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Consumption and Real Exchange Rates with Incomplete Markets and Non-traded Goods*

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ABSTRACT

This paper addresses the consumption-real exchange rate anomaly. International real business cycle models based on complete financial markets predict a unitary correlation between the real exchange rate and the ratio of home to foreign consumption when subjected to supply side shocks. In the data, this correlation is usually small and often negative. This paper shows that this anomaly can be successfully addressed by models that have an incomplete financial market structure and a non-traded as well as traded goods production sector.

JEL Classification: F31, F41.
Keywords: Consumption-real exchange rate anomaly, incomplete financial markets, non-traded goods.

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1 Introduction

One of the well known puzzles in international finance is the so called consumption-real exchange rate anomaly (see Backus and Smith, 1993 for an early paper and Chari, Kehoe and McGrattan, 2002 for a recent contribution). Most international business cycle models predict that, under the assumption of perfect financial markets along with supply disturbances, consumption across countries should be higher in the country where its price, converted into a common currency is lower. This models' feature is in sharp contrast with the empirical evidence which suggests that the consumption differential across countries does not move in any systematic pattern with its relative price (i.e. the real exchange rate).

The removal of the assumption of perfect financial markets is not sufficient in replicating the observed evidence: indeed, in their study, Chari *et al* (2002) have shown that the same anomaly in the behavior of consumption and the real exchange rate does continue to hold. In this work we explore the extent to which the introduction of non-traded goods along with a limited international financial market structure might account for the aforementioned anomaly. Our results suggest that the combination of these two factors is a promising avenue for understanding the behavior of consumption across countries and the real exchange rate. Indeed, in our model, the calibrated moments are close to reproducing the observed behavior of the data for a wide range of plausible parameters values.

There are two key features that are important in accounting for our results. By assuming that international asset trade is limited to a risk-less bond we break the link between the real exchange rate and relative consumption that would arise under perfect financial markets. While by introducing non-traded goods we allow for the possibility that, depending on the origin of the shock (i.e. traded versus non traded), the real exchange rate and relative consumption across countries can move in opposite directions.

In particular, following a positive shock to the traded goods sector in the home economy, home consumption increases with respect to consumption abroad. On the other hand, the real exchange rate appreciates if the effect coming from the relative price of non-traded to traded goods (the so-called Balassa-Samuelson effect) outweighs the terms of trade effect that would imply a depreciation of the real exchange rate. The first effect will be stronger the more dominant the shocks to the traded goods sector relative to non-traded goods sector, while the second effect will be stronger the higher the degree of home bias in preferences.

More generally, the structure of the disturbance and the specification of preferences determine the overall cross-correlation between real exchange rate and relative consumption.

Finally we check the performance of our baseline model in replicating standard international business cycle statistics. Our model overcomes the problem of an unrealistically high cross-correlation between relative consumption and the real exchange rate and comes close to matching the volatility of other key variables like, investment, the real exchange rate and the terms of trade.

Our model follows closely the ones proposed by Chari *et al* (2002) and Stockman and Tesar (1995): we construct a simple two-country stochastic dynamic open economy model in which we allow households to trade internationally in only one risk-less nominal bond, prices are flexible and households consume a final non-traded good produced with domestic as well as foreign-produced intermediate goods and a non-traded intermediate component. We allow for capital accumulation at the intermediate goods level and deviations from purchasing power parity are obtained by allowing for home-bias toward home produced intermediate goods at the production level and because of the existence of non-traded intermediate inputs.

The remainder of the paper is structured as follows: in section two, we discuss the nature of the consumption-real exchange rate anomaly and survey related contributions in the literature. Section three presents the basic structure of the model. The model is calibrated in section four, and section five outlines the basic mechanism behind our results. The results of the calibrated model are discussed in sections six and seven, respectively. Section eight concludes.

2 Data and Related Literature

Backus and Smith (1993) are the first to document the lack of correlation between relative consumption levels and the real exchange rate. Corsetti, Dedola and Leduc (2004), whose calculations we report in table 1, show that the cross-correlations obtained from Hodrick-Prescott filtered as well as first-difference filtered data for a selection of OECD countries appear to be small and often negative. The median is between -0.30 and -0.2. The data for consumption and real exchange rates are annual series from the OECD Main Economic Indicators dataset from 1973 to 2001.

[Table 1 about here]

Chari, Kehoe and McGrattan (2002) report cross-correlations for a sub-set of OECD economies from 1973 to 1994 at a quarterly frequency and find a median value of -0.07.

These results can be used to question the assumption of financial market completeness, for that assumption would imply a cross-correlation between the real exchange rate and relative consumption of close to unity.[1]

Other empirical studies have similarly questioned the assumption of financial market completeness: in particular Ravn (2001) shows that there is no role for the real exchange rate in accounting for differences in marginal utilities of consumption in different countries. In his study, he rules out non-separabilities in the utility function as possible candidates in testing for risk-sharing. In another related study, Kollmann (1995) also rejects the complete market assumption.

Starting from these premises, recent theoretical papers assume an incomplete financial market structure as a necessary condition for explaining the observed evidence. In Chari *et al* (2002) domestic and foreign agents are only allowed to trade in a non-state contingent nominal bond. Their rich model with sticky prices is unable to break the link between real exchange rate and marginal utilities of consumption. Indeed, the cross correlation between relative consumption and the real exchange rate for the incomplete market case is still perfect as in the complete market case. They conclude by saying that "*the most widely used forms of asset market incompleteness does not eliminate - or even shrink- the anomaly*".

[1] One would expect a cross-correlation equal to 1.00 only if utility is additively separable in consumption.

On the other hand, the papers by Corsetti, Dedola and Leduc (2004) and Selaive and Tuesta (2003) introduce other frictions along with asset market incompleteness and are able to get closer in replicating the empirical facts. Corsetti *et al* (2004) highlight the role of distributive trade along with market incompleteness. Assuming that bringing traded goods to the market requires non-traded distribution services can generate the low import elasticity crucial for explaining the observed patterns in the international transmission of productivity shocks and the high volatility of the real exchange rate. Their VAR analysis suggests that a positive productivity shock will improve the terms of trade, appreciate the real exchange rate and increase domestic consumption relative to the rest of the world: this pattern of transmission is compatible in their model with a relatively low price elasticity of imports. Selaive and Tuesta (2003) consider a richer structure in which prices are sticky and monetary policy is modelled through interest-rate feedback rules. They emphasize the importance of financial frictions and the role of net foreign asset position in breaking the link between real exchange rate dynamics and relative consumption levels. Another related contribution is a recent work by Ghironi and Melitz (2004). In their work a non-traded sector arises endogenously because less productive firms decide not to export their products. They find that a Balassa-Samuelson effect and a real exchange rate appreciation is generated by aggregate productivity shocks rather than sector specific ones to the traded sector.

Our contribution differs from the aforementioned works in some important aspects: differently from Corsetti *et al* (2004) there are no distribution costs and the law of one price always holds. In contrast to from Selaive and Tuesta (2003) prices are perfectly flexible. As in Chari *et al* (2002), we assume that agents consume a final consumption good, which is not traded internationally. Unlike Chari *et al* we assume that this final good contains three types of intermediate inputs: home and foreign-produced traded intermediate inputs as well as non-traded domestically produced intermediate input.

We find that our model, calibrated in a canonical fashion, generates cross-correlations between the real exchange rate and relative consumption which are not at odds with the data. We attribute this to the combination of the presence of a non-traded production sector together with a simple form of incomplete financial markets.

3 A two-sector two-country model

The structure of the model closely follows closely Chari, Kehoe and McGrattan (2002) and Stockman and Tesar (1995). There are two key modifications with respect to their baseline cases. Firstly we consider an incomplete market structure at the international level. Secondly, unlike Chari *et al*, but similar to Stockman and Tesar, we introduce non-tradeable intermediate inputs in the production process. Moreover, we focus on a perfectly competitive setting while Chari *et al* analyze an imperfectly competitive framework with staggered price setting behavior.

3.1 Consumer Behavior

We propose a two-country model with infinitely lived consumers. The world economy is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to the country H (Home), while the segment $[n, 1]$ belongs to F (Foreign). Preferences for a generic Home-consumer are described by the following utility function:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s^j, (1 - l_s^j)) \tag{1}$$

where E_t denotes the expectation conditional on the information set at date t, while β is the intertemporal discount factor, with $0 < \beta < 1$. The Home consumer obtains utility from consumption, C^j, and receive dis-utility from supplying labor, l^j.

The asset market structure in the model is relatively standard in the literature. We assume that Home individuals are assumed to be able to trade two nominal risk-less bonds denominated in the domestic and foreign currency. These bonds are issued by residents in both countries in order to finance their consumption expenditure. On the other hand, foreign residents can allocate their wealth only in bonds denominated in the foreign currency. [2] Home households face a cost (i.e. transaction cost) when they take a position in the foreign bond market. This cost depends on the net foreign asset position of the home economy as in Benigno (2001).[3] Domestic firms are assumed to be wholly owned by domestic residents, and profits are distributed equally across households. Consumers face the following budget constraint in each period t:

$$P_t C_t^j + \frac{B_{H,t}^j}{(1+i_t)} + \frac{S_t B_{F,t}^j}{(1+i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} = B_{H,t-1}^j + S_t B_{F,t-1}^j + P_t w_t l_t^j + \Pi_t^j \tag{2}$$

where $B_{H,t}^j$ and $B_{F,t}^j$ are the individual's holdings of domestic and foreign nominal risk-less bonds denominated in the local currency. i_t is the Home country nominal interest rate and $i_t^*$is the Foreign country nominal interest rate. S_t is the nominal exchange rate expressed as units of domestic currency needed to buy one unit of foreign currency, P_t is the consumer price level and w_t is the real wage. Π_t^j are dividends from holding a share in the equity of domestic firms obtained by agent j. All domestic firms are wholly owned by domestic agents and equity within these firms is evenly divided between domestic agents.

