{\omega}^a_{ss})(\Gamma(\overline{\omega}^a_{ss}) - \mu G(\overline{\omega}^a_{ss})) \tag{C-17}$$

$$N_{ss} = (1-\gamma)w^e_{ss} \tag{C-18}$$

$$\phi_{ss} = pdf(\Delta_{ss}, \sigma) \tag{C-19}$$

$$e_{ss} = (1-\phi_{ss})w^b_{ss} \tag{C-20}$$

$$\pi_{ss} = 1 \tag{C-21}$$

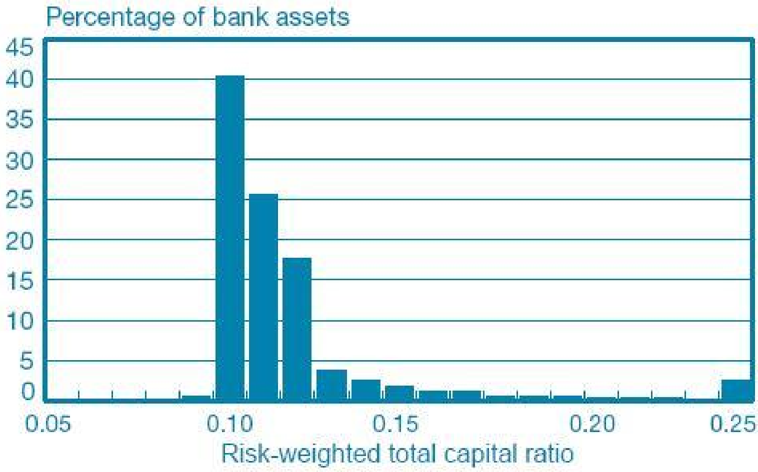
Table 1: Steady states comparison

Variable	Zhang	BGG
Capital	7.1621	7.4116
Investment	0.17905	0.1853
Output	0.86509	0.875
Consumption	0.68604	0.68964

Table 2: Notation of key variables

Symbol	Variable
K	Capital
I	Investment
Y	Output
C	Consumption
d	Bank deposit
e	Bank equity
N	Firm net worth
q	Asset price
L	Loan
R^k	Gross return on capital
R^d	Gross return on bank deposit
R^e	Gross return on bank equity
R^L	Contractual loan rate
ϕ	Bank default rate
$F(\overline{\omega}^a)$	Expected loan default rate
$F(\overline{\omega}^b)$	Realized loan default rate

