$$(\partial B_{t+1}) \quad \mathbb{E}_t\left[(1-\Gamma(\bar{\omega}_{t+1},\sigma_t))\frac{R^k_{t+1}}{R_{t+1}}+\eta_{t+1}\left[\frac{R^k_{t+1}}{R_{t+1}}(\Gamma(\bar{\omega}_{t+1},\sigma_t))-\mu G(\bar{\omega}_{t+1},\sigma_t)-1\right]\right]=0 \tag{1}$$

$$(\partial \bar{\omega}_{t+1}) \quad \mathbb{E}_t\left[\eta_{t+1}-\frac{\Gamma'(\bar{\omega}_{t+1},\sigma_t)}{\Gamma'(\bar{\omega}_{t+1},\sigma_t)-\mu G'(\bar{\omega}_{t+1},\sigma_t)}\right]=0 \tag{2}$$

By combinig the two FOCs I get

$$\left(1-\Gamma(\bar{\omega}_{t+1},\sigma_t)+\frac{\Gamma'(\bar{\omega}_{t+1},\sigma_t)}{\Gamma'(\bar{\omega}_{t+1},\sigma_t)-\mu G'(\bar{\omega}_{t+1},\sigma_t)}\right)\frac{1+R^k_{t+1}}{1+R_{t+1}}-\frac{\Gamma'(\bar{\omega}_{t+1},\sigma_t)}{\Gamma'(\bar{\omega}_{t+1},\sigma_t)-\mu G'(\bar{\omega}_{t+1},\sigma_t)}=0 \tag{3}$$

To linearize GAMMA function

$$\Gamma_t=\bar{\omega}_{t+1}\left[1-F_t\right]+G_t \tag{4}$$

which results in the following log linear form

$$\Gamma(1+\hat{\Gamma}_t)=\bar{\omega}(1-F)\left[1+\hat{\omega}_{t+1}+\widehat{1-F_t}\right]+G(1+\hat{G}_t) \tag{5}$$

Then

$$\widehat{1-F_t}=\frac{F}{F-1}\hat{F}_t \tag{6}$$

and then I proceed by applying taylor expansion on F and G which are two-variables functions