1 The Model

2 Households

$$\max_{C_t,N_t,B_t} E_t \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{1}$$

$$s.t.: C_t + Q_t \frac{B_t}{P_t} = \frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{W_t}{P_t} N_t - \frac{T_t}{P_t} \tag{2}$$

$$\lim E_t \{B_T\} \geq 0 \tag{3}$$

FOCs:

$$C_t^{-\sigma} = \lambda_t \tag{4}$$

$$-N_t^{\varphi} = \lambda_t \frac{W_t}{P_t} \tag{5}$$

$$Q_t = E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \tag{6}$$

2.1 Wage-Setting

$$\max_{W_t^*} E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[U_{C_{t+s}} N_{t+s}(h) \frac{W_t(h)}{P_{t+s}} - U(C_{t+s}, N_{t+s}) \right] \right\} \tag{7}$$

$$s.t. N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\varepsilon_{w,t}} N_t \tag{8}$$

$$\max_{W_t^*} E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[U_{C_{t+s}} \left(\frac{W_t(h)}{W_{t+s}} \right)^{-\varepsilon_{w,t+s}} N_{t+s} \frac{W_t(h)}{P_{t+s}} - U(C_{t+s}, N_{t+s}) \right] \right\} \tag{9}$$

$$\max_{W_t^*} E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[U_{C_{t+s}} \left(\frac{1}{W_{t+s}} \right)^{-\varepsilon_{w,t+s}} N_{t+s} \frac{W_t^{1-\varepsilon_{w,t+s}}(h)}{P_{t+s}} - \frac{N_{t+s}^{1+\varphi}(h)}{1+\varphi} \right] \right\} \tag{10}$$

$$\max_{W_t^*} E_t \left\{ \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[U_{C_{t+s}} \left(\frac{1}{W_{t+s}} \right)^{-\varepsilon_{w,t+s}} N_{t+s} \frac{W_t^{1-\varepsilon_{w,t+s}}(h)}{P_{t+s}} - \frac{1}{1+\varphi} \left(\frac{W_t(h)}{W_{t+s}} \right)^{-\varepsilon_{w,t+s}(1+\varphi)} N_{t+s}^{1+\varphi} \right] \right\} \tag{11}$$

Labor demand schedule:

$$N_{t+s}(h) = \left(\frac{W_t(h)}{W_{t+s}} \right)^{-\varepsilon_{w,t+s}} N_{t+s} = e^{-\varepsilon_{w,t+s} \ln\left(\frac{W_t(h)}{W_{t+s}} \right)} N_{t+s}$$

FOC:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_w)^s\left[U_{C_{t+s}}\left(1-\varepsilon_{w,t+s}\right)\left(\frac{W_t(h)}{W_{t+s}}\right)^{-\varepsilon_{w,t+s}}N_{t+s}\frac{1}{P_{t+s}}+\varepsilon_{w,t+s}\left(\frac{W_t(h)}{W_{t+s}}\right)^{-\varepsilon_{w,t+s}(1+\varphi)-1}\frac{1}{W_{t+s}}N_{t+s}^{1+\varphi}\right]\right.$$

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_w)^s\left[U_{C_{t+s}}\left(1-\varepsilon_{w,t+s}\right)N_{t+s}(h)\frac{1}{P_{t+s}}+\varepsilon_{w,t+s}N_{t+s}^{1+\varphi}(h)\frac{1}{W_t(h)}\right]\right.$$

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_w)^s N_{t+s}(h)\left[\left(1-\varepsilon_{w,t+s}\right)\frac{\lambda_{t+s}}{P_{t+s}}+\varepsilon_{w,t+s}N_{t+s}^{\varphi}(h)\frac{1}{W_t(h)}\right]\right.$$

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_w)^s N_{t+s}(h)\left[\left(1-\varepsilon_{w,t+s}\right)\frac{\lambda_{t+s}}{P_{t+s}}W_t(h)+\varepsilon_{w,t+s}N_{t+s}^{\varphi}(h)\right]\right.$$