Figure 1: Distribution of Bank Capital Ratio of U.S. Banks in 2000:4



Source: Federal Reserve Bank of Chicago

Figure 2: Impulse responses to a productivity shock

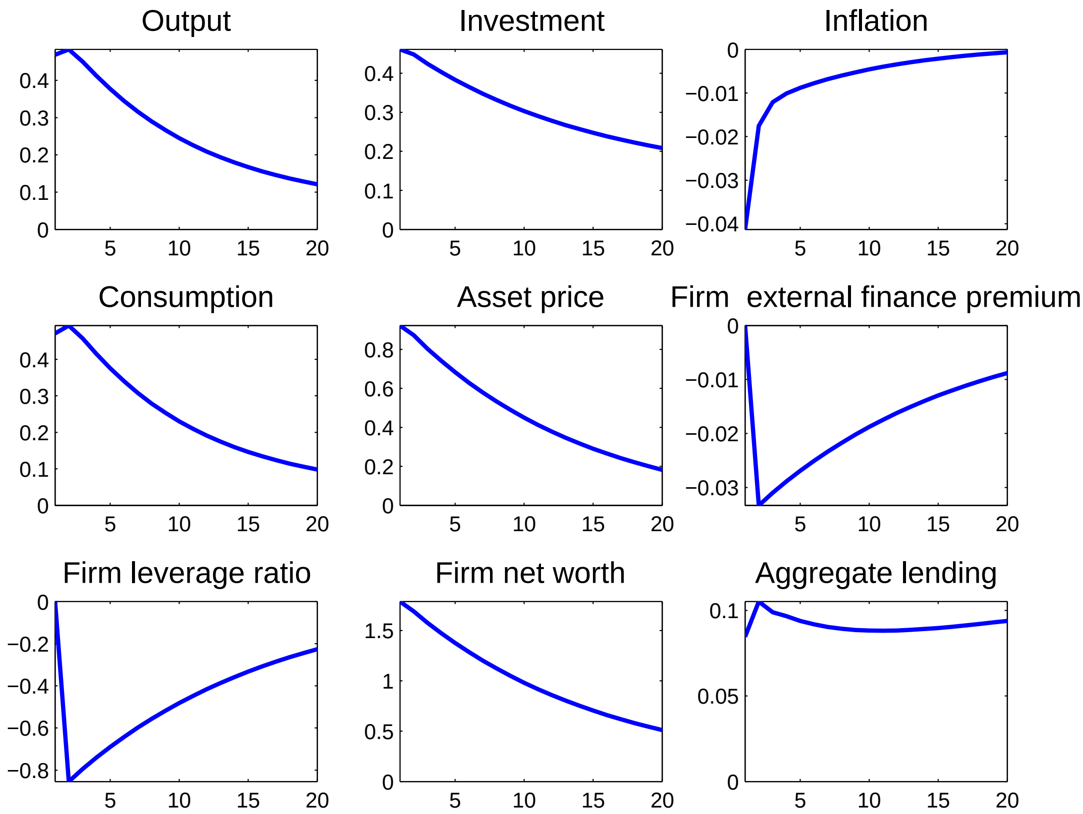


Figure 3: Impulse responses to a productivity shock

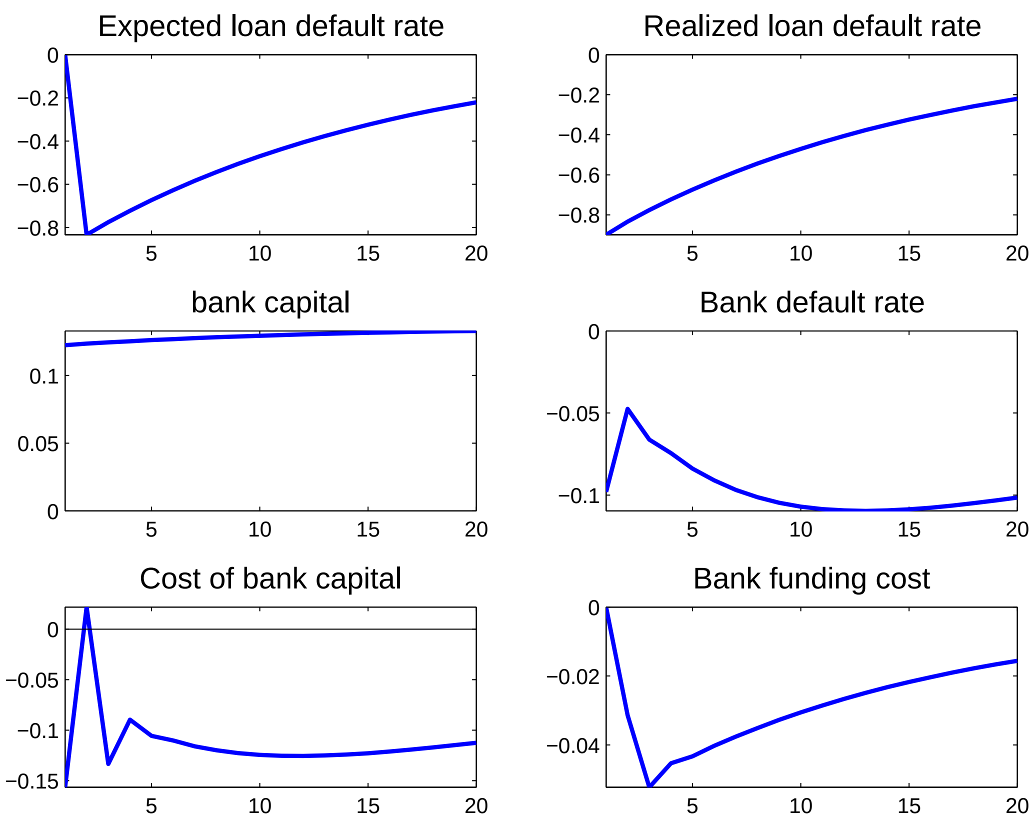


Figure 4: Impulse responses to a monetary policy shock

