Online Appendix to "Fiscal Stimulus and Distortionary Taxation"*

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Contents

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A Categorizing stimulus spending

We take our data for the stimulus components from "Table 2: Estimated cost of the conference agreement for H.R. 1, the American Recovery and Reinvestment Act of 2009, as posted on the website of the House Committee on Rules" in **?**. The annual time path for these expenditures is also taken from **?**. To split the annual data across quarters, we assume a breakdown in proportion to the aggregate series in **?**. We classify the different components are as government consumption, investment, and transfers according to Tables A.1, A.2, and A.3.

Table A.1: Categorizing the stimulus – government consumption

Item	Amount (bn USD)	Share
Dept. of Defense	4.53	0.59
Employment and Training	4.31	0.56
Legislative Branch	0.03	0
National Coordinator for Health Information Technology	1.98	0.26
National Institutes of Health	9.74	1.26
Other Agriculture, Food, FDA	3.94	0.51
Other Commerce, Justice, Science	5.36	0.69
Other Dept. of Education	2.12	0.28
Other Dept. of Health and Human Services	9.81	1.27
Other Financial Services and Gen. Govt	1.31	0.17
Other Interior and Environment	4.76	0.62
Special Education	12.2	1.58
State and Local Law Enforcement	2.77	0.36
State Fiscal Relief	90.04	11.68
State Fiscal Stabilization Fund	53.6	6.95
State, Foreign Operations, and Related Programs	0.6	0.08
Other	2.55	0.33
Consumption	209.64	27.2

Table A.2: Categorizing the stimulus –government investment

Item	Amount (bn USD)	Share
Broadband Technology Opportunities Program	4.7	0.61
Clean Water and Drinking Water State Revolving Fund	5.79	0.75
Corps of Engineers	4.6	0.6
Distance Learning, Telemedicine, and Broadband Program	1.93	0.25
Energy Efficiency and Renewable Energy	16.7	2.17
Federal Buildings Fund	5.4	0.7
Health Information Technology	17.56	2.28
Highway Construction	27.5	3.57
Innovative Technology Loan Guarantee	6	0.78
NSF	2.99	0.39
Other Energy	22.38	2.9
Other Transportation	20.56	2.67
Investment	136.09	17.66

Table A.3: Categorizing the stimulus – transfers

Item	Amount (bn USD)	Share
Assistance for the Unemployed	0.88	0.11
Economic Recovery Programs, TANF, Child Support	18.04	2.34
Health Insurance Assistance (spending)	25.07	3.25
Health Insurance Assistance (revenue)	-0.39	-0.05
Low Income Housing Program	0.14	0.02
Military Construction and Veteran Affairs	4.25	0.55
Other housing assistance	9	1.17
Other Tax Provisions	4.81	0.62
Public housing capital fund	4	0.52
Refundable Tax Credits	68.96	8.95
Student financial assistance	16.56	2.15
Supplemental Nutrition Assistance Program	19.99	2.59
Tax Provisions	214.56	27.84
Unemployment Compensation	39.23	5.09
Transfers and tax cuts	425.09	55.15

B Model appendix

Apart from the model extensions due to the introduction of government capital, rule-of-thumb consumers, and distortionary taxation, the following model appendix follows mostly the appendix of **?**, with minor changes to unify the notation.

B.1 Production

Final goods are produced in a competitive final goods sector that uses differentiated intermediate inputs, supplied by monopolistic intermediate producers.

B.1.1 Final goods producers

The representative final goods producer maximizes profits by choosing intermediate inputs $Y_t(i), i \in [0,1]$, subject to a production technology that generalizes a CES production function: Objective:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad \text{s.t.} \quad \int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p}\right) di = 1. \tag{B.1}$$

$G(\cdot)$ is the **?** aggregator, which generalizes CES demand by allowing the elasticity of demand to increase with relative prices: $G' > 0, \quad G'' < 0, \quad G(1; \tilde{\epsilon}_t^{\lambda,p}) = 1$. $\tilde{\epsilon}_t^{\lambda,p}$ is a shock to the production technology which changes the elasticity of substitution.

