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Dynamic Stochastic General Equilibrium and Business Cycles

University Paris-Dauphine
Pre-PhD Program

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Chapter 6: International Business Cycles

Content of the Lecture

Objectives of the lecture

- Understanding the theoretical foundations of international business cycle theory;
- Showing the implications of trade channels and the exchange rate on the business cycles;

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1. A two-country world

Much effort has been devoted to extending the closed economy Real Business Cycle (RBC) model to an international setting. One of the first two-country model was build by Backus et al. (1992). These new class of real business cycles models were promising but were characterized by an important number of anomalies leading to many puzzles for modern open economy theory. As underlined by Kollmann et al. (1992), in these models, each country is inhabited by a representative agent. Countries interact by trading in goods markets and in financial markets. A central assumption in this recent work is that there exist complete international asset markets.

We develop now a symmetric two-country model characterized by sticky prices, flexible exchange rate regime and tradable intermediate goods internationally. The world economy is composed of two countries $i = h, f$ equal in size where h is for home and f for foreign.

1.1 The household

Our economy i is populated by identical and infinite living households distributed over a continuum $j \in [0, 1]$. The representative household utility function reads as follows:

$$\mathcal{U}\left(C_{i,t}\left(j\right), H_{i,t}\left(j\right)\right) = \frac{C_{i,t}\left(j\right)^{1-\sigma_C}}{1-\sigma_C} - \chi \frac{H_{i,t}\left(j\right)^{1+\sigma_L}}{1+\sigma_L}$$

Utility is increasing in consumption $C_{i,t}(j)$ and decreasing in labour $H_{i,t}(j)$ where σ_C and σ_L are curvature parameters. Households in economy $i = h, f$ maximise the expected value of their utility:

$$\max_{\{C_{i,t}(j),H_{i,t}(j),b_{i,t}(j),b_{j,t}(j),K_{i,t-1}(j),I_{i,t}(j)\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \mathcal{U}\left(C_{i,t+\tau}\left(j\right), H_{i,t+\tau}\left(j\right)\right), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor and country i denotes the home country and j the foreign one such that $i \neq j \in \{h, f\}$. Households face an intertemporal problem, they find the value of consumption $C_{i,t}(j)$, hours worked $H_{i,t}(j)$, home real bonds $b_{i,t}(j)$, foreign real bonds $b_{j,t}(j)$, physical capital $K_{i,t}(j)$ and investment $I_{i,t}(j)$ which delivers the highest value of utility. They face the following budget constraint which binds every period:

$$\begin{aligned} w_{i,t}H_{i,t}\left(j\right) + z_{i,t}K_{i,t-1}\left(j\right) + \Pi_{i,t}\left(j\right) + \frac{R_{i,t-1}}{\pi_{i,t}}b_{i,t-1}\left(j\right) + e_{i,t}\frac{R_{j,t-1}}{\pi_{i,t}}b^{j}_{i,t-1}\left(j\right) \\ = wC_{i,t}\left(j\right) + I_{i,t}\left(j\right) + b_{i,t}\left(j\right) + e_{i,t}b^{j}_{i,t}\left(j\right) + T_{i,t}\left(j\right) + \frac{\chi_B}{2}\left(e_{i,t}b^{j}_{i,t}\left(j\right)\right)^2, \end{aligned} \tag{2}$$

where $w_{i,t}$ is the real wage, $z_{i,t}$ the real remuneration of capital services, $R_{i,t-1}$ is the home nominal interest rate on riskless bonds subscribed in $t-1$ while $R_{j,t-1}$ is the foreign one, $e_{i,t}$ is the nominal exchange rate (an increase of $e_{i,t}$ is associated to an appreciation of the home currency), π_t is the inflation rate $\pi_{i,t} = P_{i,t}/P_{i,t-1}$, $T_{i,t}(j)$ is a tax to finance governments and $0.5\chi_B(e_t b^{j}_{i,t}(j))^2$ is a portfolio adjustment cost on the foreign debt which generates a country premium when borrowing externally.[2] Monopolistic competition on the goods market generates real dividends $\Pi_{i,t}(j)$ which are

[2]This assumtion is deemed necessary to close open economy models, see Schmitt-Grohé and Uribe (2003) for an extensive presentation and solutions. Without country premium, the debt dynamic is not stationnary because of an unit root.

redistributed to households. The law of motion of capital also bounds the household's optimization problem:

$$K_{i,t}(j) = \left(1 - AC^{I}_{i,t}(j)\right) I_{i,t}(j) + (1-\delta) K_{i,t-1}(j), \quad (3)$$

where $K_{i,t}(j)$ is the stock of capital bought in t, $I_{i,t}(j)$ denotes investment in t while parameter δ denotes the depreciation rate of physical capital. Investment is cost, a fraction of investment is lost when setting new physical capital and reflects some congestion costs, the adjustment cost function reads as in Christiano et al. (2005):

$$AC^{I}_{i,t}(j) = 0.5\chi_I \left(\frac{I_{i,t}(j)}{I_{i,t-1}(j)} - 1\right)^2 \quad (4)$$

where χ_I denotes the cost parameter.

The solutions of the optimisation problem are obtained by maximising household's utility under budget and capital accumulation constraint. The Euler equation which determines the optimal allocation of consumption and saving is:

$$\beta \frac{R_{i,t}}{\mathbb{E}_t \pi_{i,t+1}} = \mathbb{E}_t \left\{\frac{C_{i,t+1}(j)}{C_{i,t}(j)}\right\}^{\sigma_C}, \quad (5)$$

an hour supply equation:

$$w_{i,t} = \chi H_{i,t}(j)^{\sigma_L} C_{i,t}(j)^{\sigma_C}, \quad (6)$$

the real asset price equation is given by $q_{i,t} = \lambda^{k}_{i,t}/\lambda^{c}_{i,t}$, we can rewrite first order conditions with the asset price equation:

$$q_{i,t} = 1 + q_{i,t} \frac{\partial \left[I_{i,t}(j) AC^{I}_{i,jt}(j)\right]}{\partial I_{i,t}(j)} + \beta \mathbb{E}_t \left\{\frac{\lambda^{c}_{i,t+1}}{\lambda^{c}_{i,t}} q_{i,t+1} \frac{\partial \left[I_{i,t+1}(j) AC^{I}_{i,t+1}(j)\right]}{\partial I_{i,t}(j)}\right\} \quad (7)$$

and a no-arbitrage condition between bonds and assets:

$$\frac{R_{i,t}}{\mathbb{E}_t \pi_{i,t+1}} = \mathbb{E}_t \left\{\frac{z_{i,t+1} + (1-\delta) q_{i,t+1}}{q_{i,t}}\right\} \quad (8)$$