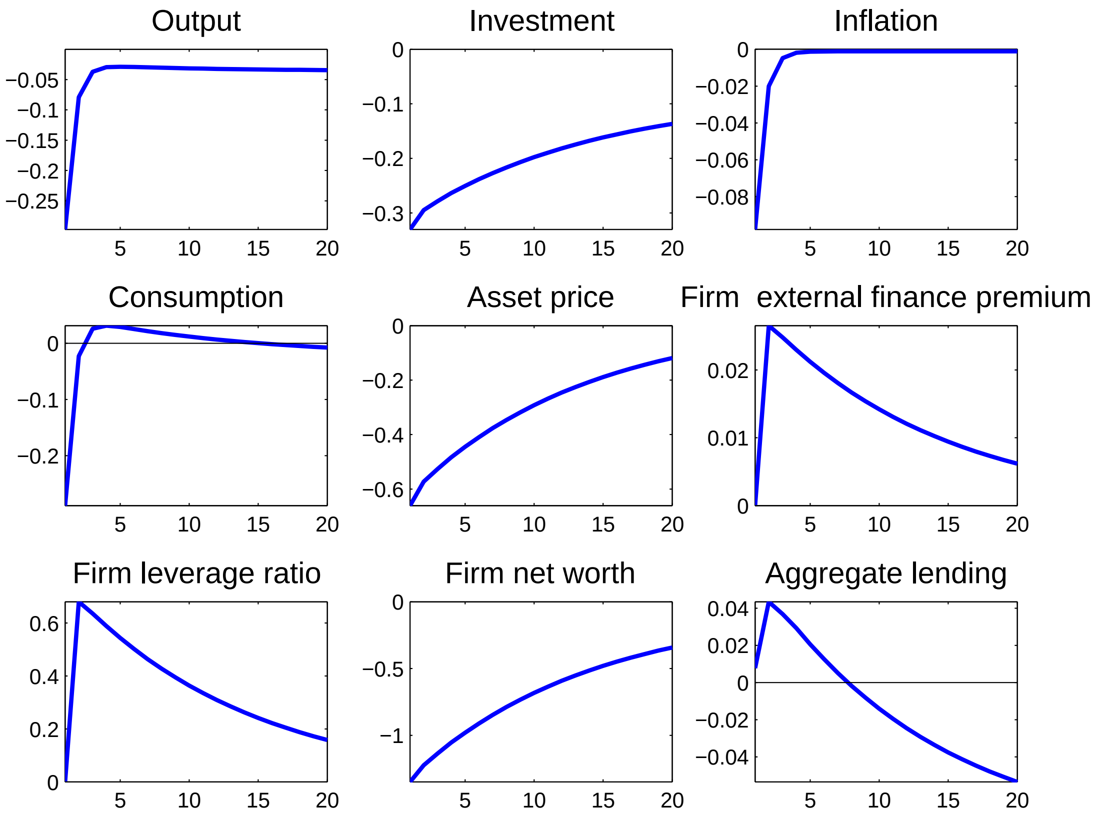


Figure 5: Impulse responses to a monetary policy shock

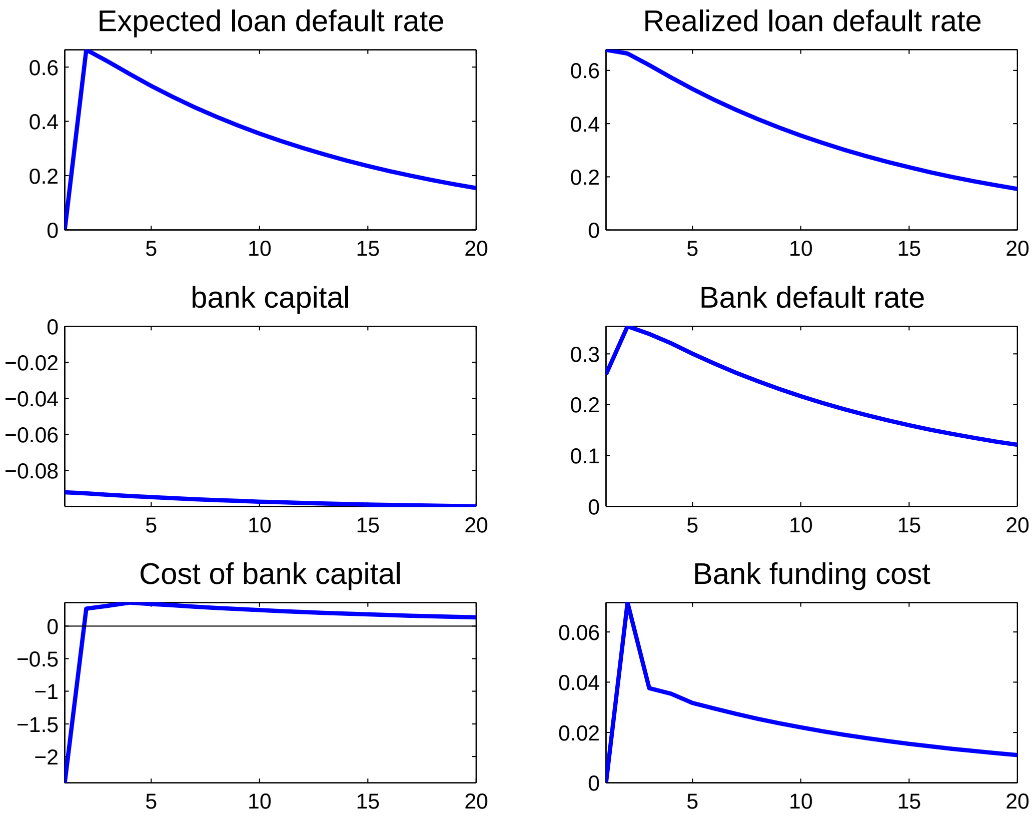


Figure 6: Impulse responses to a financial shock

