Reply: Real Business Cycles in Small Open Economy (Mendoza, 1991)

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1 Model

1.1 Production Technology and Financial Structure.

$$G\left(K_t, L_t, K_{t+1}\right) = \exp\left(e_t\right) K_t^{\alpha} L_t^{1-\alpha} - \left(\frac{\phi}{2}\right)\left(K_{t+1} - K_t\right)^2, \quad 0 < \alpha < 1,\ \phi > 0, \tag{1}$$

$$K_{t+1} = \left(1 - \delta\right) K_t + I_t, \quad 0 \leq \delta \leq 1, \tag{2}$$

$$A_{t+1} = \mathrm{TB}_t + A_t\left[1 + r^* \exp\left(n_t\right)\right], \tag{3}$$

$$C_t + I_t + \mathrm{TB}_t \leq \exp\left(e_t\right) K_t^{\alpha} L_t^{1-\alpha} - \left(\frac{\phi}{2}\right)\left(K_{t+1} - K_t\right)^2. \tag{4}$$

1.2 Preferences.

$$\mathrm{E}_t \sum_{s=t}^{\infty}\left\{u\left(C_s, L_s\right) \times \exp\left[-\sum_{\tau=t}^{s-1}\nu\left(C_\tau, L_\tau\right)\right]\right\}, \tag{5}$$

$$u\left(C_t, L_t\right) = \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\gamma} - 1}{1-\gamma}, \quad \omega > 1,\ \gamma > 1, \tag{6}$$

$$\nu\left(C_t, L_t\right) = \beta \ln\left(1 + C_t - \frac{L_t^{\omega}}{\omega}\right), \quad \beta > 0. \tag{7}$$

1.3 Solution and First Order Conditions.

Let, Θ_t as follows,

$$\Theta_s = \exp\left[-\sum_{\tau=t}^{s-1}\nu\left(C_\tau, L_\tau\right)\right] = \prod_{\tau=t}^{s-1}\exp\left[-\nu\left(C_\tau, L_\tau\right)\right], \tag{8}$$

$$\Theta_s = \exp\left[-\nu\left(C_{s-1}, L_{s-1}\right)\right]\Theta_{s-1}. \tag{9}$$

The Lagrangian associated with the maximization problem is,

$$\max_{\{A_{t+1}, C_t, K_{t+1}, L_t\}} \mathscr{L} = \mathrm{E}_t \sum_{s=t}^{\infty}\Theta_s\left\{u_s + \lambda_s\left(G_s - C_s - I_s - \mathrm{TB}_s\right) + \mu_s\left[\frac{\Theta_{s+1}}{\Theta_s} - \exp\left(-\nu_s\right)\right]\right\}, \tag{10}$$

$$\frac{\partial \mathscr{L}}{\partial A_{t+1}} : -\Theta_t \lambda_t \frac{\partial \mathrm{TB}_t}{\partial A_{t+1}} - \mathrm{E}_t \left(\Theta_{t+1} \lambda_{t+1} \frac{\partial \mathrm{TB}_{t+1}}{\partial A_{t+1}} \right) = 0, \tag{11}$$

$$\frac{\partial \mathscr{L}}{\partial C_t} = \Theta_t \left[\frac{\partial u_t}{\partial C_t} - \lambda_t + \mu_t \mathrm{exp}\left(-\nu_t\right) \frac{\partial \nu_t}{\partial C_t} \right] = 0, \tag{12}$$

$$\frac{\partial \mathscr{L}}{\partial K_{t+1}} : \Theta_t \lambda_t \left(\frac{\partial G_t}{\partial K_{t+1}} - \frac{\partial I_t}{\partial K_{t+1}} \right) + \mathrm{E}_t \left[\Theta_{t+1} \lambda_{t+1} \left(\frac{\partial G_{t+1}}{\partial K_{t+1}} - \frac{\partial I_{t+1}}{\partial K_{t+1}} \right) \right] = 0, \tag{13}$$