Finally, the no-arbitrage condition between home and foreign assets delivers the uncovered interest rate parity condition:

$$\mathbb{E}_t \left\{\frac{e_{i,t+1}}{e_{i,t}}\right\} = \frac{R_{i,t}}{R_{j,t}} \left(1 + \chi_B e_{i,t} b^{j}_{i,t}\right) \quad (9)$$

When Equation 9 binds, the expected variation of the nominal exchange rate is driven by the difference between the home and the foreign interest rate adjusted by the country risk. When domestic monetary policy tightens, investors seeking for the highest real interest rate invest in the home country, leading in turn to capital inflows and an appreciation of the domestic currency.

1.2 Firms

To introduce a monopolistic competition, the production process of goods is divided between two types of firms: intermediate and final firms. Intermediate firms (or wholesale goods producers) produce different types of goods which are imperfect substitutes (at the origin of the monopolistic competition). Final firms (or retailers) produce an homogenous good by combining home and foreign intermediate goods.

1.2.1 Final sector

The final good producers are retailers, they buy home intermediate goods $X^i_{i,t}$ and foreign ones $X^i_{j,t}$, package them into $Y_{i,t}$, and sell the final good to households. In equilibrium on the good market, $Y_{i,t}$ also denotes the aggregate demand for goods from households. On a perfectly competitive market, final producers maximise profits:

$$P_{i,t}Y_{i,t} - P^x_{i,t}X^i_{i,t} - e_{i,t}P^x_{j,t}X^i_{j,t}, \tag{10}$$

subject to a supply constraint:

$$Y_{i,t} = \left((1-\varphi)^{1/\upsilon} \left(X^i_{i,t}\right)^{(\upsilon-1)/\upsilon} + \varphi^{1/\upsilon} \left(X^i_{j,t}\right)^{(\upsilon-1)/\upsilon} \right)^{\upsilon/(\upsilon-1)}, \tag{11}$$

where Equation 11 is a CES aggregator of home and foreign intermediate good: $1-\varphi$ denote the fraction of home goods and φ the fraction of foreign goods involved in the production process with a substitutability parameter υ. Parameter υ accounts for the interest of firms in seeking for cheapest goods. The associated price index is given by:

$$P_{i,t} = \left[(1-\varphi) \left(P^x_{i,t}\right)^{1-\upsilon} + \varphi \left(e_{i,t}P^x_{j,t}\right)^{1-\upsilon} \right]^{1/(1-\upsilon)} \tag{12}$$

Here, $P^x_{i,t}$ denotes the domestic price of intermediate goods while $P^x_{j,t}$ is the price of foreign intermediate goods affected by the nominal exchange rate $e_{i,t}$.

Optimising the profits under the supply constraint, we obtain the demand for home and foreign goods from final goods in country i:

$$\underbrace{X^i_{i,t} = (1-\varphi) \left(\frac{P^x_{i,t}}{P_{i,t}} \right)^{-\upsilon} Y_{i,t}}_{\text{demand for domestic goods}} \text{ and } \underbrace{X^i_{j,t} = \varphi \left(\frac{e_{i,t}P^x_{j,t}}{P_{i,t}} \right)^{-\upsilon} Y_{i,t}}_{\text{demand for foreign goods}} \tag{13}$$

when the nominal exchange rate $e_{i,t}$ or the price of foreign intermediate goods $P^x_{j,t}$ increase, home firms reduces the demand for foreign goods $X^i_{j,t}$.

They also decide on the optimal amount of home and foreign varieties in each country i and j by solving in a second step:

$$P^x_{i,t}X^i_{i,t} - P^x_{i,t}\left(z\right)X^i_{i,t}\left(z\right) + P^x_{j,t}X^j_{j,t} - P^x_{j,t}\left(z\right)X^j_{i,t}\left(z\right) \tag{14}$$

Demands $X^i_{i,t}$ and $X^i_{j,t}$ are themselves combinations of the domestic and foreign intermediate goods according to:

$$X^i_{i,t} = \left(\int_0^1 X^i_{i,t}\left(z\right)^{1/\mu_t} \mathrm{d}z \right)^{\mu_t} \text{ and } X^j_{i,t} = \left(\int_0^1 X^j_{i,t}\left(z\right)^{1/\mu_t} \mathrm{d}z \right)^{\mu_t} \tag{15}$$

These aggregators Equation 15 implies that final producers have a technology which aggregate non-perfectly substitutable goods. This imperfect substitutability between all types of varieties i is driven by the monopolistic competition on the intermediate good market in each country. Each good z is an imperfect substitute, allowing intermediate firms to gain positive profits through a gap between their selling and producing price. The intensity of the monopolistic competition is driven by $\mu_{i,t}$ the mark-up over the producing price of intermediate firms. As in Smets and Wouters (2003), the markup $\mu_{i,t} = \exp(\varkappa\varepsilon^P_{i,t})\epsilon/\left(\epsilon-1\right)$ is assumed to be exogenously varying over time, ϵ denotes the imperfect substitutability between different goods varieties, $\varepsilon^P_{i,t}$ denotes the supply shock and $\varkappa$ is a scale parameter that normalizes the shock to unity in the log-linear form of the model as in Smets and Wouters (2007). From a microeconomic perspective, the supply shock captures changes in the margins of firms over the business cycles. From a macroeconomic perspective, this supply shock catches up shocks to the inflation equation, such as oil price shock, commodity prices shock, etc.