The cost function $\Theta(.)$ drives a wedge between the return on foreign-currency denominated bonds received by domestic and by foreign residents. We follow Benigno (2001) in rationalizing this cost by assuming the existence of foreign-owned intermediaries in the foreign asset market who apply a spread over the risk-free rate of interest when borrowing or lending to home agents in foreign currency. This spread depends on the net foreign asset position of the home

[2] We want to highlight here the fact that this asymmetry in the financial market structure is made for simplicity. The results would not change if we allow home bonds to be traded internationally. We would need to consider a further arbitrage condition.

[3] Further ways of closing open economy models are discussed in Schmitt-Grohe and Uribe (2003).

economy. We assume that profits from this activity in the foreign asset market are distributed equally among foreign residents (see Benigno, P. 2001).[4]

As in Benigno (2001), we assume that all individual belonging to the same country have the same level of initial wealth. This assumption, along with the fact that all individuals face the same labor demand and own an equal share of all firms, implies that within the same country all individuals face the same budget constraint. Thus they will choose identical paths for consumption. As a result, we can drop the j superscript and focus on a representative individual for each country.

The maximization problem of the Home individual consists of maximizing 1 subject to 2 in determining the optimal profile of consumption and bond holdings and the labor supply schedule. Households' equilibrium conditions (Home and Foreign) are described by the following equations:

$$U_C\left(C_t,(1-l_t)\right)=(1+i_t)\beta E_t\left[U_C\left(C_{t+1},(1-l_{t+1})\right)\frac{P_t}{P_{t+1}}\right] \tag{3}$$

$$U_C\left(C_t^*,(1-l_t^*)\right)=(1+i_t^*)\beta E_t\left[U_C\left(C_{t+1}^*,(1-l_{t+1}^*)\right)\frac{P_t^*}{P_{t+1}^*}\right] \tag{4}$$

$$U_C\left(C_t,(1-l_t)\right)=(1+i_t^*)\Theta\left(\frac{S_tB_{F,t}}{P_t}\right)\beta E_t\left[U_C\left(C_{t+1},(1-l_{t+1})\right)\frac{S_{t+1}P_t}{S_tP_{t+1}}\right] \tag{5}$$

$$\frac{U_l(C_t,(1-l_t))}{U_C\left(C_t,(1-l_t)\right)}=w_t \qquad \frac{U_l(C_t^*,(1-l_t^*))}{U_C\left(C_t^*,(1-l_t^*)\right)}=w_t^* \tag{6}$$

3.2 Producer Behavior

As in Chari *et al* (2002), in our economy final goods are obtained by combining intermediate goods produced in the Home and in the Foreign economy. Differently from Chari *et al* (2002) we now also consider the possibility that non-traded intermediate inputs enter in the production process for the final goods. All trade between the two countries is in intermediate goods.

We let Y be the output of final goods produced in the home country. Final goods producers combine home and foreign-produced intermediate goods to produced Y in the following manner:

$$Y=\left[\omega^{\frac{1}{\kappa}}y_T^{\frac{\kappa-1}{\kappa}}+(1-\omega)^{\frac{1}{\kappa}}y_N^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \tag{7}$$

where y_T and y_N are the intermediate traded and non-traded inputs and κ is the elasticity of intratemporal substitution between traded and non-traded intermediate goods. The traded component is in turn produced using home and foreign-produced traded goods in the following manner:

$$y_T=\left[v^{\frac{1}{\theta}}y_H^{\frac{\theta-1}{\theta}}+(1-v)^{\frac{1}{\theta}}y_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{8}$$

[4] Here we follow Benigno (2001) in assuming that the cost function $\Theta(.)$ assumes the value of 1 only when the net foreign asset position is at its steady state level, ie $B_{F,t}=\overline{B}$, and is a differentiable decreasing function in the neighbourhood of $\overline{B}$. This cost function is convenient because it allows us to log-linearise our economy properly since in steady state the desired amount of net foreign assets is always a constant $\overline{B}$. The expression for profits from financial intermediation is given by $K=\frac{B_{F,t}}{P_t^*(1+i_t^*)}\left[\frac{RS_t}{\Theta\left(\frac{S_tB_{F,t}}{P_t}\right)}-1\right]$.

where y_H and y_F are the intermediate goods produced in the Home and Foreign countries respectively. θ is the elasticity of intratemporal substitution between home and foreign-produced intermediate goods.

Final goods producers and producer of the composite traded goods are competitive and maximize their profits:

$$\max_{y_N, y_T} PY - P_T y_T - P_N y_N \tag{9}$$

$$\max_{y_H, y_F} P_T y_T - P_H y_H - P_F y_F \tag{10}$$

subject to 7 and 8 respectively. This maximization yields the following input demand functions for the home economy (similar conditions hold for Foreign producers)

$$y_N = (1-\omega)\left(\frac{P_N}{P}\right)^{-\kappa} Y, \tag{11}$$

$$y_H = \omega v \left(\frac{P_H}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\kappa} Y \quad y_F = \omega(1-v)\left(\frac{P_F}{P_T}\right)^{-\theta}\left(\frac{P_T}{P}\right)^{-\kappa} Y$$

Corresponding to the previous demand function we have the following prices indexes:

$$P_T^{1-\theta} = [vP_H^{1-\theta} + (1-v)P_F^{1-\theta}] \tag{12}$$

$$P^{1-\kappa} = [\omega P_T^{1-\kappa} + \omega P_N^{1-\kappa}] \tag{13}$$

3.2.1 Intermediate goods sectors

Firms in the traded intermediate goods sector produce goods using capital and labor services. The typical firm maximizes the following profit function:

$$\max_{l_{H,t}, x_{H,t}} P_{H_t} y_{H_t} + S_t P_{H_t}^* y_H^* - P_t w_t l_{H,t} - P_t x_{H,t} \tag{14}$$

where $l_{H,t}$ is the total labor supply employed in the domestic traded intermediate sector, $x_{H,t}$ denotes investment in the traded domestic sector. Since we assume that the law of one price holds for traded goods, we can rewrite the maximization problem as:

$$\max_{l_{H,t}, x_{H,t}} P_{H_t}(y_{H_t} + y_H^*) - P_t w_t l_{H,t} - P_t x_{H,t}$$

subject to:

$$y_{H_t} + y_{H_t}^* = F(k_{H,t-1}, l_{H,t}) = (A_t l_{H,t})^\alpha (k_{H,t-1})^{1-\alpha} \tag{15}$$

$$k_{H,t} = (1-\delta)k_{H,t-1} + x_{Ht} - \phi\left(\frac{x_{Ht}}{k_{Ht-1}}\right) k_{Ht-1}$$

where $\phi(.)$ is the cost for installing investment goods.[5] The first-order conditions are given by the following equations:

$$P_t w_t = \alpha P_{H,t}(A_t)^\alpha \left(\frac{k_{H,t-1}}{l_{H,t}}\right)^{1-\alpha} \tag{16}$$

[5] The function $\phi(.)$ has the following form: $\phi\left(\frac{x_t}{k_{t-1}}\right) = \frac{b(\frac{x_t}{k_{t-1}}-\delta)^2}{2}$ so that $\phi'\left(\frac{x_t}{k_{t-1}}\right) = b(\frac{x_t}{k_{t-1}} - \delta)$ and $\phi''\left(\frac{x_t}{k_{t-1}}\right) = b$ away from steady state and in steady state: $\phi\left(\frac{x}{k}\right) = \phi'\left(\frac{x_t}{k_{t-1}}\right) = 0$.

$$U_C\left(C_t,(1-l_t)\right)=\left[1-\phi'\left(\frac{x_{H,t}}{k_{H,t-1}}\right)\right]\mathrm{E}\beta U_C\left(C_{t+1},(1-l_{t+1})\right)w_{t+1}\frac{f_{k_{t+1}}}{f_{l_{t+1}}}+ \tag{17}$$

$$\mathrm{E}_t\beta\frac{1-\phi'\left(\frac{x_{H,t}}{k_{H,t-1}}\right)}{1-\phi'\left(\frac{x_{H,t+1}}{k_{H,t}}\right)}U_C\left(C_{t+1},(1-l_{t+1})\right)\left[(1-\delta)-\phi\left(\frac{x_{H,t+1}}{k_{H,t}}\right)+\phi'\left(\frac{x_{H,t+1}}{k_{H,t}}\right)\frac{x_{H,t+1}}{k_{H,t}}\right]$$

where f_{k_t} is the marginal product of capital and $f_{l_{t+1}}$ the marginal product of labor and w_{t+1} is the real wage.

A similar problem holds for the non-traded goods sector:

$$\max_{l_{N,t},x_{N,t}} P_{N_t}y_{N_t}-P_tw_tl_{N,t}-P_tx_{N,t} \tag{18}$$

$$y_{N_t}=F(k_{t-1},l_{N,t})=(A_{N,t}l_{N,t})^{\alpha}(k_{N,t-1})^{1-\alpha} \tag{19}$$

$$k_{N,t}=(1-\delta)k_{N,t-1}+x_t-\phi\left(\frac{x_{N,t}}{k_{N,t-1}}\right)k_{N,t-1} \tag{20}$$

And the corresponding first order conditions are given by:

$$P_tw_t=\alpha P_{N,t}(A_{N,t})^{\alpha}(\frac{k_{N,t-1}}{l_{N,t}})^{1-\alpha} \tag{21}$$

$$U_C\left(C_t,(1-l_t)\right)=\left[1-\phi'\left(\frac{x_{N,t}}{k_{Nt-1}}\right)\right]\mathrm{E}\beta U_C\left(C_{t+1},(1-l_{t+1})\right)w_{t+1}\frac{f_{k_{t+1}}}{f_{l_{t+1}}}+ \tag{22}$$

$$\mathrm{E}_t\beta\frac{1-\phi'\left(\frac{x_{N,t}}{k_{N,t-1}}\right)}{1-\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)}U_C\left(C_{t+1},(1-l_{t+1})\right)\left[(1-\delta)-\phi\left(\frac{x_{N.t+1}}{k_{N,t}}\right)+\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)\frac{x_{N,t+1}}{k_{N,t}}\right]$$

where f_{k_t} is the marginal product of capital and $f_{l_{t+1}}$ the marginal product of labor and w_{t+1} is the real wage.

