4 Steady state:

We start of with few standard values of the variables as follows:

$$\Pi^P = 1$$

$$L^p = \frac{1}{3}$$

$$L^i = \frac{1}{3}$$

$$L = \frac{1}{3}$$

$$R^d = \frac{1}{\beta^p}$$

$$R^b = R$$

$$R^{bH} = \left(\frac{\epsilon^{bH}}{\epsilon^{bH} - 1}\right)(R^b - 1) + 1$$

$$R^{bE} = \left(\frac{\epsilon^{bE}}{\epsilon^{bE} - 1}\right)(R^b - 1) + 1$$

$$R^d = \left(\frac{\epsilon^d}{\epsilon^d - 1}\right)(R - 1) + 1 \quad \text{Define } R \text{ using this equation}$$

$$Q^k = 1$$

$$\frac{I}{K} = \delta$$

Now lets derive the R^k. we start combining FOC for capital and firm loan for entrepreneurs i.e.

$$(1 - \delta)s^e m^e + \beta^e \lambda^e[(1 - \delta) + R^k] = \lambda^e$$
$$\lambda^e = \beta^e \lambda^e R^{bE} + s^e R^{bE}$$

dividing both the equations by λ^e, we get,

$$(1 - \delta)\frac{s^e}{\lambda^e} m^e + \beta^e[(1 - \delta) + R^k] = 1$$
$$1 = \beta^e R^{bE} + \frac{s^e}{\lambda^e} R^{bE}$$

Therefore we have

$$\frac{s^e}{\lambda^e} = \frac{1 - \beta^e R^{bE}}{R^{bE}}$$

using this in the first equation, we get,

$$
\begin{aligned}
(1-\delta)\frac{1-\beta^e R^{bE}}{R^{bE}} m^e + \beta^e[(1-\delta) + R^k] &= 1 \\
\beta^e[(1-\delta) + R^k] &= 1 - (1-\delta)\frac{1-\beta^e R^{bE}}{R^{bE}} m^e \\
(1-\delta) + R^k &= \frac{1}{\beta^e} - (1-\delta)\frac{m^e}{\beta^e}\frac{1-\beta^e R^{bE}}{R^{bE}}
\end{aligned}
$$

Finally, we have

$$
R^k = \frac{1}{\beta^e} - (1-\delta)\frac{m^e}{\beta^e}\frac{1-\beta^e R^{bE}}{R^{bE}} - (1-\delta)
$$

$$
MC = \frac{\epsilon^P}{\epsilon^P - 1}
$$

$$
\frac{Y}{K} = \frac{R^k}{\alpha}\frac{1}{MC}
$$

$$
\frac{I}{Y} = \frac{I/K}{Y/K}
$$

$$
\frac{C}{Y} = 1 - I/Y
$$

$$
w = \mu(1-\alpha)\left(\frac{\mu}{1-\mu}\right)^{-(1-\mu)}\left[MC\left(\frac{R^k}{\alpha}\right)^{-\alpha}\right]^{\frac{1}{(1-\alpha)}}
$$

where $w = (w^p)^\mu (w^i)^{1-\mu}$. Given the calibration for respective labors, we have

$$
\frac{w^p}{w^i} = \frac{\mu}{1-\mu}
$$

We also have

$$
w = \left(\frac{w^p}{w^i}\right)^\mu w^i
$$

Therefore the impatient labor's wage in ss

$$
w^i = w\left(\frac{w^p}{w^i}\right)^{-\mu} = w\left(\frac{\mu}{1-\mu}\right)^{-\mu}
$$

$$
w^p = w\left(\frac{\mu}{1-\mu}\right)^{1-\mu}
$$

Therefore the marginal rate of substitution becomes

$$
MRS^p = w^p\left(\frac{\epsilon^w - 1}{\epsilon^w}\right) \qquad MRS^i = w^i\left(\frac{\epsilon^w - 1}{\epsilon^w}\right)
$$

Notice that,

$$
\begin{aligned}
wL &= (w^p)^{\mu}(w^i)^{1-\mu}(L^p)^{\mu}(L^i)^{1-\mu} \\
&= (w^p L^p)^{\mu}(w^i L^i)^{1-\mu} \\
&= (w^p L^p)^{\mu}(w^i L^i)^{1-\mu} \\
&= \left(\mu(1-\alpha)\frac{R^k}{\alpha}K\right)^{\mu}\left((1-\mu)(1-\alpha)\frac{R^k}{\alpha}K\right)^{1-\mu} \\
&= \mu^{\mu}(1-\mu)^{1-\mu}(1-\alpha)\frac{R^k}{\alpha}K
\end{aligned}
$$

Therefore,

$$
\frac{K}{L} = \frac{w}{\left[\mu^{\mu}(1-\mu)^{1-\mu}(1-\alpha)\frac{R^k}{\alpha}\right]}
$$

Now lets get the consumptions,

$$
C^p = \frac{MRS^p}{(L^p)^{\varphi}} \qquad C^i = \frac{MRS^i}{(L^i)^{\varphi}}
$$

To obtain the total consumption, we use the resource constrain in ss,

$$
\frac{C}{L} = (Y/K)(K/L) - \delta(K/L)
$$

$$
C = (C/L)L
$$

$$
C^e = C - C^p - C^i
$$

Given the consumptions, we can find the respective lagrange multipliers as,

$$
\lambda^P = 1/C^p \qquad \lambda^i = 1/C^i \qquad \lambda^e = 1/C^e
$$

From the FOC of the housing of patient hhs,

$$
H^p = \frac{J}{(1-\beta^p)\lambda^p Q^h}
$$

From the FOC of mortgage loan we get,

$$
s^i = \lambda^i \frac{(1-\beta^i R^{bH})}{R^{bH}}
$$

From the FOC of housing for impatient hhs we get,

$$
H^i = \frac{J}{(1-\beta^i)\lambda^i Q^h - s^i m^i Q^h}
$$

Given relation $\bar{H} = H^p + H^i$, we can get Q^h as following,

$$
Q^h = \frac{J}{\bar{H}}\left[\frac{1}{(1-\beta^p)\lambda^p} + \frac{1}{\{(1-\beta^i) - m^i(\frac{1}{R^{bH}} - \beta^i)\}\lambda^i}\right]
$$

Remember to define Q^h in the dynare code before H^p, H^i.

The steady state demand for mortgage loan can be calculated from the constraint as,

$$B^i = \frac{H^i m^i Q^h}{R^{bH}}$$

From the FOC of firm loan,

$$s^e = \lambda^e \frac{(1 - \beta^e R^{bE})}{R^{bE}}$$

The steady state demand for firm loan can be calculated from the constraint as,

$$B^e = (1 - \delta)(K/L)L \frac{m^e}{R^{bE}}$$

Since $B = B^e + B^i$ and $\frac{K^b}{B} = v_i$, we have

$$K^b = v_i B$$

Notice that this steady state is calculated for $\delta^{kB} = 0$. Hence $J^b = 0$ [2]. We can find out the steady state demand for deposit as,

$$D = \frac{1}{R^d - 1} \left(\{R^{bH} - 1\} B^i + \{R^{bE} - 1\} B^e \right)$$

[2] $J^b = \delta^{kB} K^b$ following the bank capital accumulation equation in steady state.