Denote the Lagrange multiplier on the constraint by Ξ_t^f. If a positive solution to equation (B.1) exists it satisfies the following conditions:

$$\begin{aligned}
[Y_t] \quad & P_t = \Xi_t^f \frac{1}{Y_t} \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p}\right) \frac{Y_t(i)}{Y_t} di, \\
[Y_t(i)] \quad & P_t(i) = \Xi_t^f \frac{1}{Y_t} G'\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p}\right).
\end{aligned}$$

From these two equations, we obtain an expression for the aggregate price index and intermediate inputs. The price index is given by:

$$P_t = \int_0^1 \frac{Y_t(i)}{Y_t} P_t(i) di. \tag{B.2}$$

Solving for intermediate input demands:

$$Y_t(i) = Y_t G'^{-1}\left(\frac{P_t(i)Y_t}{\Xi_t^f}\right) = Y_t G'^{-1}\left(\frac{P_t(i)}{P_t}\int_0^1 G'\left(\frac{Y_t(j)}{Y_t};\tilde{\epsilon}_t^{\lambda,p}\right)\frac{Y_t(j)}{Y_t}dj\right). \tag{B.3}$$

For future reference, note that the relative demand curves $\mathrm{y}_t(i) \equiv \frac{Y_t(i)}{Y_t}$ are downward sloping in the relative price $\frac{P_t(i)}{P_t}$ with an decreasing elasticity as the relative quantity increases. For simplicity, the dependence of the $G(\cdot)$ aggregator on the shock $\tilde{\epsilon}_t^{\lambda,p}$ is suppressed:

$$\eta_p(\mathrm{y}_t(i)) \equiv -\frac{P_t(i)}{Y_t(i)}\frac{d\mathrm{y}_t(i)}{dP_t(i)}\Big|_{dY_t=d\Xi_t^f=0} = -\frac{G'(\mathrm{y}_t(i))}{\mathrm{y}_t(i)G''(\mathrm{y}_t(i))} \tag{B.4}$$

$$\begin{aligned}\hat{\eta}_p(\mathrm{y}_t(i)) &\equiv \frac{P_t(i)}{\eta_p(\mathrm{y}_t(i))}\frac{d\eta_p(\mathrm{y}_t(i))}{dP_t(i)} = 1+\eta_p+\eta_p\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)\\ &= 1+\eta_p(\mathrm{y}_t(i))\left(2+\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)-1\right)\\ &= 1+\eta_p(\mathrm{y}_t(i))\left(\frac{2+\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)}{1-\eta_p(\mathrm{y}_t(i))^{-1}}(1-\eta_p(\mathrm{y}_t(i))^{-1})-1\right)\\ &\equiv 1+\frac{1+\lambda^p(\mathrm{y}_t(i))}{\lambda^p(\mathrm{y}_t(i))}\left(\frac{1}{[1+\lambda^p(\mathrm{y}_t(i))]A_p(\mathrm{y}_t(i))}-1\right),\end{aligned} \tag{B.5}$$

where the last line defines the markup λ_t^p and the parameter A_p as

$$\lambda_t^p(\mathrm{y}_t(i)) \equiv \frac{1}{\eta_p(\mathrm{y}_t(i))-1}, \qquad A_p(\mathrm{y}_t(i)) \equiv \frac{\lambda^p(\mathrm{y}_t(i))}{2+\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)}.$$

The model will be parameterized in terms of $\hat{\epsilon}(1)$, the change in the own-price elasticity of demand along the balanced growth path. To that end, it is convenient to solve for A_p in terms of the markup and the $\hat{\epsilon}$:

$$A_p(\mathrm{y}) = \frac{1}{\lambda^p(\mathrm{y})\hat{\eta}_p(\mathrm{y})+1}. \tag{B.6}$$

Finally, note that in the Dixit-Stiglitz case $G(\mathrm{y}) = \mathrm{y}^{\frac{1}{1+\lambda^p}}$ so that the elasticity of demand is constant at $\eta_p(\mathrm{y}) = \frac{1}{\lambda^p}+1\forall\mathrm{y}$ and consequently $\hat{\eta}_p = 0$.

B.1.2 Intermediate goods producers

There is a unit mass of intermediate producers, indexed by $i \in [0,1]$. Each producer is the monopolistic supplier of good i. They rent capital services

K_t^{eff} and hire labor n_t to maximize profits intertemporally, taking as given rental rates R_t^k and wages W_t. Given a Calvo-style pricing friction, their profit-maximization problem is dynamic.

Production is subject to a fixed cost and the gross product is produced using a Cobb-Douglas technology at the firm level. Government capital K_t^g increases total factor productivity in each firm, but is subject to a congestion effect as overall production increases, similar to the congestion effects in the AK model in **?**. Firms fail to internalize the effect of their decisions on public sector productivity. Net output is therefore given by:

$$Y_t(i) = \tilde{\epsilon}_t^a \left(\frac{K_{t-1}^g}{\int_0^1 Y_t(j)dj + \Phi\mu^t} \right)^{\frac{\zeta}{1-\zeta}} K_t^{eff}(i)^{\alpha} [\mu^t n_t(i)]^{1-\alpha} - \mu^t \Phi, \tag{B.7}$$

where $\Phi\mu^t$ represent fixed costs which grow at the rate of labor augmenting technical progress and $K_t(i)^{eff}$ denotes the capital services rented by firm i. $\tilde{\epsilon}_t^a$ denotes a stationary TFP process.