3.3 Current account

One important implication of the incomplete market framework is that it allows us to characterize the dynamic of the current account. By aggregating the individual budget constraints in the home country, we obtain:

$$P_tC_t+\frac{S_tB_t^F}{(1+i_t^*)}\frac{1}{\Theta(\frac{S_tB_t^F}{P_t})}=S_tB_{t-1}^F+P_tw_ll_t+\Pi_t \tag{23}$$

where we have applied the assumption that home bonds are in zero net supply and only held by Home residents. The aggregate profits in the home economy are given by:

$$\Pi_t=P_{H_t}\left(y_{H_t}+y_{H_t}^*\right)-P_tw_tl_{H_t}-P_tx_{H_t}+ \tag{24}$$
$$P_{N_t}y_{N_t}-P_tw_tl_{N_t}-P_tx_{N_t}$$

From which substituting the economy-wide constraint on labor and investment ($l = l_H + l_N$ and $x = x_H + x_N$) we obtain:

$$C_t + \frac{S_t B_t^F}{P_t(1+i_t^*)} \frac{1}{\Theta(\frac{S_t B_t^F}{P_t})} = \frac{S_t B_{t-1}^F}{P_t} + \frac{P_{H_t}}{P_t}\left(y_{H_t} + y_{H_t}^*\right) + \frac{P_{N_t}}{P_t} y_{N_t} - x_t \quad (25)$$

or after substituting in the final goods sector constraint ($Y = C + x$):

$$\frac{S_t B_t^F}{P_t(1+i_t^*)} \frac{1}{\Theta(\frac{S_t B_t^F}{P_t})} = \frac{S_t B_{t-1}^F}{P_t} + \frac{P_{H_t}}{P_t}\left(y_{H_t} + y_{H_t}^*\right) + \frac{P_{N_t}}{P_t} y_{N_t} - Y_t \quad (26)$$

a similar equation holds for the Foreign economy.

3.4 Monetary policy

Since we are characterizing a nominal model we need to specify a monetary policy rule. In what follows we assume that the monetary authorities in both countries follow a strategy of setting producer price inflation equal to zero.

3.5 Solution technique

Before solving our model, we log-linearize around the steady state to obtain a set of equations describing the equilibrium fluctuations of the model. The log-linearization yields a system of linear difference equations which we list in the appendix and can be expressed as a singular dynamic system of the following form:

$$\mathbf{A}\mathrm{E}_t\mathbf{y}(t+1 \mid t) = \mathbf{B}\mathbf{y}(t) + \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{y}(t)$ is ordered so that the non-predetermined variables appear first and the predetermined variables appear last, and $\mathbf{x}(t)$ is a martingale difference sequence. There are four shocks in $\mathbf{C}$: shocks to the Home traded and non-traded intermediate goods sectors' productivity and shocks to the Foreign traded and non-traded intermediate goods sectors' productivity. The variance-covariance as well as the autocorrelation matrices associated with these shocks are described in table 2. Given the parameters of the model, which we describe in the next section, we solve this system using the King and Watson (1998) solution algorithm.

4 Calibration

The calibration of our model parameters as well as the process for technological innovations follows the calibration suggested by Stockman and Tesar (1995) where applicable.[6]

We assume that the Home and Foreign economy are of equal size and are calibrated in a symmetric fashion. Following Stockman and Tesar we choose the following functional form for the utility function:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{1}{1-\rho} (C_s^j)^{1-\rho} (1 - l_s^j)^{\eta}) \right] \quad (27)$$

[6] We follow Stockman and Tesar (1995) because their analysis uses a model similar to ours, and is one of the very few published papers to provide estimated productivity shock processes for a symmetric two-country two-sector model.

In choosing the parameters of utility function, we set β to match a 4% annual discount rate. The coefficient of constant relative risk aversion, or the inverse of the intertemporal elasticity of substitution, ρ, is set to 2, as in Stockman and Tesar. As in Stockman and Tesar we set the inverse of the intertemporal elasticity of substitution in leisure, η to 3.17 and assume along with most real business cycle studies that agents devote around 80% of their time endowment to leisure and the remaining 20% to work.

We calibrate the parameters pertaining to the final goods producing sector in the following way. The share of tradable intermediate goods in the final consumption good, ω is 0.4, while the share of home-produced intermediate inputs in the tradable intermediate input, v is 0.7. This calibration suggests significant home bias in consumption, and is in line with other recent studies, see Corsetti et al (2004). Following Stockman and Tesar, we assume a unitary elasticity of substitution between home and foreign-produced traded goods, θ, and an elasticity of substitution between traded and non-traded goods, κ of 0.44 in the production of the final consumption good.

We assume that the share of labor input in intermediate good production, α in our Cobb-Douglas production function, is the same across sectors. We choose the average of the shares reported for the traded and non-traded sectors as in Stockman and Tesar: $\alpha = 0.585$. We assume that capital stocks depreciate at a rate of 10% per annum. We choose the adjustment cost parameter in investment, d, so as to ensure a volatility of investment relative that of GDP in excess of 3.

The two remaining parameters relate to our specification of incomplete markets. We follow Benigno, P. (2001) in choosing a 10 basis point spread (per quarter) of the domestic interest rate on foreign assets over the foreign rate, such that on an annualized basis $\varepsilon \equiv -\Theta'(\bar{b})\bar{Y} = 0.004$, while the steady-state ratio of net foreign assets to GDP, $\bar{a} = \frac{\bar{b}}{\bar{Y}}$ is assumed to be equal to zero.

The structure of our shock processes is also taken from Stockman and Tesar and is reproduced in table 2. The shocks to technology are assumed to follow a first order autoregressive process:

$$\mathbf{A}_{t+1} = \Omega \mathbf{A}_t + \mu_t$$

where $\mathbf{A}$ is a vector of shocks: $[A_H,\ A_F,\ A_N,\ A_{N^*}]$ and Ω is a 4×4 matrix describing the autoregressive components of the shocks. The innovations to $\mathbf{A}$ are $[\mu_H,\ \mu_F,\ \mu_N,\ \mu_{N^*}]$ and the variance-covariance matrix is $V[\mu]$.

[Table 2 about here]

In Stockman and Tesar's estimated shock processes the variance of traded goods sector shocks is about twice that of shocks to non-traded productivity. Shocks to the traded sector's supply side are less persistent than shocks affecting the non-traded sector. The cross-correlation between shocks implied by $V[\mu]$ can be summarized as follows:

$$Corr(\mu_i, \mu_j) = \begin{bmatrix} 1 & 0.33 & 0.46 & 0.19 \\ 0.33 & 1 & 0.19 & 0.46 \\ 0.46 & 0.19 & 1 & 0.14 \\ 0.19 & 0.46 & 0.14 & 1 \end{bmatrix}$$

5 Relative consumption and the real exchange rate: the role of incomplete markets and sectorial shocks

Before we analyze the characteristics of our calibrated model in terms of second moments, this section looks at impulse responses for the real exchange rate and relative consumption following productivity shocks. Our impulse responses are derived under the assumption that all elements of the autocorrelation matrix Ω are set to zero and that the variance-covariance matrix $V[\mu]$ of the shocks takes the form of an identity matrix.

In this section we want to highlight the roles of market incompleteness, the importance of the non-traded goods sector as well as the sectorial origin of the disturbance. Our two-country, two-sectors model with no departures from the law of one price, implies that the real exchange rate can be expressed as a combination of the terms of trade and relative prices of traded versus non-traded goods in the home and the foreign economy. In log-linear terms we have:

$$\widehat{RS}_t = (v - v^*)\hat{T}_t + (\omega - 1)\hat{R}_t + (1 - \omega^*)\hat{R}^*_t \tag{28}$$

As in Benigno and Thoenissen (2003), we can decompose movements in the real exchange rate into two channels: the home-bias channel, $(v - v^*)\hat{T}_t$ where $\hat{T}$ represents the terms of trade (i.e. the relative price of foreign to home-produced traded goods) in deviation from its steady state value and $(v - v^*)$ is the difference between the home and foreign share of home-produced intermediate input in the traded component of final output; and what we call the internal real exchange rate $\left[(\omega - 1)\hat{R}_t + (1 - \omega^*)\hat{R}^*_t\right]$ where $\hat{R}$ and $\hat{R}^*$ are deviations from steady state of the relative price of non-traded to traded goods (P_N/P_T) at home and abroad, respectively. This expression shows that by allowing for home bias, $v > v^*$ the terms of trade enters directly into the dynamics of the real exchange rate via the home bias channel.

We start by considering a framework in which markets are complete as in Stockman and Tesar (1995) (see Figures 1 and 2). In the top panel we show the percentage deviation of the real exchange rate, the internal real exchange rate as well as the home bias channel following a positive productivity shock to the traded goods sector in the presence of Arrow-Debreu securities. The bottom panel shows the response of the relative consumption measured as a difference between the log-deviations of Home and Foreign consumption from their steady state levels. Since markets are complete, the real exchange rate and relative consumption are linked by the following risk sharing condition (here in log-linear terms):

$$\widehat{RS}_t = \hat{U}_C(C^*_t, (1 - l^*_t)) - \hat{U}_C(C_t, (1 - l_t)). \tag{29}$$

Risk-sharing equates the ratio of marginal utilities of consumption with the real exchange rate. For most types of preferences, this risk-sharing relationship implies a cross-correlation between the real exchange rate and relative consumption close to unity no matter what is the source of the disturbance. This theoretical result is illustrated in our figures 1 and 2. Figure 1, which corresponds to our baseline calibration except for the shock matrices, shows the response of our model to a one percent deviation to traded sector productivity.