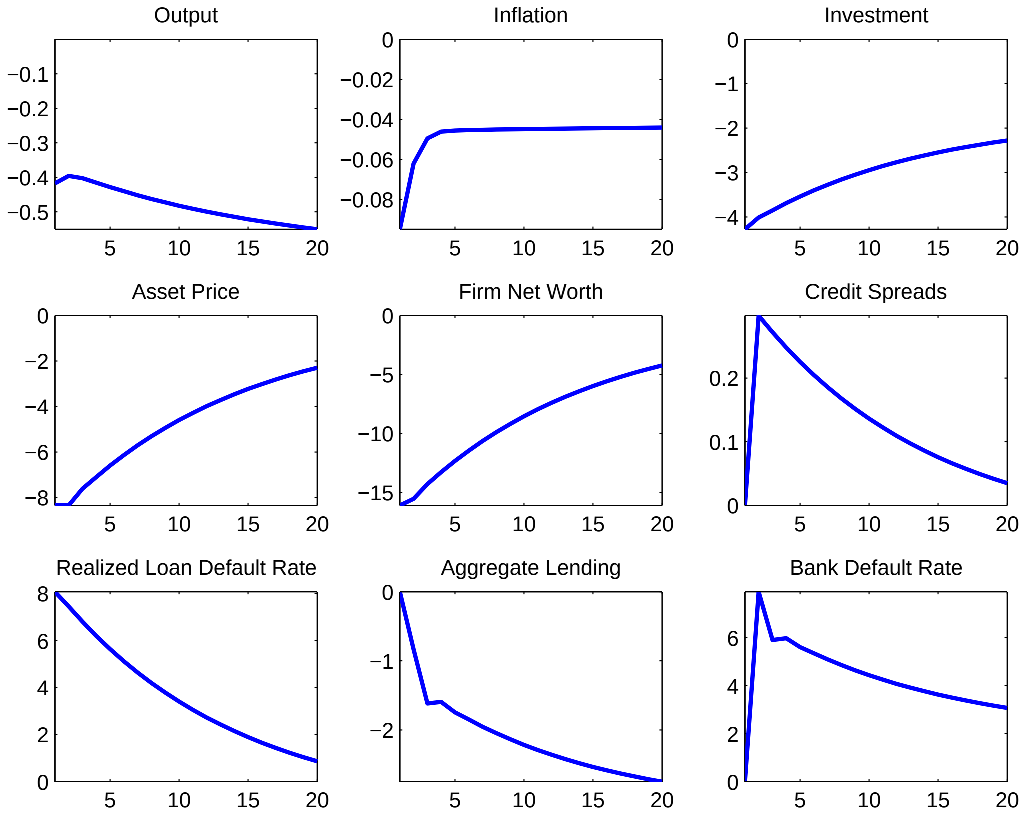


Figure 7: Impulse responses to a monetary policy shock

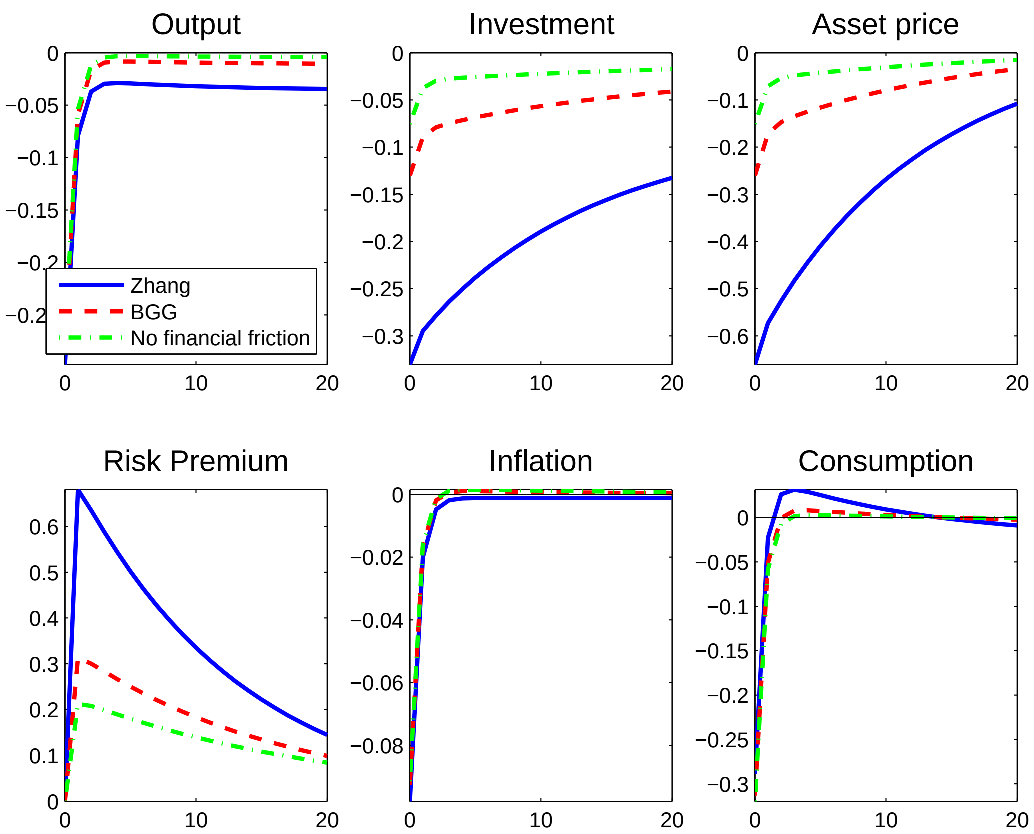


Figure 8: Impulse responses to a productivity shock