3 Detrending

Define:

$$\tilde{C}_t=\frac{C_t}{A_{t-1}},\tilde{\lambda}_t=\frac{\lambda_t}{A_{t-1}^{-\sigma}},\tilde{W}_t=\frac{W_t}{A_{t-1}^{-\sigma}},\tilde{A}_t=\frac{A_t}{A_{t-1}}$$

$$\tilde{C}_t^{-\sigma}=\tilde{\lambda}_t$$
$$-N_t^{\varphi}=\tilde{\lambda}_t\tfrac{\tilde{W}_t}{P_t}$$
$$Q_t=E_t\frac{\lambda_{t+1}\frac{A_t^{-\sigma}}{A_t^{-\sigma}}}{\lambda_t\frac{A_{t-1}^{-\sigma}}{A_{t-1}^{-\sigma}}}\frac{1}{\pi_{t+1}}=E_t\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\frac{A_t^{-\sigma}}{A_{t-1}^{-\sigma}}\frac{1}{\pi_{t+1}}\Leftrightarrow Q_t=E_t\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\tilde{A}_t^{-\sigma}\frac{1}{\pi_{t+1}}$$

4 Steady State

5 Linearizing

Wage equation

LHS:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_w)^s N_{t+s}(h)\left(1-\varepsilon_{w,t+s}\right)\tfrac{\lambda_{t+s}}{P_{t+s}}W_t(h)\right\}$$
$$\approx E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\left(1-\varepsilon_w\right)\tfrac{\lambda}{P}W(h)\left\{\hat{n}_{t+s}(h)+\hat{\lambda}_{t+s}+\hat{w}_t(h)-\hat{p}_{t+s}\right\}$$
$$-E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\varepsilon_w\tfrac{\lambda}{P}W(h)\hat{\varepsilon}_{w,t+s}$$
$$=\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\left(1-\varepsilon_w\right)\tfrac{\lambda}{P}W(h)\hat{w}_t(h)+E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\left(1-\varepsilon_w\right)\tfrac{\lambda}{P}W(h)\left\{\hat{n}_{t+s}(h)+\hat{\lambda}_{t+s}-\hat{p}_{t+s}\right\}$$
$$-E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\varepsilon_w\tfrac{\lambda}{P}W(h)\hat{\varepsilon}_{w,t+s}$$
$$=\tfrac{1}{1-\beta\theta_w}N(h)\left(1-\varepsilon_w\right)\tfrac{\lambda}{P}W(h)\hat{w}_t(h)+E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\left(1-\varepsilon_w\right)\tfrac{\lambda}{P}W(h)\left\{\hat{n}_{t+s}(h)+\hat{\lambda}_{t+s}-\hat{p}_{t+s}\right\}$$
$$-E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\varepsilon_w\tfrac{\lambda}{P}W(h)\hat{\varepsilon}_{w,t+s}$$

RHS:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_w)^s\varepsilon_{w,t+s}N_{t+s}^{1+\varphi}(h)\right\}$$
$$\approx E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\varepsilon_w N^{1+\varphi}(h)\left\{\hat{\varepsilon}_{w,t+s}+(1+\varphi)\,\hat{n}_{t+s}(h)\right\}$$

Equating:

$$E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\varepsilon_w N^{1+\varphi}(h)\left\{\hat{\varepsilon}_{w,t+s}+(1+\varphi)\,\hat{n}_{t+s}(h)\right\}$$
$$=\tfrac{1}{1-\beta\theta_w}\varepsilon_{w,t+s}N^{1+\varphi}(h)\hat{w}_t(h)+E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\varepsilon_{w,t+s}N^{1+\varphi}(h)\left\{\hat{n}_{t+s}(h)+\hat{\lambda}_{t+s}-\hat{p}_{t+s}\right\}$$
$$-E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s N(h)\varepsilon_w\tfrac{\lambda}{P}\tfrac{1-\varepsilon_w}{1-\varepsilon_w}W(h)\hat{\varepsilon}_{w,t+s}$$
$$\Leftrightarrow E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\varepsilon_w N^{1+\varphi}(h)\left\{\hat{\varepsilon}_{w,t+s}+(1+\varphi)\,\hat{n}_{t+s}(h)\right\}$$
$$=\tfrac{1}{1-\beta\theta_w}\varepsilon_{w,t+s}N^{1+\varphi}(h)\hat{w}_t(h)+E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\varepsilon_{w,t+s}N^{1+\varphi}(h)\left\{\hat{n}_{t+s}(h)+\hat{\lambda}_{t+s}-\hat{p}_{t+s}\right\}$$
$$-E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\varepsilon_{w,t+s}N^{1+\varphi}(h)\tfrac{\varepsilon_w}{1-\varepsilon_w}\hat{\varepsilon}_{w,t+s}$$
$$\Leftrightarrow E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\hat{\varepsilon}_{w,t+s}+(1+\varphi)\,\hat{n}_{t+s}(h)\right\}$$
$$=\tfrac{1}{1-\beta\theta_w}\hat{w}_t(h)+E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\hat{n}_{t+s}(h)+\hat{\lambda}_{t+s}-\hat{p}_{t+s}\right\}$$
$$-E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\tfrac{\varepsilon_w}{1-\varepsilon_w}\hat{\varepsilon}_{w,t+s}$$
$$\Leftrightarrow\tfrac{1}{1-\beta\theta_w}\hat{w}_t(h)=E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\hat{\varepsilon}_{w,t+s}+(1+\varphi)\,\hat{n}_{t+s}(h)-\hat{n}_{t+s}(h)-\hat{\lambda}_{t+s}+\hat{p}_{t+s}+\tfrac{\varepsilon_w}{1-\varepsilon_w}\hat{\varepsilon}_{w,t+s}\right\}$$
$$\Leftrightarrow\tfrac{1}{1-\beta\theta_w}\hat{w}_t(h)=E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\varphi\hat{n}_{t+s}(h)-\hat{\lambda}_{t+s}+\hat{p}_{t+s}+\tfrac{1}{1-\varepsilon_w}\hat{\varepsilon}_{w,t+s}\right\}$$

Labor demand schedule

$$N(h)\hat{n}_{t+s}(h) = \ln\left(\frac{W(h)}{W}\right)\left(\frac{W(h)}{W}\right)^{-\varepsilon_w} N\varepsilon_w\hat{\varepsilon}_{w,t+s} - \varepsilon_w\left(\frac{W(h)}{W}\right)^{-\varepsilon_w} N\left\{\hat{w}_t(h) - \hat{w}_{t+s}\right\} + \left(\frac{W(h)}{W}\right)^{-\varepsilon_w} N\hat{n}_{t+s}$$
$$\Leftrightarrow \hat{n}_{t+s}(h) = -\varepsilon_w\left(\hat{w}_t(h) - \hat{w}_{t+s} - \ln\left(\frac{W(h)}{W}\right)\hat{\varepsilon}_{w,t+s}\right) + \hat{n}_{t+s}$$
$$\ln\left(\frac{W(h)}{W}\right) = 0?$$

Combined

$$\hat{n}_{t+s}(h) = \hat{n}_{t+s} - \varepsilon_w\left(\hat{\varepsilon}_{w,t+s} + \hat{w}_t(h) - \hat{w}_{t+s}\right)$$
$$\frac{1}{1-\beta\theta_w}\hat{w}_t(h) = E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\varphi\hat{n}_{t+s}(h) - \hat{\lambda}_{t+s} + \hat{p}_{t+s} + \frac{1}{1-\varepsilon_w}\hat{\varepsilon}_{w,t+s}\right\}$$