Finally, we find the intermediate demand function for each variety i is:

$$X_{i,t}^{i}\left(z\right)=\left(\frac{P_{i,t}^{x}\left(z\right)}{P_{i,t}^{x}}\right)^{-\frac{\mu_{i,t}}{\mu_{i,t}-1}}X_{i,t}^{i},\ \ \forall z \tag{16}$$

$$X_{i,t}^{j}\left(z\right)=\left(\frac{P_{j,t}^{x}\left(z\right)}{P_{ij,t}^{x}}\right)^{-\frac{\mu_{j,t}}{\mu_{j,t}-1}}X_{i,t}^{j},\ \ \forall z \tag{17}$$

The zero-profit condition on intermediate goods sectors gives the following price index:

$$P_{i,t}^{x}=\left(\int_{0}^{1}P_{i,t}^{x}\left(z\right)^{\frac{1}{1-\mu_{i,t}}}\mathrm{d}z\right)^{1-\mu_{i,t}}\text{ and }P_{j,t}^{x}=\left(\int_{0}^{1}P_{j,t}^{x}\left(z\right)^{\frac{1}{1-\mu_{j,t}}}\mathrm{d}z\right)^{1-\mu_{j,t}} \tag{18}$$

1.2.2 Intermediate sector

In each country, firms are homogeneous and distributed on a unitary interval $z\in[0,1]$ and have the following production technology:

$$X_{i,t}\left(z\right)=\varepsilon_{i,t}^{A}K_{i,t-1}\left(z\right)^{\alpha}H_{i,t}\left(z\right)^{1-\alpha} \tag{19}$$

where $X_{i,t}\left(z\right)$ is the intermediate sector production, $K_{i,t-1}\left(z\right)$ and $H_{i,t}\left(z\right)$ are input factors, *i.e.* the quantity of physical capital and hours worked involved in the production process of firms. Finally, $\varepsilon_{i,t}^{A}$ denotes the productivity shock process.

Intermediate goods producers solve a two-stage problem. In the first stage, taken the real input prices $w_{i,t}$ and $z_{i,t}$ as given, firms rent inputs $H_{i,t}^{d}\left(i\right)$ and $K_{i,t}^{u}\left(i\right)$ in a perfectly competitive factor markets in order to minimize costs subject to the production constraint (19).

The first stage can be expressed as a profit maximisation problem:

$$\max_{\{Y_{i,t}(z),K_{i,t-1}(z),H_{i,t}(z)\}}mc_{i,t}\left(z\right)X_{i,t}\left(z\right)-z_{t}K_{i,t-1}\left(z\right)-w_{i,t}H_{i,t}\left(z\right)+\lambda_{t}\left[\varepsilon_{i,t}^{A}K_{i,t-1}\left(z\right)^{\alpha}H_{i,t}\left(z\right)^{1-\alpha}-X_{i,t}\left(z\right)\right]$$

where $mc_{i,t}\left(z\right)$ denotes the real marginal cost of producing one additional good. First order conditions are:

$$\left(\partial X_{i,t}\left(z\right)\right):\lambda_{t}=mc_{i,t}\left(z\right) \tag{20}$$

$$\left(\partial K_{i,t-1}\left(z\right)\right):z_{i,t}=\alpha\lambda_{t}\frac{X_{i,t}\left(z\right)}{K_{i,t-1}\left(z\right)} \tag{21}$$

$$\left(\partial H_{i,t}\left(z\right)\right):w_{i,t}=\left(1-\alpha\right)\lambda_{t}\frac{X_{i,t}\left(z\right)}{H_{i,t}\left(z\right)}. \tag{22}$$

Replacing the lagrange multiplier, costs are minimised when the following condition holds:

$$\left(1-\alpha\right)K_{i,t-1}\left(z\right)z_{i,t}=\alpha H_{i,t}\left(z\right)w_{i,t} \tag{23}$$

and the real marginal cost is determined by combining the production function in Equation 19 with the three minimizing conditions on inputs in Equation 20:

$$mc_{i,t}\left(z\right)=\frac{1}{\varepsilon_{i,t}^{A}}\left(\frac{z_{i,t}}{\alpha}\right)^{\alpha}\left(\frac{w_{i,t}}{1-\alpha}\right)^{\left(1-\alpha\right)} \tag{24}$$

In the second stage problem, the firms decides its selling price on a Calvo basis with nominal rigidities. This nominal rigidity prevent firms to optimally set prices. There is a fraction of firms θ that is not allowed to reset price, prices then evolves according to $P_{i,t}^{x}\left(z\right)=P_{i,t-1}^{x}\left(z\right)$ while for the other remain share of firms $1-\theta$, they are able to set their selling price such that $P_{i,t}^{x}\left(z\right)=P_{i,t}^{x*}\left(z\right)$, where $P_{i,t}^{x*}\left(z\right)$ denotes the optimal price set by the representative firm. Price law of motion can be summarised by:

$$P_{i,t}^{x}=\begin{cases} P_{i,t}^{x*}\left(z\right) & \text{with probability } 1-\theta \\ P_{i,t-1}^{x}\left(z\right) & \text{with probability } \theta \end{cases}$$

Due to the nominal rigidity, the price sets now by the firm may last more than one period, which will affect the expected profits of the firms. Firms allowed to choose their new price $P_{i,t}^{x*}\left(z\right)$ will consider the perspective of being price constrained in the future. The firm allowed to reset its selling price with a probability $1-\theta$ maximises the following expected real sum of discounted profits:

$$\max_{P_{it}^{*}}\mathbb{E}_{t}\sum_{\tau=0}^{+\infty}\frac{\lambda_{i,t+\tau}^{c}}{\lambda_{i,t}^{c}}\left(\beta\theta\right)^{\tau}\left[\frac{P_{i,t}^{x*}\left(z\right)}{P_{i,t+\tau}}-\frac{MC_{i,t+\tau}}{P_{i,t+\tau}}\right]X_{i,t+\tau}\left(z\right) \tag{25}$$

