Note that

$$G_t(\bar{\omega}_{t+1}, \sigma_t) = \int_0^{\bar{\omega}_{t+1}} \omega \, dF_t(\omega) = \Phi\left(\frac{\ln(\bar{\omega}_{t+1}) + 0.5 \times \sigma_t^2}{\sigma_t} - \sigma_t\right) \quad (1)$$

where Φ denotes cumulative standard density function,

using Taylor linearization

$$G_t(\bar{\omega}_{t+1}, \sigma_t) = G(\bar{\omega}, \sigma) + G_{\bar{\omega}}(\bar{\omega}, \sigma) \times (\bar{\omega}_{t+1} - \bar{\omega}) + G_{\sigma}(\bar{\omega}, \sigma) \times (\sigma_t - \sigma) \quad (2)$$

reordering

$$G_t(\bar{\omega}_{t+1}, \sigma_t) - G(\bar{\omega}, \sigma) = G_{\bar{\omega}}(\bar{\omega}, \sigma) \times (\bar{\omega}_{t+1} - \bar{\omega}) + G_{\sigma}(\bar{\omega}, \sigma) \times (\sigma_t - \sigma) \quad (3)$$

where $G_{\bar{\omega}}$ y G_{σ} involve partial derivatives of Φ, so probability densities will appear

dividing by $G(\bar{\omega}, \sigma)$

$$\frac{G_t(\bar{\omega}_{t+1}, \sigma_t) - G(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} = \frac{G_{\bar{\omega}}(\bar{\omega}, \sigma) \times (\bar{\omega}_{t+1} - \bar{\omega}) + G_{\sigma}(\bar{\omega}, \sigma) \times (\sigma_t - \sigma)}{G(\bar{\omega}, \sigma)} \quad (4)$$

$$\frac{G_t(\bar{\omega}_{t+1}, \sigma_t) - G(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} = \frac{G_{\bar{\omega}}(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} \times \bar{\omega} \times \frac{\bar{\omega}_{t+1} - \bar{\omega}}{\bar{\omega}} + \frac{G_{\sigma}(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} \times \sigma \times \frac{\bar{\sigma}_t - \sigma}{\sigma} \quad (5)$$

then

$$\hat{G}_t = \bar{\omega} \times \frac{G_{\bar{\omega}}(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} \times \hat{\bar{\omega}}_{t+1} + \sigma \times \frac{G_{\sigma}(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} \times \hat{\sigma}_t \quad (6)$$

where $\hat{x}_t = (x_t - x)/x$.

with respect to $\Gamma_t(\bar{\omega}_{t+1}, \sigma_t)$ we have to

$$\Gamma_t(\bar{\omega}_{t+1}, \sigma_t) = \bar{\omega}_{t+1}[1 - F_t(\bar{\omega}_{t+1})] + G_t(\bar{\omega}_{t+1}, \sigma_t) \quad (7)$$

where

$$F_t(\bar{\omega}_{t+1}, \sigma_t) = \Phi\left(\frac{\ln(\bar{\omega}_{t+1}) + 0.5 \times \sigma_t^2}{\sigma_t}\right) \quad (8)$$

and the procedure is similar.

Since

$$\hat{F}_t = \bar{\omega} \times \frac{F_{\bar{\omega}}(\bar{\omega}, \sigma)}{F(\bar{\omega}, \sigma)} \times \hat{\bar{\omega}}_{t+1} + \sigma \times \frac{F_{\sigma}(\bar{\omega}, \sigma)}{F(\bar{\omega}, \sigma)} \times \hat{\sigma}_t \tag{9}$$

it's needed

$$F(\bar{\omega}, \sigma) = \Phi\left(\frac{\ln(\bar{\omega}) + 0.5 \times \sigma^2}{\sigma}\right) \tag{10}$$

$$F_{\bar{\omega}}(\bar{\omega}, \sigma) = \frac{1}{\bar{\omega}\sigma}\phi\left(\frac{\ln(\bar{\omega}) + 0.5 \times \sigma^2}{\sigma}\right) \tag{11}$$

$$F_{\sigma}(\bar{\omega}, \sigma) = \Big(- \sigma^{-2}\ln(\bar{\omega}) + 0.5\Big) \times \phi\left(\frac{\ln(\bar{\omega}) + 0.5 \times \sigma^2}{\sigma}\right) \tag{12}$$

where ϕ denotes a standard normal probability density function