The home country enjoys the productivity increase and domestic consumption rises. Foreign consumption also rises. One reason is the presence of state-contingent bonds, which transfer resources from Home to Foreign in the case of a Home productivity increase. Moreover, because the terms of trade depreciate in response to a positive domestic supply shock, purchasing power is further transferred from Home to Foreign agents. The net effect on relative consumption is that home consumption rises by more than foreign consumption, causing relative consumption to rise. Since relative consumption and the real exchange rate are linked through the above risk sharing condition, the real exchange rate appreciates as relative consumption rises.

In figure 2 we do the same experiment for a home productivity shock to the non-traded goods sector: as in the previous example, home consumption increases (because of the increase in the non-traded goods component) and risk-sharing operates via a depreciation of the real exchange rate that improves the purchasing power of foreign consumers. In both cases the dynamics of relative consumption are linked to that of the real exchange rate via the risk-sharing mechanism associated with Arrow-Debreu securities.

In both of these cases, the model with Arrow-Debreu securities generates cross-correlations between the real exchange rate and relative consumption close to unity (relative consumption and the real exchange rate move in the same direction). This behavior, which is at odds with the evidence reported in section 2, is referred to as the Backus-Smith puzzle or the consumption-real exchange rate anomaly. In our next experiments we examine to what extent the removal of the assumption of market completeness will break the link between relative consumption and the real exchange rate.

One consequence of the incomplete financial market structure is presence of current account dynamics. For illustrative purposes, we rewrite our log-linearized current account equation for the case in which there are no investment dynamics and the steady-state net foreign asset position is zero:

$$\beta\hat{b}_t = \hat{b}_{t-1} + (1-\theta)(v-1)\omega\hat{T}_t \tag{30}$$
$$+\theta\omega\,(1-v)\,\widehat{RS}_t + \omega(v-1)\left(\hat{C}_t - \hat{C}_t^*\right) + (\theta-\kappa)(\omega-1)\omega(v-1)\hat{R}_t$$
$$+(\theta-\kappa)\,(\omega^*-1)\,\omega\,(1-v)\,\hat{R}_t^*$$

where $\hat{b}$ is the deviation of foreign currency-denominated bond holdings from their steady state, relative to domestic GDP. In a bond economy there are only limited opportunities for sharing risk between countries. Non-state contingent bonds offer one avenue for risk diversification. The other way to share risk is through changes in the terms of trade. Following a positive supply side shock to the home economy, home agents become richer and demand more goods of all types. As a risk-sharing mechanism the terms of trade depreciate improving the purchasing power of foreign consumers.

In a two-sector economy, a positive supply shock affects two relative prices: the terms of trade and the relative price of non-traded goods. If the supply shock occurs in the traded goods sector, the relative price of non-traded goods tends to increase - this effect is sometimes called the Balassa-Samuelson effect - which contributes towards an appreciation of the real exchange rate and switches demand from home non-traded to traded goods. The terms of trade on the other hand still depreciates thereby switching demand from foreign to home-produced traded goods.

Figures 3 and 4 show the response of our key variables following a productivity shock to the Home traded (figure 3) and Home non-traded (figure 4) sectors. In our model and for our calibration, the terms of trade depreciates (rises) following a positive productivity shock to home-produced traded goods. Whereas an increase in productivity raises domestic output and consumption, part of the increase in consumption is shared with foreign agents via the terms of trade depreciation. In our model and calibration, this effect is outweighed by an increase in the relative price of non-traded to traded goods, so that the real exchange rate appreciates in response to an increase in home traded sector productivity. Because risk-sharing opportunities are limited, consumption increases by more at Home than in Foreign. However, consumption and the real exchange rate move in opposite directions indicating a negative cross-correlation between the two variables. Both the rise in relative consumption, and the appreciation of the real exchange rate contribute towards an initial current account deficit. When the source of the disturbance arises in the non-traded goods sector the relative price of non-traded to traded goods and the terms of trade will increase causing the real exchange rate to depreciate. For this shock and calibration, the current account improves. In general, the response of the current account depends on both the response of relative output and relative prices. Figure 4 suggests that predominance of non-traded shocks will result in large and positive cross-correlations between relative consumption and the real exchange rate.

In figure 5 we review Chari *et al*'s (2002) findings: by setting ω to 1 (i.e. absence of non-traded goods) and $\nu = 0.984$ as in their calibration. In this case the only determinant of the real exchange rate is given by the terms of trade (see equation 28). On impact a positive productivity shock in the Home economy will imply a real exchange rate depreciation caused by a relative decrease in the price of home produced goods. Because risk-sharing opportunities are limited under incomplete financial markets, home consumption rises by more than foreign consumption. The implied cross-correlation between the real exchange rate and relative consumption is large and positive.

[Figures 1 - 5 about here]

6 Characteristics of the calibrated model

Having analyzed the impulse responses for $\Omega = 0$ and $V[\mu] = I$, in this section, we analyze the second moments generated by our model using the calibration in table 2 for model parameters as well as shock processes. Table 3 summarizes a selection of second moments from the data and compares these with moments generated by the artificial model economies under different calibrations. Both the actual data, taken from Corsetti, Dedola and Leduc (2004) as well as the artificial model economy data are of annual frequency, and logged as well as Hodrick-Prescott filtered.

[Table 3 about here]

The column headed *baseline model* in table 3 shows a selection of second moments generated by our model under the calibration proposed in table 2. The numbers in the bottom rows of table 3 show that for our baseline calibration our model generates a negative correlation between the real exchange rate and relative consumption. We also find a large positive correlation between the real exchange rate and the terms of trade, as suggested by the data.

As pointed out above, our chosen calibration is close to that proposed by Stockman and Tesar. The only parameter not usually determined in the literature and one which does not appear to have a counterpart in Stockman and Tesar's model is d the adjustment cost parameter in investment. The choice of d determines, amongst other things, the volatility of investment relative to GDP. We chose a low value of d (1.00) to generate a relatively volatile series for aggregate investment. In the baseline model the standard deviation of investment is 3.08 times that of GDP. This value is slightly above the one reported in the data by Stockman and Tesar (2.18) but below the one reported by Corsetti *et al* (4.25). If investment is volatile, then so is GDP. As a result consumption, employment, and to an lesser extent the real exchange rate, are all too smooth relative to GDP as the first five reported statistics of table 3 show.[7][8]

Where our calibrated model fares better are the relative volatilities of the real exchange rate and the terms of trade, as well as the cross-correlations between GDP and net exports and as mentioned above, the cross-correlation between the real exchange rate and, respectively, relative consumption and the terms of trade. Two broadly accepted stylized facts of international real business cycles are that the trade balance is counter-cyclical and that the correlation between the real exchange rate and relative consumption is significantly below unity, as evidence reported in table 1 suggests. Our model generates a correlation between GDP and net exports of -0.30 and a correlation between the real exchange rate and relative consumption of -0.65. Compared to the data, net exports are not quite as counter-cyclical as in the data, whereas the model's prediction of the correlation between relative consumption and the real exchange rate is towards the higher end of evidence suggested in table 1. The correlation between the real exchange rate and the terms of trade is 0.92.

As in Stockman and Tesar consumption is more highly correlated with its foreign counterpart than is output, which is in contrast to what is found in the data. This finding is sometimes called the quantity anomaly. As in the data, we find that employment is more correlated across borders than investment, but our generated correlations are higher than the data suggests.

7 Sensitivity analysis

In this section we examine how the properties of our reported moments change when we alter some to the key parameters of the model. The column of table

[7]For our symmetric calibration, the standard deviation of foreign variables are the same as that of home country variables. Hence we do not report statistics for the foreign economy in table 3.

[8]An alternative calibration strategy, followed by Chari et al (2002) is to choose d so as to match the relative volatility of consumption. As in Chari et al, this yields an investment series that is too smooth at around 1.5 time the volatility of GDP. The real exchange rate and the terms of trade, on the other hand, are more volatile than in our baseline calibration. The cross-correlation between the real exchange rate and relative consumption is still negative, the trade balance is still counter-cyclical and the terms of trade are positively correlated with the real exchange rate.

3 headed *Arrow-Debreu* reports the selection of second moments for an economy with a full set of state-contingent Arrow-Debreu securities. We include this specification to show that whereas incomplete markets help to address the consumption-real exchange rate anomaly, they do not do so at the expense of other moments of the model. Indeed, the real exchange rate is even less volatile under complete financial markets than in our baseline model - 0.71 times as volatile as GDP as opposed to 2.91 times for the baseline model. As expected the correlation between the real exchange rate and relative consumption is close to unity (0.94).[9]

Next, we consider economies with market incompleteness. Consumption home-bias is a well documented phenomenon whereby the share of home-produced traded goods in consumption is greater at home than abroad. By assuming zero home-bias the real exchange rate can only deviate from purchasing power parity (PPP) through the presence of non-traded goods (see equation 28).

Equation 28 shows that by allowing for home bias, $v > v^*$ the terms of trade enters directly into the dynamics of the real exchange rate. Assume the domestic economy is hit by a positive shock to its traded intermediate goods sector. Such a shock causes the domestic relative price of non-traded to traded goods to increase relative to the foreign non-traded to traded price ratio, i.e. $\left[(\omega - 1)\hat{R}_t + (1 - \omega^*)\hat{R}_t\right] < 0$. This causes the real exchange rate to appreciate. At the same time, such a shock lowers the price of the domestically produced intermediate traded goods relative to the price of the foreign-produced intermediate traded good, causing the terms of trade to depreciate. This depreciation of the terms of trade partly off-sets the appreciation of the real exchange rate. Thus for a given supply shock to the traded goods sector, the real exchange rate appreciates less in the presence of consumption home-bias, which should lead to a higher correlation between the real exchange rate and relative consumption. The column headed *Home-bias* reports selected second moments for a calibration assuming a higher degree of home bias where $v = 0.85$ and $v = 0.15$. This calibration is suggested by Chari *et al* (2002). As our intuition suggests the cross-correlation between the real exchange rate and relative consumption rises - from -0.65 to -0.09. Because of the greater weight attached to the home bias channel in the determination of real exchange rate dynamics, we observe that the real exchange rate is both more volatile and more highly correlated with the terms of trade.