To see the implications of the congestion costs, consider the symmetric case that $Y_t(i) = Y_t, K_t^{eff}(i) = K_t^{eff} \forall i$, which is the case along the symmetric balanced growth path and in the flexible economy. We then obtain the following aggregate production function:

$$Y_t = \epsilon_t^a K_{t-1}^{g}{}^{\zeta} K_t^{eff\,\alpha(1-\zeta)} [\mu^t n_t]^{(1-\alpha)(1-\zeta)} - \mu^t \Phi, \quad \epsilon_t^a \equiv (\tilde{\epsilon}_t^a)^{1-\zeta}. \tag{B.8}$$

Choose units such that $\bar{\epsilon}^a \equiv 1$.

To solve a firm's profit maximization problem, note that it is equivalent to minimizing costs (conditional on operating) and then choosing the quantity optimally. Consider the cost-minimization problem first:

$$\min_{K_t(i), n_t(i)} W_t n_t(i) + R_t^k K_t(i) \text{ s.t. (B.7).}$$

Denote the Lagrange multiplier on the production function by MC_t: Producing a marginal unit more raises costs (the objective) by MC_t. The static FOC are necessary and sufficient, given $Y_t(i)$:

$$[n_t(i)] \quad MC_t(i)(1-\alpha)\frac{Y_t(i) + \mu^t\Phi}{n_t(i)} = W_t,$$
$$[K_t(i)] \quad MC_t(i)\alpha\frac{Y_t(i) + \mu^t\Phi}{K_t(i)} = R_t^k.$$

The FOC can be used to solve for the optimal capital-labor ratio in production and marginal costs:

$$\frac{k_t(i)}{n_t(i)} = \frac{\alpha}{1-\alpha}\frac{w_t}{r^k_t}, \tag{B.9}$$

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\frac{W_t^{1-\alpha}(R^k_t)^{\alpha}\mu^{-(1-\alpha)t}}{\left(\frac{K^g_{t-1}}{Y_t+\mu^t\Phi}\right)^{\frac{\zeta}{1-\zeta}}\epsilon^a_t}, \tag{B.10}$$

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\frac{w_t^{1-\alpha}(r^k_t)^{\alpha}}{\left(\frac{\mu k^g_{t-1}}{y_t+\Phi}\right)^{\frac{\zeta}{1-\zeta}}\epsilon^a_t},$$

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\frac{w_t^{1-\alpha}(r^k_t)^{\alpha}}{\left(\frac{\mu k^g_{t-1}}{y_t+\Phi}\right)^{\frac{\zeta}{1-\zeta}}\epsilon^a_t}, \tag{B.11}$$

where lower-case letters denote detrended, real variables, as applicable:

$$k_t \equiv K_t\mu^{-t}, y_t \equiv Y_t\mu^{-t}, w_t \equiv \frac{W_t}{\mu^t P_t}, r^k_t \equiv \frac{R^k_t}{P_t}, mc_t \equiv \frac{MC_t}{P_t}.$$

For future reference, it is useful to detrend the FOC:

$$w_t = mc_t(i)(1-\alpha)\frac{y_t(i)+\Phi}{n_t(i)}, \tag{B.12a}$$

$$r^k_t = mc_t(i)\alpha\frac{y_t(i)+\Phi}{k_t(i)}. \tag{B.12b}$$

Given the solution to the static cost-minimization problem, the firm maximizes the present discounted value of its profits by choosing quantities optimally, taking as given its demand function (B.3), the marginal costs of production (B.10), and the Calvo-style price-setting friction. The Calvo-friction implies that a firm can re-set its price in each period with probability $1-\zeta_p$ and otherwise indexes its price to an average of current and past inflation $\prod_{l=1}^{s}\pi_{t+l-1}^{\iota_p}\bar{\pi}^{1-\iota_p}$. In each period t that the firm can change its prices it chooses:

$$P^*_t(i) = \arg\max_{\tilde{P}_t(i)}\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\frac{\bar{\beta}^s\xi_{t+s}P_t}{\xi_t P_{t+s}}\Big[\tilde{P}_t(i)\Big(\prod_{l=1}^{s}\pi_{t+l-1}^{\iota_p}\bar{\pi}^{1-\iota_p}\Big) - MC_{t+s}(i)\Big]Y_{t+s}(i),$$

subject to (B.3) and (B.10). $\frac{\bar{\beta}^s\xi_{t+s}}{\xi_t}$ denotes the (noncredit-constrained) representative household's stochastic discount factor and $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes period t inflation.