Since firms are owned by households, they discount the expected profits using the same discount factor than households $(\beta^{\tau}\lambda_{i,t+\tau}^{c}/\lambda_{i,t}^{c})$. The firm face the downward sloping constraint Equation 16 from final good producers (re-arranged because of nominal rigidities):

$$X_{i,t+\tau}\left(z\right)=\left(\frac{P_{i,t}^{x*}\left(z\right)}{P_{i,t}^{x}}\right)^{-\frac{\mu_{i,t+\tau}}{\mu_{i,t+\tau}-1}}X_{i,t+\tau}^{i},\quad\tau>0 \tag{26}$$

The first order condition, which defines the optimal price setting given the probability θ to be price constrained in the future, is determined by:

$$\mathbb{E}_{t}\sum_{\tau=0}^{+\infty}\frac{\lambda_{i,t+\tau}^{c}}{\lambda_{i,t}^{c}}\frac{\left(\beta\theta\right)^{\tau}}{\mu_{i,t+\tau}-1}\left[\frac{P_{i,t}^{x*}\left(z\right)}{P_{i,t+\tau}}-\mu_{i,t+\tau}mc_{i,t+\tau}\right]X_{i,t+\tau}\left(z\right)=0. \tag{27}$$

1.3 Authorities

To close the model, each country has a central bank which sets its interest rate according to a standard Taylor rule:

$$\log\left(\frac{R_{i,t}}{\overline{R}}\right)=\rho\log\left(\frac{R_{i,t-1}}{\overline{R}}\right)+\left(1-\rho\right)\phi_{\pi}\log\left(\frac{\pi_{i,t}}{\overline{\pi}}\right)+\log\left(\varepsilon_{i,t}^{R}\right), \tag{28}$$

where $R_{i,t}$ is the nominal interest rate, $\pi_{i,t}$ is the inflation rate and $\varepsilon_{i,t}^{R}$ is an AR(1) monetary policy shock. Finally, parameters $\overline{R}$ and $\overline{\pi}$ are long term values for the interest rate and the inflation rate (symmetric between countries). The central bank reacts to the deviation of the inflation rate from its steady state value in a proportion ϕ_{π}, the central bank also smooths her rate at a degree ρ.

Turning to fiscal authorities, the real budget constraint for the government is:

$$\int_0^1 T_{i,t}\left(j\right) \mathrm{d}j = G_{i,t} \tag{29}$$

where G_t denote public spending and $\int_0^1 T_{i,t}\left(j\right)\mathrm{d}j$ is the aggregate tax from all households in country i. Spending are assumed to be exogenously determined $G_{i,t} = g^y \bar{Y} \varepsilon_{i,t}^G$ and represents a demand shock, such that $\bar{G} = g^y \bar{Y}$, where g^y is the fixed spending-to-GDP ratio. Variable $\varepsilon_{i,t}^G$ denotes the demand shock which affects output.

1.4 Aggregation and Equilibrium conditions

Our economy is populated by two agents: households j and firms z. Their interactions, summarized by market clearing conditions for goods, labour and physical capital markets, can be expressed as follows. First, the demand (or the resource constraint) is given by:

$$Y_{i,t} = (\underbrace{\int_0^1 C_{i,t}\left(j\right) \mathrm{d}j}_{\text{aggregate consumption}} + \underbrace{\int_0^1 I_{i,t}\left(j\right) \mathrm{d}j}_{\text{aggregate investment}} + \underbrace{g^y \bar{Y} \varepsilon_{i,t}^G}_{\text{aggregate public spending}} + 0.5 \chi_B e_{i,t} b_{i,t}^j\left(j\right))$$

Letting Y_t, C_t and I_t denote the aggregate supply, consumption and investment, the resource constraint boils down to:

$$Y_{i,t} = C_{i,t} + I_{i,t} + g^y \bar{Y} \varepsilon_{i,t}^G + 0.5 \chi_B e_{i,t} b_{i,t}^j\left(j\right). \tag{30}$$

The equilibrium on the intermediate goods market:

$$\underbrace{X_{i,t}}_{\text{aggregate supply}} = \underbrace{\left(1-\varphi\right)\left(\frac{P_{i,t}^x}{P_{i,t}}\right)^{-\upsilon} \Delta_{i,t} Y_{i,t}}_{\text{home demand}} + \underbrace{\varphi\left(\frac{e_{j,t} P_{i,t}^x}{P_{j,t}}\right)^{-\upsilon} \Delta_{j,t} Y_{j,t}}_{\text{foreign demand}} \tag{31}$$

where $X_{i,t} = \int_0^1 X_{i,t}\left(z\right)\mathrm{d}z$ is the aggregate supply, $\Delta_{i,t}$ denote price dispersion term implied by nominal rigidities (here they are neglected as they clears up to a first order approximation). Given these nominal rigidities, the aggregate price index in Equation 18 is divided between sticky versus re-set prices:

$$P_{i,t}^x = \left(\left(1-\theta\right)\left(P_{i,t}^{x*}\right)^{\frac{1}{1-\mu_{i,t}}} + \theta\left(P_{i,t-1}^x\right)^{\frac{1}{1-\mu_{i,t}}}\right)^{1-\mu_{i,t}} \tag{32}$$

The equilibrium on the labour market is:

$$H_{i,t} = \underbrace{\int_0^1 H_{i,t}\left(z\right) \mathrm{d}z}_{\text{labour demand}} = \underbrace{\int_0^1 H_{i,t}\left(j\right) \mathrm{d}j}_{\text{labour supply}},$$

and for physical capital:

$$K_{i,t} = \underbrace{\int_0^1 K_{i,t}\left(z\right) \mathrm{d}z}_{\text{capital demand}} = \underbrace{\int_0^1 K_{i,t}\left(j\right) \mathrm{d}j}_{\text{capital supply}}.$$