Next, we examine the role of the intratemporal elasticity of substitution between home and foreign produced goods, θ. This parameter determines the degree to which the terms of trade respond to supply shocks. The greater is θ, the more substitutable are home and foreign-produced traded intermediate goods in the production of final goods. In the limit, as θ becomes very large, the two types of tradable intermediate goods will be perfect substitutes with a constant terms of trade. In the column headed $\theta = 0.5$ ($\theta = 1.5$) we examine the consequences of changing θ form its baseline value of 1.00 to 0.5 and 1.5 (values in brackets). As noted above, the volatility of the terms of trade relative to the volatility of GDP varies inversely with θ. The real exchange rate becomes more (less) volatile the higher (lower) is θ relative to the baseline case. For the lower value of θ, the correlation of the real exchange rate and relative consumption is

[9] Only when preferences are additively separable in consumption and leisure or when labour supply is completely inelastic, does the correlation equals 1.00.

about zero. For the higher value of θ the correlation decreases relative to the baseline to -0.87. The remaining moments are not greatly affected by varying θ.

The column headed $\kappa = \mathit{0.74}$ explores an alternative calibration of the intratemporal elasticity of substitution between traded and non-traded goods. The value we use in our baseline calibration is that estimated by Stockman and Teasar (1995). Their estimate is based on a sample of developed as well as developing countries. Mendoza (1991) provides an estimate of this elasticity for a sample of industrialized countries, which is somewhat higher at $\kappa = 0.74$. Our analysis suggests that a higher intratemporal elasticity of substitution between traded and non-traded goods results in a lower relative volatility of the real exchange rate and the terms of trade, but leaves most of the other variables of interest relatively unchanged. Indeed, the correlation between the real exchange rate and relative consumption is even more negative under this calibration.

The column headed $\omega = \mathit{0.5}$ explores an alternative calibration of the share of traded intermediate goods in the final good. Equation (28) shows that ω and ω^* determine the weight attached to deviations from steady state of the internal real exchange rate, $\left[(\omega - 1)\hat{R}_t + (1 - \omega^*)\hat{R}_t\right]$, in the dynamics of the real exchange rate. *A priori* one would expect that a larger ω reduces the influence of this real exchange rate channel thereby reducing the overall volatility of the real exchange rate. Table 3 confirms this presumption. For the higher value of ω we observe a decline in the volatility of the real exchange rate and the terms of trade. The cross-correlations between the real exchange rate and relative consumption on the one hand, and the terms of trade on the other, do not change significantly under this calibration, compared to the baseline.

We argue in the introduction that one of the reasons Chari, Kehoe and McGrattan (2002) find such a high cross-correlation between the real exchange rate and relative consumption in their bond economy model is the omission of non-traded goods. The column headed *Chari et al* tests this proposition. We present a calibration that approximates that of Chari *et al.* First, Chari *et al's* model does not have a non-traded intermediate goods sector. To capture this in our model we set the share of traded intermediate inputs in final goods production equal to unity, i.e. $\omega = \omega^* = 1$. Next, we rewrite the shock processes in such a way that the variance and autocorrelation of non-traded shocks is zero and any spill overs only take place between the home and foreign traded goods sectors. [10] We also set the degree of home-bias as in Chari *et al*, $v = 1 - v^* = 0.984$. This calibration produces a cross-correlation between the real exchange rate and GDP of 0.81. The most of the remainder of the moments are close to those of our baseline model, except for the relative volatility of the real exchange rate and the terms of trade which are now substantially less volatile than GDP.

[10] The autocorrelation matrix becomes: $\Omega = \begin{bmatrix} 0.154 & 0 & 0 & 0 \\ 0 & 0.154 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and the variance covariance matrix: $V[\mu] = \begin{bmatrix} 3.62 & 1.21 & 0 & 0 \\ 1.21 & 3.62 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

8 Conclusion

In this paper, we address the consumption-real exchange rate anomaly. This anomaly refers to the propensity of international business cycle models based on complete financial markets to generate cross-correlations between the real exchange rate and relative consumption close to unity. In the data, this correlation is close to zero or even negative. We show that if a canonical international business cycle model, similar to the one proposed by Chari *et al* (2002) includes both an incomplete financial markets structure as well as a non-traded goods sector, then such a model, calibrated in a standard way will generate cross-correlations between the real exchange rate and relative consumption close to those in the data.

The presence of a non-traded goods sector allows the real exchange rate to appreciate (decrease) in response to a productivity shock to the domestic traded goods sector - the familiar Balassa-Samuelson effect - while limited risk-sharing opportunities cause consumption in the domestic economy to increases by more than consumption in the foreign economy following such a shock. The result is a negative cross-correlation between the real exchange rate and relative consumption.

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A Summary of equations

In this appendix, we list all the equations of the model in the order in which they appear in our code.

- Home and foreign euler equations, the UIP condition, home and foreign consumption-labor effort trade-off and current account:

$$\rho E_t \hat{C}_{t+1} + E_t \eta \frac{l}{1-l} \hat{l}_{t+1} = \rho \hat{C}_t + \eta \frac{l}{1-l} \hat{l}_t + \hat{\imath}_t - E_t \pi_{t+1} \tag{A1}$$

$$\rho E_t \hat{C}^*_{t+1} + E_t \eta \frac{l}{1-l} \hat{l}^*_{t+1} = \rho \hat{C}^*_t + \eta \frac{l}{1-l} \hat{l}^*_t + \hat{\imath}^*_t - E_t \pi^*_{t+1} \tag{A2}$$

$$E_t \Delta \hat{s}_{t+1} = \hat{\imath}_t - \hat{\imath}^*_t + \varepsilon \hat{b}_t \tag{A3}$$

$$-\hat{C}_t + \hat{w}_t = \eta \frac{l}{1-l} \hat{l}_t \tag{A4}$$

$$-\hat{C}^*_t + \hat{w}^*_t = \eta \frac{l}{1-l} \hat{l}^*_t \tag{A5}$$

$$\beta(1+\bar{a}\varepsilon)\hat{b}_t = \hat{b}_{t-1} + \bar{a}\left(\beta i_t^* + \Delta s_t - \pi_t\right) + (1-\theta)(v-1)\bar{\gamma}\hat{T}_t \tag{A6}$$
$$+\theta\left(\bar{\gamma} - v\omega\right)\widehat{RS}_t + \omega(v-1)\hat{Y}_t + \left[\bar{\gamma} - v\omega\right]\hat{Y}_t^*$$
$$+(\omega-1)\left((1-\kappa)\omega(v-1) + (1-\theta)\left[\bar{\gamma} - v\omega\right]\right)\hat{R}_t$$
$$+(\theta-\kappa)\left(\omega^*-1\right)\left[\bar{\gamma} - v\omega\right]\hat{R}_t^*$$

- The firms' optimality conditions for investment, capital and labor input in traded and non-traded sectors home...

$$\rho E_t\hat{C}_{t+1} + E_t\eta\frac{l}{1-l}\hat{l}_{t+1} = \rho\hat{C}_t + \eta\frac{l}{1-l}\hat{l}_t \tag{A7}$$
$$+(1+\beta(\delta-1))\left[\hat{w}_{t+1} + \hat{l}_{H_{t+1}} - \hat{k}_{H_t}\right]$$
$$-b\delta\left[\hat{x}_{H_t} - \hat{k}_{H_{t-1}}\right] + b\delta\beta\left[\hat{x}_{H_{t+1}} - \hat{k}_{H_t}\right]$$

$$\rho E_t\hat{C}_{t+1} + E_t\eta\frac{l}{1-l}\hat{l}_{t+1} = \rho\hat{C}_t + \eta\frac{l}{1-l}\hat{l}_t \tag{A8}$$
$$+(1+\beta(\delta-1))\left[\hat{w}_{t+1} + \hat{l}_{N_{t+1}} - \hat{k}_{N_t}\right]$$
$$-b\delta\left[\hat{x}_{N_t} - \hat{k}_{N_{t-1}}\right] + b\delta\beta\left[\hat{x}_{N_{t+1}} - \hat{k}_{N_t}\right]$$

$$\widehat{k}_{H_t} = (1-\delta)\widehat{k}_{H_{t-1}} + \delta\widehat{x}_{H_t} \tag{A9}$$

$$\widehat{k}_{N_t} = (1-\delta)\widehat{k}_{N_{t-1}} + \delta\widehat{x}_{N_t} \tag{A10}$$

$$\hat{w}_t = (v-1)\hat{T}_t + (\omega-1)\hat{R}_t + \alpha\hat{A}_{H_t} + (\alpha-1)\hat{l}_{H_t} + (1-\alpha)k_{H_{t-1}} \tag{A11}$$

$$\hat{w}_t = \omega\hat{R}_t + \alpha\hat{A}_{N_t} + (\alpha-1)\hat{l}_{N_t} + (1-\alpha)k_{N_{t-1}} \tag{A12}$$