combined

$$\frac{1}{1-\beta\theta_w}\hat{w}_t(h) = E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\varphi\left(\hat{n}_{t+s} - \varepsilon_w\left(\hat{\varepsilon}_{w,t+s} + \hat{w}_t(h) - \hat{w}_{t+s}\right)\right) - \hat{\lambda}_{t+s} + \hat{p}_{t+s} + \frac{1}{1-\varepsilon_w}\hat{\varepsilon}_{w,t+s}\right\}$$
$$\frac{1}{1-\beta\theta_w}\hat{w}_t(h) = E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\varphi\left(\hat{n}_{t+s} - \varepsilon_w\left(\hat{w}_t(h) - \hat{w}_{t+s}\right)\right) - \hat{\lambda}_{t+s} + \hat{p}_{t+s} + \left(-\varphi\varepsilon_w + \frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t+s}\right\}$$
$$\frac{(1+\varphi\varepsilon_w)(\hat{w}_t(h)-\hat{w}_t)}{1-\beta\theta_w} = E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\begin{array}{l}\varphi\left(\hat{n}_{t+s} + \varepsilon_w\left(\hat{w}_{t+s} - \hat{w}_t\right)\right) - \hat{w}_t - \hat{\lambda}_{t+s} + \hat{p}_{t+s} \\ + \left(-\varphi\varepsilon_w + \frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t+s}\end{array}\right\}$$
$$\frac{(1+\varphi\varepsilon_w)(\hat{w}_t(h)-\hat{w}_t)}{1-\beta\theta_w} = E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\begin{array}{l}\varphi\hat{n}_{t+s} - \hat{w}_{t+s} + \varphi\varepsilon_w\left(\hat{w}_{t+s} - \hat{w}_t\right) + \hat{w}_{t+s} - \hat{w}_t - \hat{\lambda}_{t+s} + \hat{p}_{t+s} \\ + \left(-\varphi\varepsilon_w + \frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t+s}\end{array}\right\}$$
$$\frac{(1+\varphi\varepsilon_w)(\hat{w}_t(h)-\hat{w}_t)}{1-\beta\theta_w} = E_0\sum_{s=0}^{\infty}(\beta\theta_w)^s\left\{\begin{array}{l}\varphi\hat{n}_{t+s} - \hat{w}_{t+s} + (1+\varphi\varepsilon_w)\left(\hat{w}_{t+s} - \hat{w}_t\right) - \hat{\lambda}_{t+s} + \hat{p}_{t+s} \\ + \left(-\varphi\varepsilon_w + \frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t+s}\end{array}\right\}$$

TRICK

$$(1+\varphi\varepsilon_w)\left(\hat{w}_t(h)-\hat{w}_t\right)1-\beta\theta_w = E_0\sum_{s=0}^{\infty}\left(\beta\theta_w\right)^s\left\{\begin{array}{l}\varphi\hat{n}_{t+s}-\hat{w}_{t+s}+(1+\varphi\varepsilon_w)\sum_{i=0}^{s-1}\pi^w_{t+i+1}-\hat{\lambda}_{t+s}+\hat{p}_{t+s}\\ +\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t+s}\end{array}\right\}$$

$$and:$$

$$\sum_{s=0}^{\infty}\left(\beta\theta_w\right)^s\sum_{i=0}^{s-1}\pi^w_{t+i+1}=\frac{\beta\theta_w}{1-\beta\theta_w}\sum_{s=0}^{\infty}\left(\beta\theta_w\right)^s\pi^w_{t+i+1}$$

$$\frac{(1+\varphi\varepsilon_w)(\hat{w}_t(h)-\hat{w}_t)}{1-\beta\theta_w}=E_0\sum_{s=0}^{\infty}\left(\beta\theta_w\right)^s\left\{\begin{array}{l}\varphi\hat{n}_{t+s}-\hat{w}_{t+s}+(1+\varphi\varepsilon_w)\frac{\beta\theta_w}{1-\beta\theta_w}\pi^w_{t+i+1}-\hat{\lambda}_{t+s}+\hat{p}_{t+s}\\ +\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t+s}\end{array}\right\}$$