As in open economy models, we solve the model from the home country perspective, this assumption strongly improves the clarity of the model's presentation and formulation. This way, from the home country perspective the nominal exchange rate e_t is:

$$e_t = e_{h,t} \text{ and } e_t = \frac{1}{e_{f,t}} \tag{33}$$

In a perfectly closed world, the net supply of foreign asset clears:

$$b^f_{h,t} + b^h_{f,t} = 0 \tag{34}$$

Then, we can substitute $b^h_{f,t}$ by $-b^f_{h,t}$. The net accumulation of foreign foreign asset in the home country h is given by:

$$b^f_{h,t} = \frac{e_t}{e_{t-1}} \frac{R_{f,t-1}}{\pi_t} b^f_{h,t-1} + \underbrace{\frac{P^x_{i,t}}{P_{i,t}} X_{i,t} - Y_{i,t}}_{\text{trade balance}}$$

The current account, from the home country perspective, is determined by foreign assets variations:[3]

$$CA_t = b^f_{h,t} - b^f_{h,t-1} \tag{35}$$

1.5 Disturbances

Our shock processes are characterized by four disturbances (in logs):

$$\begin{aligned}
\text{productivity} : \log\left(\varepsilon^A_{i,t}\right) &= \rho_A \log\left(\varepsilon^A_{i,t-1}\right) + \eta^A_{i,t} \text{ with } \eta^A_{i,t} \sim \mathcal{N}\left(0, \sigma^2_A\right) \\
\text{spending} : \log\left(\varepsilon^G_{i,t}\right) &= \rho_G \log\left(\varepsilon^G_{i,t-1}\right) + \eta^G_{i,t} \text{ with } \eta^G_{i,t} \sim \mathcal{N}\left(0, \sigma^2_G\right) \\
\text{cost-push} : \log\left(\varepsilon^P_{i,t}\right) &= \rho_P \log\left(\varepsilon^P_{i,t-1}\right) + \eta^P_{i,t} \text{ with } \eta^P_{i,t} \sim \mathcal{N}\left(0, \sigma^2_P\right) \\
\text{monetary policy} : \log\left(\varepsilon^R_{i,t}\right) &= \rho_R \log\left(\varepsilon^R_{i,t-1}\right) + \eta^R_{i,t} \text{ with } \eta^R_{i,t} \sim \mathcal{N}\left(0, \sigma^2_R\right)
\end{aligned}$$

1.6 Summary of the linear model and parametrisation

Here after, we summarize the binding equations of the model considering a first order approximation.

Our model is expressed in real terms with, $p^x_{i,t} = P^x_{i,t}/P_{i,t}$, the real exchange rate $rer_{i,t} = P_{j,t}e_{i,t}/P_{i,t}$, the real asset price $q_{i,t} = Q_{i,t}/P_{i,t}$

While the exchange rate is not stationary, a simple way to deal with this is to express the nominal in variations with $\Delta e_t = e_t/e_{t-1}$. From the home country perspective, the real exchange rate can be rewritten as a law of motion:

$$\frac{rer_t}{rer_{t-1}} = \Delta e_t \frac{\pi_{f,t}}{\pi_{h,t}} \tag{36}$$

While the price index $p^x_{i,t}$ can be expressed differently:

$$\frac{p^x_{i,t}}{p^x_{i,t-1}} = \frac{\pi^x_{i,t}}{\pi_{i,t}}$$

[3] Here again, the sum of current accounts surpluses/deficits in the world must equal 0. In a two-country set-up, $CA_{h,t} + CA_{f,t} = 0$, i.e. a deficit in one country is compensated by a surplus in the other economy.

The equilibrium on the intermediate good market (dropping $\Delta_{i,t}$) in each country can be rewritten in real terms:

$$X_{h,t} = (1-\varphi)\left(p^x_{h,t}\right)^{-\upsilon} Y_{h,t} + \varphi \left(\frac{p^x_{h,t}}{rer_t}\right)^{-\upsilon} Y_{f,t} \tag{37}$$

$$X_{f,t} = (1-\varphi)\left(p^x_{f,t}\right)^{-\upsilon} Y_{f,t} + \varphi \left(rer_t p^x_{f,t}\right)^{-\upsilon} Y_{h,t} \tag{38}$$

While the price indexes defined in Equation 12 divided by are $P_{i,t}$:

$$1 = (1-\varphi)\left(p^x_{h,t}\right)^{1-\upsilon} + \varphi \left(rer_t p^x_{f,t}\right)^{1-\upsilon} \tag{39}$$

$$1 = (1-\varphi)\left(p^x_{f,t}\right)^{1-\upsilon} + \varphi \left(\frac{p^x_{h,t}}{rer_t}\right)^{1-\upsilon} \tag{40}$$

Finally, we assume that net foreign assets are zero in the long run ($i.e.$ current accounts are balanced in steady state), implying that $\bar{b}^f_h = 0$. To log-linearize the net foreign assets with a zero steady state, we assume as in open economy literature that net foreign assets are expressed in terms of consumption goods, the log-linearisation of $b^f_{h,t}$ is thus $b^f_{h,t} \simeq \bar{C}(1+\hat{b}^f_{h,t})$ where $\hat{b}^f_{h,t}$ denotes the variation of real net foreign assets.

$$\text{Euler} : \hat{c}_{i,t} = \mathbb{E}_t \hat{c}_{i,t+1} - \frac{1}{\sigma_C} \left(\hat{r}_{i,t} - \mathbb{E}_t \widehat{\pi}_{i,t+1} \right) \text{ for } i = h, f \tag{41}$$

$$\text{labour supply} : \hat{w}_{i,t} = \sigma_L \hat{h}_{i,t} + \sigma_C \hat{c}_{i,t} \text{ for } i = h, f \tag{42}$$

$$\text{no-arbitrage} : \hat{r}_{i,t} - \mathbb{E}_t \pi_{i,t+1} = \left(\frac{\bar{Z}}{\bar{R}} \right) \mathbb{E}_t \hat{z}_{i,t+1} + \frac{(1-\delta)}{\bar{R}} \mathbb{E}_t \hat{q}_{i,t+1} \text{ for } i = h, f \tag{43}$$