...and foreign

$$\rho E_t\hat{C}_{t+1}^* + E_t\eta\frac{l}{1-l}\hat{l}_{t+1}^* = \rho\hat{C}_t^* + \eta\frac{l}{1-l}\hat{l}_t^* \tag{A13}$$
$$+(1+\beta(\delta-1))\left[\hat{w}_{t+1}^* + \hat{l}_{F_{t+1}}^* - \hat{k}_{F_t}^*\right]$$
$$-b\delta\left[\hat{x}_{F_t}^* - \hat{k}_{F_{t-1}}^*\right] + b\delta\beta\left[\hat{x}_{F_{t+1}}^* - \hat{k}_{F_t}^*\right]$$

$$\rho E_t\hat{C}_{t+1}^* + E_t\eta\frac{l}{1-l}\hat{l}_{t+1}^* = \rho\hat{C}_t^* + \eta\frac{l}{1-l}\hat{l}_t^* \tag{A14}$$
$$+(1+\beta(\delta-1))\left[\hat{w}_{t+1}^* + \hat{l}_{N_{t+1}}^* - \hat{k}_{N_t}^*\right]$$
$$-b\delta\left[\hat{x}_{N_t}^* - \hat{k}_{N_{t-1}}^*\right] + b\delta\beta\left[\hat{x}_{N_{t+1}}^* - \hat{k}_{N_t}^*\right]$$

$$\hat{k}^*_{F_t} = (1-\delta)\hat{k}_{F_{t-1}} + \delta\hat{x}_{F_t} \tag{A15}$$

$$\hat{k}^*_{N_t} = (1-\delta)\hat{k}^*_{N_{t-1}} + \delta\hat{x}^*_{N_t} \tag{A16}$$

$$\hat{w}^*_t = v\hat{T}_t - \widehat{RS}_t + (\omega-1)\hat{R}_t + \alpha\hat{A}_{F_t} + (\alpha-1)\hat{l}_{F_t} + (1-\alpha)\hat{k}_{F_{t-1}} \tag{A17}$$

$$\hat{w}^*_t = \omega^*\hat{R}^*_t + \alpha\hat{A}^*_{N_t} + (\alpha-1)\hat{l}^*_{N_t} + (1-\alpha)\hat{k}^*_{N_{t-1}} \tag{A18}$$

- Production constraints - home ...

$$\begin{aligned} &-\theta(v-1)\widehat{T}_t + \frac{\omega v}{\bar{\gamma}}\widehat{Y}_t + \frac{\bar{\gamma}-v\omega}{\bar{\gamma}}\left[\theta\widehat{RS}_t + \widehat{Y}^*_t\right] \\ &+\left[\frac{\omega v}{\bar{\gamma}}\kappa(1-\omega) + \frac{\bar{\gamma}-v\omega}{\bar{\gamma}}\theta(1-\omega)\right]\hat{R}_t \\ &+\frac{\bar{\gamma}-v\omega}{\bar{\gamma}}(\theta-\kappa)(\omega^*-1)\hat{R}^*_t \\ &= \left((1-\alpha)\widehat{k}_{H_{t-1}} + \alpha\widehat{l}_{H_t} + \alpha\widehat{A}_{H_t}\right) \end{aligned} \tag{A19}$$

$$-\kappa\omega\hat{R}_t + \hat{Y}_t = \left((1-\alpha)\widehat{k}_{N_{t-1}} + \alpha\widehat{l}_{N_t} + \alpha\widehat{A}_{N_t}\right) \tag{A20}$$

and foreign

$$\begin{aligned} &-v\theta\hat{T}_t + \frac{(1-v)v^*\omega}{\omega(1-v)v^* + (1-v^*)\left[\bar{\gamma}-v\omega\right]}\left[-\kappa(\omega-1)\hat{R}_t + \hat{Y}_t\right] \\ &+\frac{(1-v^*)\left[\bar{\gamma}-v\omega\right]}{\omega(1-v)v^* + (1-v^*)\left[\bar{\gamma}-v\omega\right]}\left[\begin{array}{c}(\theta-\kappa)(\omega^*-1)\hat{R}^*_t \\ -\theta(\omega-1)\hat{R}_t + \theta\widehat{RS}_t + \hat{Y}^*_t\end{array}\right] \\ &= \left((1-\alpha^*)\widehat{k_F}^*_{t-1} + \alpha^*\widehat{l_F}^*_t + \alpha^*\widehat{A}^*_{F_t}\right) \end{aligned} \tag{A21}$$

$$-\kappa\omega^*\hat{R}^*_t + \hat{Y}^*_t = \left((1-\alpha)\widehat{k}_{N^*_{t-1}} + \alpha\widehat{l}_{N^*_t} + \alpha\widehat{A}_{N^*_t}\right) \tag{A22}$$

- Economy wide constraints

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{C}_t + \frac{\bar{x}_H}{\bar{Y}}\hat{x}_{H_t} + \frac{\bar{x}_H}{\bar{Y}}\hat{x}_{N_t} \tag{A23}$$

where

$$\frac{\bar{x}_H}{\bar{Y}} = \delta\left(\frac{1-\alpha}{1/\beta-(1-\delta)}\right)\bar{\gamma}$$
$$\frac{\bar{x}_N}{\bar{Y}} = \delta\left(\frac{1-\alpha}{1/\beta-(1-\delta)}\right)(1-\omega)$$

$$\hat{Y}_t^* = \frac{\bar{C}^*}{\bar{Y}^*}\hat{C}_t^* + \frac{\bar{x}_F}{\bar{Y}^*}\hat{x}_{F_t} + \frac{\bar{x}_N^*}{\bar{Y}^*}\hat{x}_{N_t}^* \tag{A24}$$

where

$$\frac{\bar{x}_F}{\bar{Y}^*} = \delta\left(\frac{1-\alpha}{1/\beta-(1-\delta)}\right)\left[\frac{\omega(1-v)\omega^* v^* + \omega^*(1-v^*)(\bar{\gamma}-v\omega)}{(\bar{\gamma}-v\omega)}\right]$$
$$\frac{\bar{x}_N^*}{\bar{Y}^*} = \delta\left(\frac{1-\alpha}{1/\beta-(1-\delta)}\right)(1-\omega^*)$$

$$\hat{l}_t = \frac{\bar{l}_H}{\bar{l}}\hat{l}_{H_t} + \frac{\bar{l}_N}{\bar{l}}\hat{l}_{N_t} \tag{A25}$$

where

$$\frac{\bar{l}_N}{\bar{l}} = \frac{1-\omega}{1+\bar{a}(\beta-1)}$$
$$\frac{\bar{l}_H}{\bar{l}} = \frac{\omega+\bar{a}(\beta-1)}{1+\bar{a}(\beta-1)}$$

$$\hat{l}_t^* = \frac{\bar{l}_F^*}{\bar{l}}\hat{l}_{H_t} + \frac{\bar{l}_N^*}{\bar{l}}\hat{l}_{N_t}^* \tag{A26}$$

where

$$\frac{\bar{l}_N^*}{\bar{l}} = \frac{(1-\omega^*)\left[(1-v)\omega+\bar{a}(\beta-1)\right]}{(1-\omega^* v^*)\left[(1-v)\omega+\bar{a}(\beta-1)\right]+\omega(1-v)v^*\omega^*}$$
$$\frac{\bar{l}_F^*}{\bar{l}} = 1-\frac{(1-\omega^*)\left[(1-v)\omega+\bar{a}(\beta-1)\right]}{(1-\omega^* v^*)\left[(1-v)\omega+\bar{a}(\beta-1)\right]+\omega(1-v)v^*\omega^*}$$

- The real exchange rate and the terms of trade

$$\widehat{RS}_t = (v-v^*)\hat{T}_t + (\omega-1)\hat{R}_t + (1-\omega^*)\hat{R}_t^* \tag{A27}$$

$$\hat{T}_t = \hat{T}_{t-1} + \Delta s_t + \pi_t^{F^*} - \pi_t^H \tag{A28}$$

- Inflation when monetary authorities in both countries set producer inflation to zero:

$$\pi_t = \omega(1-v)\pi_t^{F^*} + \omega(1-v)\Delta s_t \tag{A29}$$
$$\pi_t^* = \omega^* v^* \pi_t^H - \omega^* v^* \Delta s_t \tag{A30}$$

where

$$\pi_t^H = -\left(\frac{1-\omega}{\omega v+(1-\omega)}\right)\left[\hat{A}_{H,t}-\hat{A}_{N,t}\right] \tag{A31}$$
$$\pi_t^{F^*} = -\left(\frac{(1-\omega^*)}{\omega^*(1-v^*)+1-\omega^*}\right)\left[\hat{A}_{F,t}-\hat{A}_{N,t}^*\right] \tag{A32}$$

A31 and A32 are derived by setting equations (16) and (21) from the text equal to one another, dividing by the lagged value of the resulting expression and linearising. We make use of the fact that if the two sectors have the same labour intensity, α they will have the same capital - effective labor ratio.

In addition to these 32 equations of the model we also have 6 lag identities for $\{\hat{b}_{t-1},\ \hat{T}_{t-1},\ \hat{k}_{H_{t-1}},\ \hat{k}^*_{F_{t-1}},\ \hat{k}_{N_{t-1}},\ \hat{k}^*_{N_{t-1}}\}$ as well as two constructed variables which are not needed to solve the model, but are convenient since we are interested in their moments. First we define the consumption differential:

$$CC^*_t = \hat{C}_t - \hat{C}^*_t \tag{A39}$$

and then the current account:

$$CA_t = \hat{b}_t - \hat{b}_{t-1} \tag{A40}$$

Excluding the last two equations, we have a system of 34 variables in 34 equations, where the variables $\{\hat{b}_{t-1},\ \hat{T}_{t-1},\ \hat{k}_{H_{t-1}},\ \hat{k}^*_{F_{t-1}},\ \hat{k}_{N_{t-1}},\ \hat{k}^*_{N_{t-1}}\}$ are predetermined variables and the rest are treated like 'jump' variables.

Figure 1: The real exchange rate and its components (top panel) and relative consumption (bottom panel) following a positive productivity shock to the domestic traded goods sector in the presence of Arrow-Debreu securities.