$$\frac{\partial \mathscr{L}}{\partial L_t} : \Theta_t \left[\frac{\partial u_t}{\partial L_t} + \lambda_t \frac{\partial G_t}{\partial L_t} + \mu_t \mathrm{exp}\left(-\nu_t\right) \frac{\partial \nu_t}{\partial L_t} \right] = 0, \tag{14}$$

$$\frac{\partial \mathscr{L}}{\partial \Theta_{t+1}} : \mu_t + \mathrm{E}_t \left\{ u_{t+1} + \lambda_{t+1} \left[G_{t+1} - C_{t+1} - I_{t+1} - \mathrm{TB}_{t+1} \right] - \mu_{t+1} \mathrm{exp}\left(-\nu_{t+1}\right) \right\} = 0,$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_t} : \Theta_t \left(G_t - C_t - I_t - \mathrm{TB}_t \right) = 0, \tag{15}$$

$$\frac{\partial \mathscr{L}}{\partial \mu_t} : \Theta_t \left[\frac{\Theta_{t+1}}{\Theta_t} - \mathrm{exp}\left(-\nu_t\right) \right] = 0. \tag{16}$$

From equation (16) we can notice that

$$\frac{\Theta_{t+1}}{\Theta_t} = \mathrm{exp}\left(-\nu_t\right).$$

$$u_{c,t} \equiv \frac{\partial u_t}{\partial C_t} = \left(C_t - \frac{L_t^{\omega}}{\omega} \right)^{-\gamma},$$

$$u_{l,t} \equiv \frac{\partial u_t}{\partial L_t} = -L_t^{\omega-1} u_{c,t}$$

$$\nu_{c,t} \equiv \frac{\partial \nu_t}{\partial C_t} = \beta \left(1 + C_t - \frac{L_t^{\omega}}{\omega} \right)^{-1}, \tag{17}$$

$$\nu_{l,t} \equiv \frac{\partial \nu_t}{\partial L_t} = -L_t^{\omega-1} \frac{\partial \nu_t}{\partial C_t}. \tag{18}$$

Replacing the functional forms we have the following FOCs

$$\lambda_t = \mathrm{E}_t \left\{ \mathrm{exp}\left(-\nu_t\right) \left[1 + r^* \mathrm{exp}\left(n_t\right) \right] \lambda_{t+1} \right\}, \tag{19}$$

$$\lambda_t = u_{c,t} + \mu_t \mathrm{exp}\left(-\nu_t\right) \nu_{c,t} \tag{20}$$

$$\lambda_t \left[\phi \left(K_{t+1} - K_t \right) + 1 \right] = \mathrm{E}_t \left\{ \mathrm{exp}\left(-\nu_t\right) \left[\alpha \mathrm{exp}\left(e_{t+1}\right) \left(\frac{L_{t+1}}{K_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] \lambda_{t+1} \right\}, \tag{21}$$

$$L_t = \left[\left(1 - \alpha\right) \exp\left(e_t\right) K_t^{\alpha} \right]^{\frac{1}{\omega+\alpha-1}}, \tag{22}$$

$$\mu_t = \mathrm{E}_t \left[\mu_{t+1} \mathrm{exp}\left(-\nu_{t+1}\right) - u_{t+1} \right], \tag{23}$$

$$G_t = C_t + I_t + \mathrm{TB}_t. \tag{24}$$

1.4 Steady-State

$$L = \left[(1-\alpha)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\omega-1}}.$$

$$K = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} L,$$

$$G = K^{\alpha} L^{1-\alpha},$$

$$I = \delta K,$$

$$C = (1 + r^*)^{1/\beta} + \frac{L^{\omega}}{\omega} - 1,$$

$$\mathrm{TB} = G - C - I,$$

$$A = -\frac{\mathrm{TB}}{r^*}$$

$$\lambda = u_c + \mu \exp\left(-\nu\right) \nu_c,$$

$$\mu = -\left(\frac{1 + r^*}{r^*}\right) u,$$