$$\text{capital motion} : \delta \hat{\imath}_{i,t} = \hat{k}_{i,t} - (1-\delta) \hat{k}_{i,t-1} \text{ for } i = h, f \tag{44}$$

$$\text{asset price} : \hat{q}_{i,t} = \chi_I \left(\hat{\imath}_{i,t} - \hat{\imath}_{i,t-1} \right) - \beta \chi_I \left(\mathbb{E}_t \hat{\imath}_{i,t+1} - \hat{\imath}_{i,t} \right) \text{ for } i = h, f \tag{45}$$

$$\text{production} : \hat{x}_{i,t} = \hat{\varepsilon}^A_{i,t} + \alpha \hat{k}_{i,t-1} + (1-\alpha) \hat{h}_{i,t} \text{ for } i = h, f \tag{46}$$

$$\text{relative price} : \hat{p}^x_{i,t} - \hat{p}^x_{i,t-1} = \hat{\pi}^x_{i,t} - \hat{\pi}_{i,t} \text{ for } i = h, f \tag{47}$$

$$\text{home equilibrium} : \hat{x}_{h,t} = (1-\varphi) \left(\hat{y}_{h,t} - \upsilon \hat{p}^x_{h,t} \right) + \varphi (\hat{y}_{f,t} - \upsilon (\hat{p}^x_{h,t} - \widehat{rer}_t)) \tag{48}$$

$$\text{foreign equilibrium} : \hat{x}_{f,t} = (1-\varphi) \left(\hat{y}_{f,t} - \upsilon \hat{p}^x_{f,t} \right) + \varphi (\hat{y}_{h,t} - \upsilon (\hat{p}^x_{f,t} + \widehat{rer}_t)) \tag{49}$$

$$\text{home price index} : (1-\varphi) \hat{p}^x_{h,t} + \varphi (\widehat{rer}_t + \hat{p}^x_{f,t}) = 0 \tag{50}$$

$$\text{foreign price index} : (1-\varphi) \hat{p}^x_{f,t} + \varphi (\hat{p}^x_{h,t} - \widehat{rer}_t) = 0 \tag{51}$$

$$\text{marginal cost} : \widehat{mc}_{i,t} = \alpha \hat{z}_{i,t} + (1-\alpha) \hat{w}_{i,t} - \hat{\varepsilon}^A_{i,t} \text{ for } i = h, f \tag{52}$$

$$\text{cost minimisation} : \hat{w}_{i,t} + \hat{h}_{i,t} = \hat{z}_{i,t} + \hat{k}_{i,t-1} \text{ for } i = h, f \tag{53}$$

$$\text{NK-PC} : \widehat{\pi}^x_{i,t} = \beta \mathbb{E}_t \widehat{\pi}^x_{i,t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta} \widehat{mc}_{i,t} + \hat{\varepsilon}^P_{i,t} \text{ for } i = h, f \tag{54}$$

$$\text{gdp} : \bar{Y} \hat{y}_{i,t} = \bar{C} \hat{c}_{i,t} + \bar{I} \hat{\imath}_{i,t} + g^y \bar{Y} \hat{\varepsilon}^G_{i,t} \text{ for } i = h, f \tag{55}$$

$$\text{monetary policy} : \hat{r}_{i,t} = \rho \hat{r}_{i,t-1} + (1-\rho) \phi_\pi \widehat{\pi}_{i,t} + \hat{\varepsilon}^R_{i,t} \text{ for } i = h, f \tag{56}$$

$$\text{real exchange rate} : \widehat{rer}_t - \widehat{rer}_{t-1} = \Delta \hat{e}_t + \hat{\pi}_{f,t} - \hat{\pi}_{h,t} \tag{57}$$

$$\text{parity condition} : \mathbb{E}_t \Delta \hat{e}_{t+1} = r_{h,t} - r_{f,t} + \chi_B b^f_{h,t} \tag{58}$$

$$\text{NFA} : \hat{b}^f_{h,t} = \bar{R} \hat{b}^f_{h,t-1} + \frac{\bar{X}}{\bar{C}} \left(\hat{p}^x_{h,t} + \hat{x}_{h,t} - \hat{y}_{h,t} \right) \tag{59}$$

$$\text{current account} : \widehat{ca}_t = \hat{b}^f_{h,t} - \hat{b}^f_{h,t-1} \tag{60}$$

$$\text{productivity} : \hat{\varepsilon}^A_{i,t} = \rho_A \hat{\varepsilon}^A_{i,t-1} + \hat{\eta}^A_{i,t} \tag{61}$$

$$\text{spending} : \hat{\varepsilon}^G_{i,t} = \rho_G \hat{\varepsilon}^G_{i,t-1} + \hat{\eta}^G_{i,t} \tag{62}$$

$$\text{cost-push} : \hat{\varepsilon}^P_{i,t} = \rho_P \hat{\varepsilon}^P_{i,t-1} + \hat{\eta}^P_{i,t} \tag{63}$$

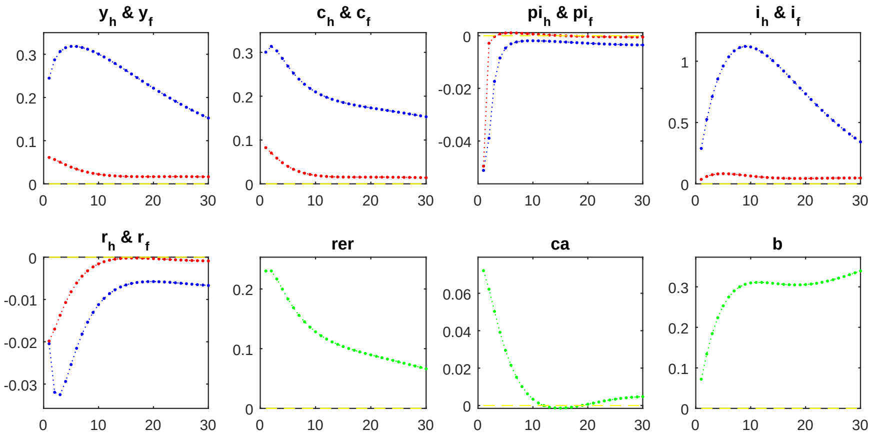
$$\text{policy shock} : \hat{\varepsilon}^R_{i,t} = \rho^R \hat{\varepsilon}^R_{i,t-1} + \hat{\eta}^R_{i,t} \tag{64}$$