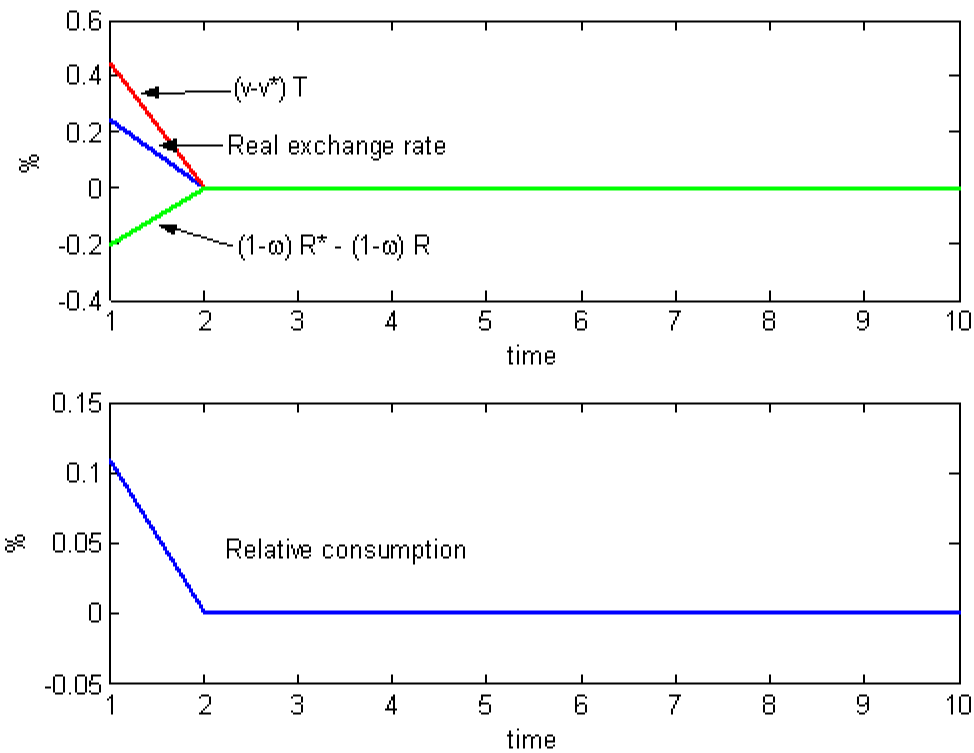

Figure 2: The real exchange rate and its components (top panel) and relative consumption (bottom panel) following a positive productivity shock to the domestic non-traded goods sector in the presence of Arrow-Debreu securities.

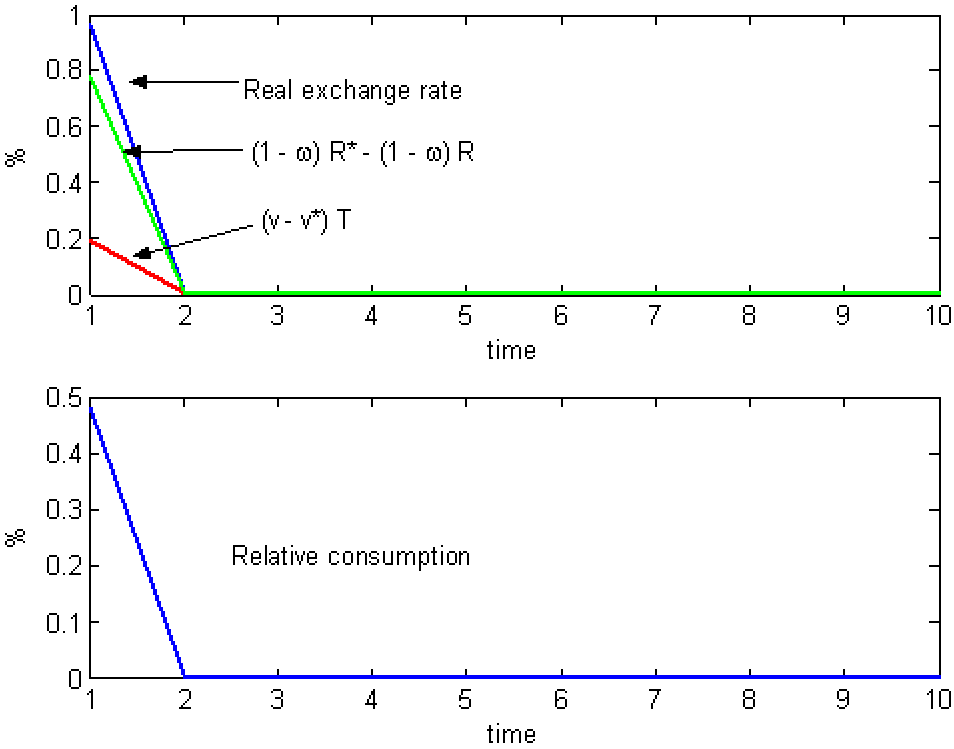

Figure 3: The real exchange rate and its components (top panel) and relative consumption and the current account (bottom panel) following a positive productivity shock to the domestic traded goods sector where financial markets are incomplete.

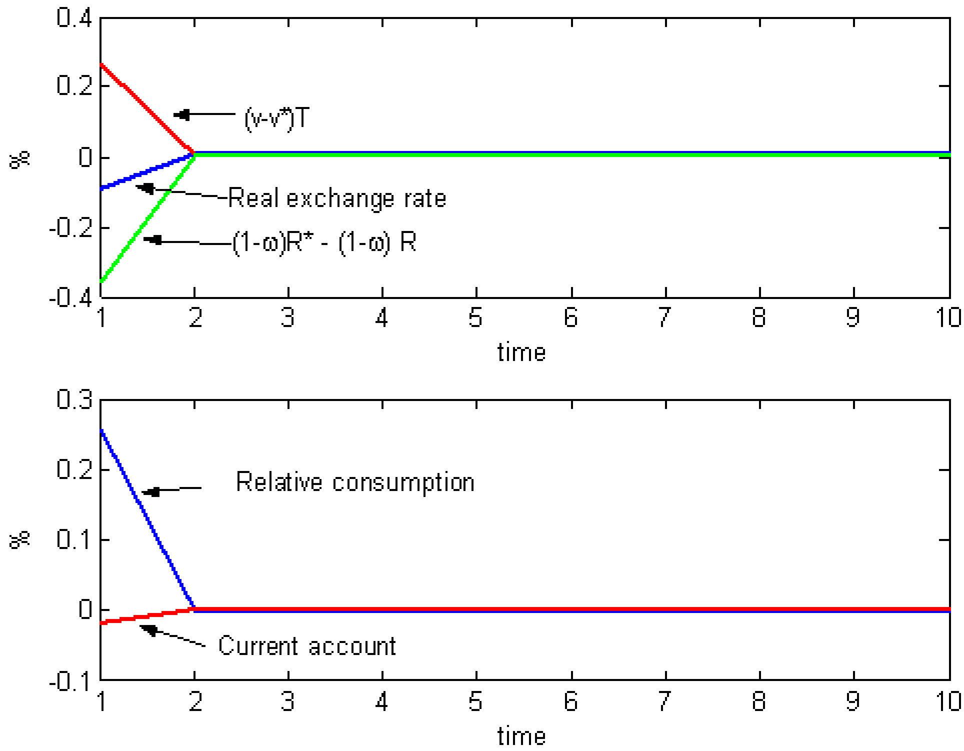

Figure 4: The real exchange rate and its components (top panel) and relative consumption and the current account (bottom panel) following a positive productivity shock to the domestic non-traded goods sector where financial markets are incomplete.

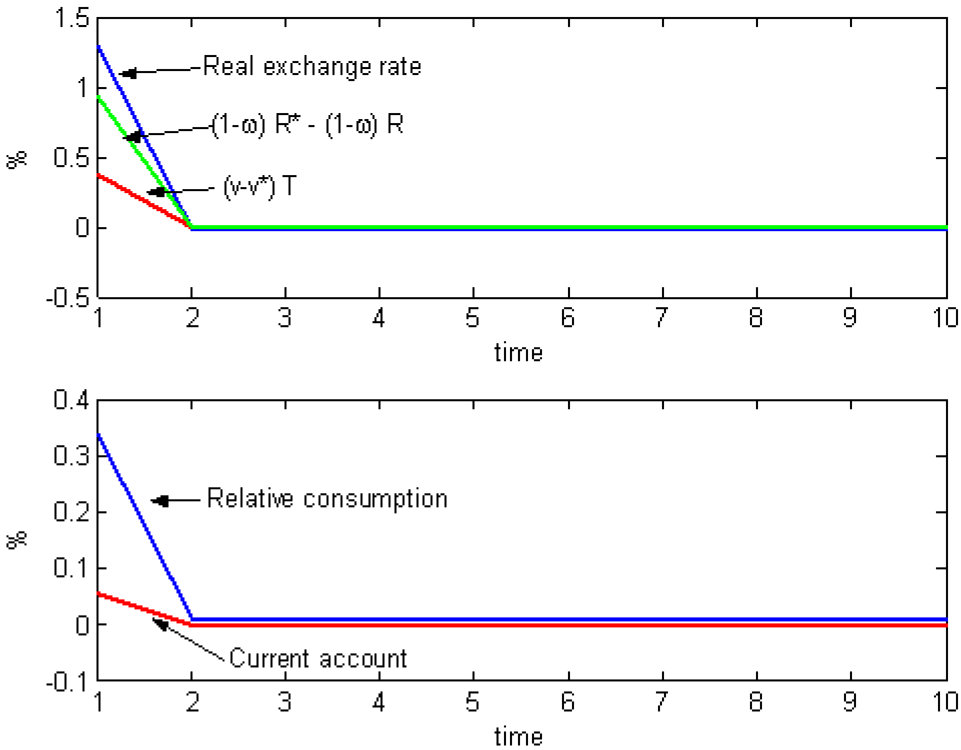

Figure 5: The real exchange rate and its components (top panel) and relative consumption and the current account (bottom panel) following a positive productivity shock to the domestic traded goods sector where financial markets are incomplete - the Chari et al case.