$$\hat{w}_t(h)-\hat{w}_t=\frac{1-\beta\theta_w}{1+\varphi\varepsilon_w}E_0\sum_{s=0}^{\infty}\left(\beta\theta_w\right)^s\left\{\varphi\hat{n}_{t+s}-\hat{w}_{t+s}-\hat{\lambda}_{t+s}+\hat{p}_{t+s}+\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t+s}\right\}$$

$$+\beta\theta_w\sum_{s=0}^{\infty}(\beta\theta_w)^s\pi^w_{t+i+1}$$

$$WritingInDifferencedFORM...CHECK$$

$$\hat{w}_t(h)-\hat{w}_t=\beta\theta_w\left(\hat{w}_{t+1}(h)-\hat{w}_{t+1}\right)+\frac{1-\beta\theta_w}{1+\varphi\varepsilon_w}\left(\varphi\hat{n}_t-\hat{w}_t-\hat{\lambda}_t+\hat{p}_t+\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t}\right)+\beta\theta_w\pi^w_{t+1}$$

$$substitute:$$

$$\hat{w}_t(h)-\hat{w}_t=\frac{\theta_w}{1-\theta_w}\left(\hat{w}_t-\hat{w}_{t-1}\right)=\frac{\theta_w}{1-\theta_w}\pi^w_t$$

$$\frac{\theta_w}{1-\theta_w}\pi^w_t=\beta\theta_w E_t\left(\frac{\theta_w}{1-\theta_w}\pi^w_{t+1}\right)+\frac{1-\beta\theta_w}{1+\varphi\varepsilon_w}\left(\varphi\hat{n}_t-\hat{w}_t-\hat{\lambda}_t+\hat{p}_t+\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t}\right)+\beta\theta_w E_t\pi^w_{t+1}$$

$$\pi^w_t=\beta\theta_w E_t\left(\pi^w_{t+1}\right)+\frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varphi\varepsilon_w)\theta_w}\left(\varphi\hat{n}_t-\hat{w}_t-\hat{\lambda}_t+\hat{p}_t+\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t}\right)+\beta\left(1-\theta_w\right)E_t\pi^w_{t+1}$$

$$\pi^w_t=\beta E_t\pi^w_{t+1}+\frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varphi\varepsilon_w)\theta_w}\left(\varphi\hat{n}_t-\hat{w}_t-\hat{\lambda}_t+\hat{p}_t+\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t}\right)$$

$$\pi^w_t=\beta E_t\pi^w_{t+1}+\frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varphi\varepsilon_w)\theta_w}\left(\varphi\hat{n}_t-\hat{w}^R_t-\hat{\lambda}_t+\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t}\right)$$

$$\pi^w_t=\beta E_t\pi^w_{t+1}+\frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varphi\varepsilon_w)\theta_w}\left(\varphi\hat{n}_t-\hat{w}^R_t-\hat{\lambda}_t+\left(-\varphi\varepsilon_w+\frac{1}{1-\varepsilon_w}\right)\hat{\varepsilon}_{w,t}\right)$$

$$\pi^w_t=\beta E_t\pi^w_{t+1}+\frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varphi\varepsilon_w)\theta_w}\left(\varphi\hat{n}_t-\hat{w}^R_t-\hat{\lambda}_t+\frac{1-\varphi\varepsilon_w+\varphi\varepsilon_w^2}{1-\varepsilon_w}\hat{\varepsilon}_{w,t}\right)$$

for:

$$\left(\frac{W(h)}{W}\right)=0:$$

$$\pi^w_t=\beta E_t\pi^w_{t+1}+\frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varphi\varepsilon_w)\theta_w}\left(\varphi\hat{n}_t-\hat{w}^R_t-\hat{\lambda}_t+\frac{1}{1-\varepsilon_w}\hat{\varepsilon}_{w,t}\right)$$

References