Description	Name	Value
Discount factor	β	0.993
Depreciation rate	δ	0.025
Capital share in production	α	0.20
Consumption Risk	σ_C	1.4
Labour disutility	σ_L	2
Calvo lottery	θ	0.65
Investment cost	χ_I	5.5
Spending-to-GDP ratio	g^y	0.18
Monetary policy smoothing	ρ	0.80
Inflation reaction	ϕ_π	2
Bond adjustment cost	χ_B	0.01
Intermediate goods openness	φ	0.15
Substitution goods	ν	1.5
Productivity AR(1) term	ρ_A	0.95
Spending AR(1) term	ρ_G	0.97
Cost-push AR(1) term	ρ_P	0.90
Monetary Policy AR(1) term	ρ_R	0.12
Productivity shock std	$\sigma_{h,t}^A$	0.45%
Spending shock std	$\sigma_{h,t}^G$	0.50%
Cost-push shock std	$\sigma_{h,t}^P$	0.50%
Monetary Policy shock std	$\sigma_{h,t}^R$	0.25%

Table 1: Calibration on quartly basis

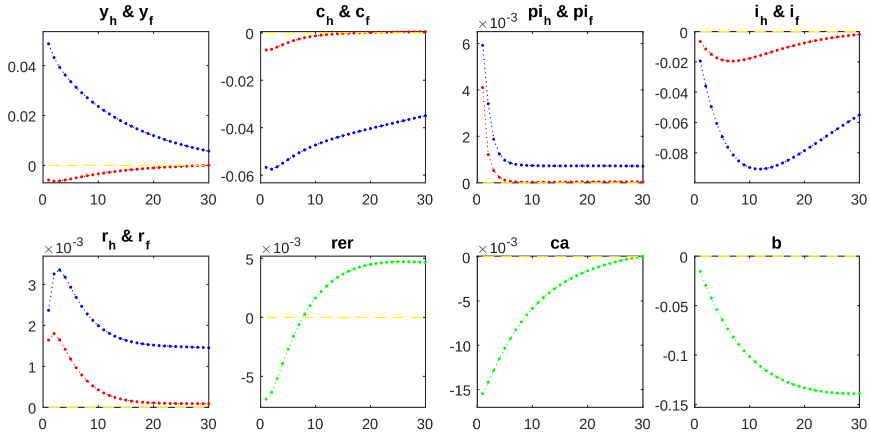
1.7 System responses

subsection 1.7 reports the system response after a productivity shock in the home country. A drop in inflation and an increase in output characterizes the productivity shock. Firms are more productive, they produce more with less inputs. In reaction, the central bank lowers its interest rate to revive inflation, leading in turn to an appreciation of the real exchange rate as the home real interest rate is higher than the foreign one. Since firms are more competitive, they export more which in turn generates current account surpluses. In addition, the appreciation of the exchange rate lowers the imports which amplifies trade balance surpluses. Finally, the decline of the interest rate strongly encourages investment. The foreign economy is slightly affected through the goods market and the exchange rate.



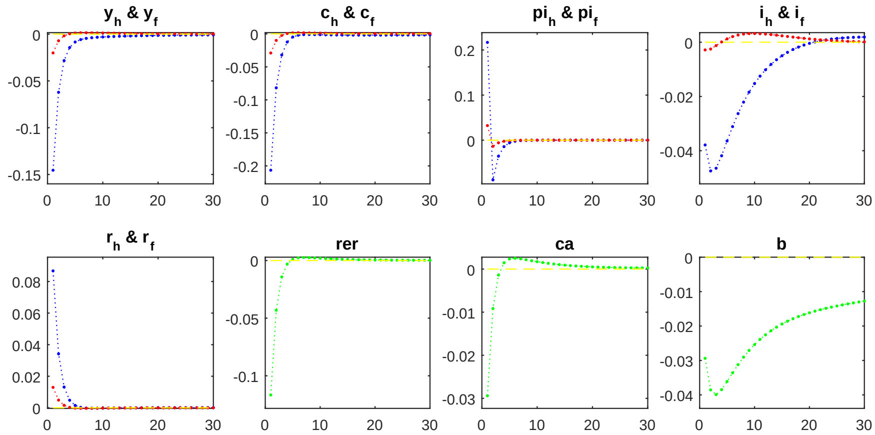
System response to a domestic productivity shock $\eta^{A}_{h,t}$, blue (red) is for the home (foreign) economy while world variables are in green.

subsection 1.7 reports the consequences of a public spending shock occurring in the domestic economy. This shock standardly increases the gdp and inflation via a demand effect. To fight inflation, the central bank rises its interest rate leading to an appreciation of the domestic currency. Both the central bank and the exchange rate reduce the effectiveness of fiscal policy stimulus. The demand shock also boosts imports and deteriorates the current account/trade balance. In the short run, the foreign economy is negatively affected by the rise of inflation of imported goods followed by a monetary policy tightening.