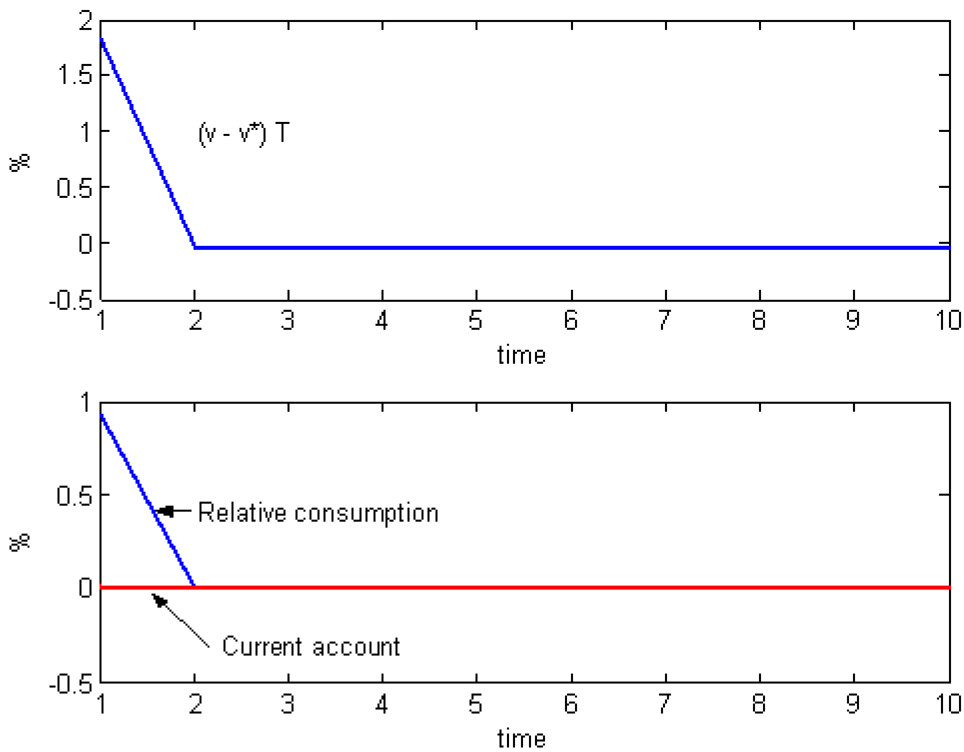

Table 1: Selected Cross-correlations between real exchange rate and relative consumption. Source: Corsetti et al (2004)

	Correlations			
	HP-Filtered		First-Difference	
Country	US	OECD	US	OECD
Australia	-0.01	0.05	-0.09	-0.13
Austria	-0.35	-0.54	-0.20	-0.30
Belgium	-0.12	0.15	-0.11	0.19
Canada	-0.41	-0.10	-0.20	0.02
Denmark	-0.16	-0.27	-0.20	-0.21
EU	-0.30	-0.10	-0.23	-0.04
Finland	-0.27	-0.64	-0.40	-0.55
France	-0.18	0.12	-0.21	-0.01
Germany	-0.27	-0.17	-0.13	0.01
Italy	-0.26	-0.51	-0.27	-0.31
Japan	0.09	0.27	0.04	0.08
South Korea	-0.73	-0.50	-0.79	-0.63
Mexico	-0.73	-0.77	-0.68	-0.74
Netherlands	-0.41	-0.20	-0.30	-0.19
New Zealand	-0.25	-0.37	-0.27	-0.28
Portugal	-0.56	-0.73	-0.48	-0.67
Sweden	-0.52	-0.39	-0.34	-0.29
Spain	-0.60	-0.66	-0.41	-0.38
Switzerland	0.16	0.53	0.09	0.32
Turkey	-0.31	-0.25	-0.34	-0.17
United Kingdom	-0.47	0.08	-0.40	-0.04
United States	-	-0.30	-	-0.31
Median	-0.30	-0.27	-0.27	-0.21

Table 2: Parameter values

Preferences	$\beta = 0.96$, $\rho = 2$, $\eta = 3.17$
Final goods technology	$\omega = \omega^* = 0.4$, $v = v^* = 0.7$, $\theta = 1$, $\kappa = 0.44$
Intermediate goods technology	$\alpha = 0.585$, $\delta = 0.1$, d adjusted
Financial markets	$\varepsilon = 0.004$, $\bar{a} = 0$
Shocks	$\Omega = \begin{bmatrix} 0.154 & -0.199 & 0.040 & 0.262 \\ -0.199 & 0.154 & 0.262 & 0.040 \\ -0.150 & -0.110 & 0.632 & 0.125 \\ -0.110 & -0.150 & 0.125 & 0.632 \end{bmatrix}$
	$V[\mu] = \begin{bmatrix} 3.62 & 1.21 & 1.23 & 0.51 \\ 1.21 & 3.62 & 0.51 & 1.23 \\ 1.23 & 0.51 & 1.99 & 0.27 \\ 0.51 & 1.23 & 0.27 & 1.99 \end{bmatrix}$

Table 3: Data and Models - Baseline model and sensitivity analysis

	Data Corsetti	Baseline Model	Arrow Debreu	Home-bias $v = 0.85$ $v^{*} = 0.15$	$\theta = 0.5$ $(\theta = 1.5)$	$\kappa = 0.74$	$\omega = 0.5$	Chari et al $\omega = \omega^{*} = 1$ $v = 0.984$
Standard deviations relative to GDP								
Real exchange rate	3.28	2.91	0.71	2.97	2.87 (3.00)	2.60	2.07	0.68
Terms of trade	1.78	2.04	1.21	2.14	2.56 (1.83)	1.94	1.62	0.70
Consumption	0.92	0.21	0.21	0.21	0.2 (0.23)	0.22	0.20	0.17
Investment	4.25	3.08	3.13	3.11	3.11 (3.06)	3.07	3.10	3.19
Employment	1.09	0.46	0.46	0.49	0.47 (0.47)	0.46	0.45	0.50
Cross-correlations between home and foreign								
GDP	0.49	0.43	0.41	0.46	0.48 (0.38)	0.39	0.37	0.34
Consumption	0.32	0.53	0.50	0.64	0.83 (0.28)	0.43	0.40	0.37
Investment	0.08	0.43	0.34	0.44	0.46 (0.41)	0.41	0.40	0.34
Employment	0.32	0.60	0.56	0.45	0.76 (0.44)	0.58	0.66	0.35
Cross-correlation between GDP and								
Net exports	-0.51	-0.30	-0.28	-0.08	-0.49 (-0.31)	-0.34	-0.38	-0.53
Cross-correlation between real exchange rate								
and relative consumption	-0.45	-0.65	0.94	-0.09	0.00 (-0.87)	-0.71	-0.68	0.81
and terms of trade	0.60	0.92	0.44	0.95	0.91 (0.94)	0.92	0.89	1.00

ABOUT THE CDMA

The **Centre for Dynamic Macroeconomic Analysis** was established by a direct grant from the University of St Andrews in 2003. The Centre funds PhD students and a programme of research centred on macroeconomic theory and policy. Specifically, the Centre is interested in the broad area of dynamic macroeconomics but has a particular interest in a number of specific areas such as: characterising the key stylised facts of the business cycle; constructing theoretical models that can match these actual business cycles; using these models to understand the normative and positive aspects of the macroeconomic policymakers' stabilisation problem; the problem of financial constraints and their impact on short and long run economic outcomes. The Centre is also interested in developing numerical tools for analysing quantitative general equilibrium macroeconomic models (such as developing efficient algorithms for handling large sparse matrices). Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

THE CDMA INAUGURAL CONFERENCE 2004

The Inaugural CDMA Conference was held in St. Andrews on the 17th and 18th of September 2004. The list of delegates attending, and the group photo, can be found here.

PAPERS PRESENTED AT THE CONFERENCE, IN ORDER OF PRESENTATION:

Title	Author(s) (presenter in bold)
A Critique of rule-of-thumb/indexing Microfoundations for inflation persistence	**Richard Mash (Oxford)**
Fiscal and Monetary Policy Interactios in a New Keynesian Model with Liquidity Constraints	**V. Anton Muscatelli (Glasgow)**, Patrizio Tirelli (Milano-Bicocca) and Carmine Trecroci (Brescia)
Inflation Persistence as Regime Change in a Classical Macro Model	**Patrick Minford (Cardiff and CEPR)**, Eric Nowell (Liverpool), Prakriti Sofat (Cardiff) and Naveen Srinivasan (Cardiff)
Habit Formation and Interest Rate Smoothing	**Luisa Corrado (Rome 'Tor Vergata')** and Sean Holly (Cambridge)
A Model of Job and Worker Flow	**Nobuhiro Kiyotaki (LSE)** and Richard Lagos (FRB of Minneapolis and New York)
The Specification of Monetary Policy Inertia in Empirical Taylor Rules	**John Driffill (Birkbeck, London)** and Zeno Rotondi (Ferrera and Capitalia)
Inequality and Industrialization	**Parantap Basu (Durham)** and Alessandra Guariglia (Nottingham)
Public Expenditures, Bureaucratic Corruption and Economic Development	**Keith Blackburn (Manchester)**, Niloy Bose (Wisconsin) and M. Emanrul Haque (Nottingham)
On the Consumption-Real Exchange Rate Anomaly	Gianluca Benigno (LSE and CEPR) and **Christoph Thoenissen (St Andrews)**
The Issue of Persistence in DGE Models with Heterogeneous Taylor Contracts	**Huw Dixon (York)** and Engin Kara (York
Performance of Inflation Targeting Based on Constant Interest Rate Projections	**Seppo Honkapohja (Cambridge)** and Kaushik Mitra (Royal Holloway, London)

See also the CDMA Working Paper series.