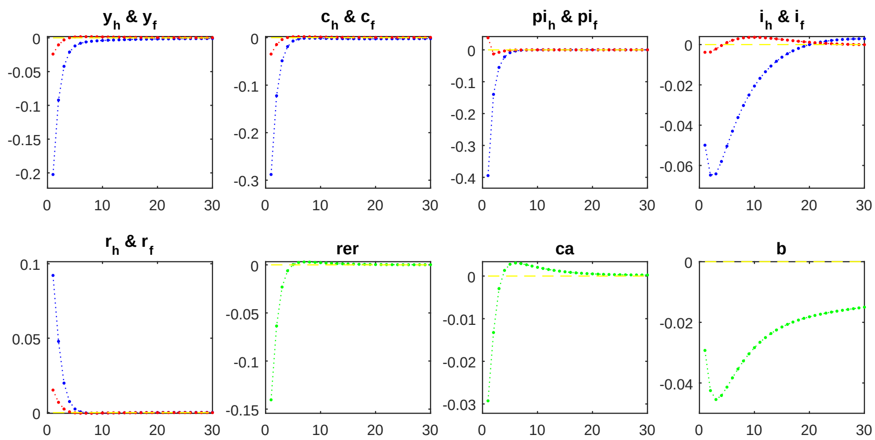
System response to a domestic spending shock $\eta^{G}_{h,t}$, blue (red) is for the home (foreign) economy while world variables are in green.

subsection 1.7 report the macroeconomic effect of a cost-push shock in the home country. Final firms increase their selling prices which in turn reduces real output through a demand effect. Higher prices reduce the demand for home made goods by domestic and foreign agents. In response to inflation, central banks increase their interest rates which conducts inflation to drop significantly after two periods. A domestic shift of the inflation rate leads to a depreciation of the real exchange rate. Domestic are also less competitive on international goods markets, their sales abroad tend to diminish which deteriorate the domestic external position through a negative trade balance and current account deficits. Finally, the monetary policy tightening in both countries reduces investment internationally and acceleration the economic depression.



System response to a domestic cost-push shock $\eta^{P}_{h,t}$, blue (red) is for the home (foreign) economy while world variables are in green.

subsection 1.7 draw the world response after a monetary policy shock in the home country. This shock standardly pushes households to postpone consumption later leading to a drop of consumption and in turn of output. Similarly, investment is depressed by the rise in the cost of investment financing. The increase of the monetary policy rate appreciate the nominal domestic currency making home goods less competitive on international markets. Two opposite effects affect the foreign economy. On the first hand, the foreign economy observes an increase of its competitiveness compared to the home economy because of the appreciation of the currency, this should lead foreign exports to grow and stimulate the foreign production. On the other hand, the reduction of the home demand affects the imports of the foreign country and depresses output. Overall, foreign output is negatively affected by a domestic monetary policy shock which allows us to conclude that the demand slowdown effect is more important than the improvement of the competitiveness effect.



System response to a domestic monetary policy shock $\eta^{R}_{h,t}$, blue (red) is for the home (foreign) economy while world variables are in green.

Exercise 1: Optimal policy in open economies

Let's first concentrate on the perspective of an international coordinations of monetary policies in the world. A never ending debate started during the 80s dealing with the possibility of central banks to coordinate policies in order to enhance the global welfare. Rogoff (1985) shows that a coordinated scheme could be counterproductive. This result was reinforced by DSGE model literature showing that trade and financial market integration was too low to make policy coordination effective and necessary. Benigno (2002) shows that trade linkages plays an important role while Sutherland (2002) points out the key aspect of financial integration in the decision to coordinate monetary policies internationally. We can use our model to address this question.

Let's assume that the home country is the US and the foreign one is the Eurozone. In each country, the loss function is given by:

$$\mathcal{L}_h = var\left(\pi_{h,t}\right) + var\left(Y_{h,t}\right) + 0.10 var\left(r_{h,t}\right)$$
$$\mathcal{L}_f = var\left(\pi_{f,t}\right) + var\left(Y_{f,t}\right) + 0.10 var\left(r_{f,t}\right)$$

1. Write the optimisation problem in Dynare.
2. Calibrate the EZ shocks symmetrically like for the US economy.
3. Fill the table, the second column corresponds to a situation where the US minimises first the loss function followed by the EZ, the last row is the same story in the other way round (EZ first then US).
4. According to your result, should we coordinate with the US or not?
5. Is the intermediate goods trade channel an important feature for deciding to coordinate monetary policies?

Objective:	$\mathcal{L}_h + \mathcal{L}_f$			$\mathcal{L}_h$ then $\mathcal{L}_f$			$\mathcal{L}_f$ then $\mathcal{L}_h$		
	ρ	ϕ_π	$\mathcal{L}$	ρ	ϕ_π	$\mathcal{L}$	ρ	ϕ_π	$\mathcal{L}$
Trade autarky $\varphi = 0$									
US - Home									
EZ - Foreign									
Benchmark $\varphi = 0.10$									
US - Home									
EZ - Foreign									
Perfect trade integration $\varphi = 0.50$									
US - Home									
EZ - Foreign									

Exercise 2: Two-country model business cycles statistics

(If not done yet, calibrate the foreign shocks symmetrically in the same fashion as in the home economy.)

1. It has been widely discussed that open economy DSGE model failed at capturing the co-movement of output between countries (output puzzle). Assuming that EZ-US's GDPs correlation is 25%, fix the model:

   (a) Calibrate the correlation of productivity shocks to get the right cross-country correlation. (See Dynare's manual for the command).

   (b) Introduce a (new) common productivity shock. Find the appropriate calibration to replicate the right correlation.

2. Another well-know puzzle is the BKK puzzle (in reference to Backus et al. (1992)'s model) Empirically, the correlation of consumption across countries is higher than output correlation. But BKK's model reveals that consumption is less correlation than output. Do you find such a result with our new Keynesian model?

Exercise 3: Two-country model business cycles statistics

1. Given the following policymakers' preferences:

$$\mathcal{L}_h = var\left(\pi_{h,t}\right) + var\left(Y_{h,t}\right) + 0.10 var\left(r_{h,t}\right)$$
$$\mathcal{L}_f = var\left(\pi_{f,t}\right) + var\left(Y_{f,t}\right) + 0.10 var\left(r_{f,t}\right)$$

   Is it optimal for a central bank to target the exchange rate? Minimise the joint-loss function $\mathcal{L}_h + \mathcal{L}_f$.

2. Let's now assume that the foreign central bank adopts a currency board regime (i.e. the central bank sets its interest rate using the rule $R_{f,t} = R_{h,t}$). Is it optimal at a global level? Minimise the joint-loss function $\mathcal{L}_h + \mathcal{L}_f$. How are the business cycles affected?

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