To solve the problem, it is useful to define $\chi_{t,t+s}$ such that in the absence of further price adjustments prices evolve as $P_{t+s}(i) = \chi_{t,t+s} P_t^*(i)$:

$$\chi_{t,t+s} = \begin{cases} 1 & s = 0, \\ \prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p} & s = 1, \ldots, \infty. \end{cases}$$

Using the definition $\mathrm{y}_{t+s}(i) = \frac{Y_{t+s}(i)}{Y_{t+s}}$ yields therefore:

$$\frac{d(Y_{t+s}(i)[P_{t+s}(i) - MC_{t+s}(i)])}{d\tilde{P}_t(i)} = \mathrm{y}_{t+s}(i) Y_{t+s} \left(\chi_{t,t+s}[1 - \eta_p(\mathrm{y}_{t+s}(i))] + \eta_p \frac{MC_{t+s}(i)}{P_t(i)} \right).$$

The first order condition is then given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} \mathrm{y}_{t+s}(i) Y_{t+s} \left([1 - \eta_p(\mathrm{y}_{t+s}(i))] \chi_{t,t+s} + \eta_p \frac{MC_{t+s}(i)}{P_t(i)} \right) = 0 \tag{B.13}$$

For future reference, it is useful to rewrite the FOC as follows:

$$\frac{P_t^*(i)}{P_t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s \frac{\xi_{t+s}}{\lambda_p(\mathrm{y}_{t,t+s}(i)) \xi_t} y_{t,t+s}(i) \frac{\eta_p(\mathrm{y}_{t,t+s}(i))}{\eta_p(\mathrm{y}_{t,t+s}(i)) - 1} mc_{t+s}(i)}{\mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s \frac{\xi_{t+s}}{\lambda_p(\mathrm{y}_{t,t+s}(i)) \xi_t} \frac{\chi_{t,t+s}}{\prod_{l=1}^{s} \pi_{t+s}} y_{t,t+s}(i)} \tag{B.14}$$

where $\mathrm{y}_{t,t+s}(i) = G'^{-1}\left(\frac{P_t^* \chi_{t,t+s} Y_{t+s}}{\Xi_{t+s}^f} \right)$, $Y_{t,t+s}(i) = \mathrm{y}_{t,t+s}(i) Y_{t+s}$.

Noting that measure $1 - \zeta_p$ of firms changes prices in each period and that each firm faces a symmetric problem, the expression for the aggregate price index (B.2) can be expressed recursively as a weighted average of adjusted and indexed prices:

$$P_t = (1 - \zeta_p) P_t^* G'^{-1}\left(\frac{P_t^* Y_t}{\Xi_t^f} \right) + \zeta_p \pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1} G'^{-1}\left(\frac{\pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1} Y_t}{\Xi_t^f} \right). \tag{B.15}$$

This expression uses that price distribution of nonadjusting firms at t is the same as that of all firms at time $t-1$, adjusted for the shrinking mass due to price adjustments. The optimal price equals the average price along the deterministic balanced growth path, which is normalized to unity:

$$\bar{P}^* = \bar{P} = 1.$$

Similarly, along the deterministic growth path, the price is a constant markup over marginal cost:

$$\frac{\bar{P}^*}{\bar{P}} = \frac{\eta_p}{\eta_p - 1}\overline{mc} = (1+\bar{\lambda}_p)\overline{mc} = 1 \tag{B.16}$$

Finally, the assumption of monopolistic competition in the presence of free entry requires zero profits along the balanced growth path. Real and detrended profits of intermediate producer i are given by:

$$\Pi_t^p(i) = \frac{P_t(i)}{P_t}y_t(i) - w_t n_t(i) - r_t^k k_t(i) = \frac{P_t(i)}{P_t}y_t(i) - mc_t(i)[y_t(i) + \mu^t\Phi]$$

Integrating over all $i \in [0,1]$ and using the definition of the price index (B.2) yields:

$$\Pi_t^p = y_t - w_t\int_0^1 n_t(i)di - r_t^k\int_0^1 k_t(i)di \tag{B.17a}$$

$$= y_t - mc_t\left(\int_0^1 y_t(i)di + \Phi\right) = y_t - mc_t\left(y_t\int_0^1 \frac{P_t(i)}{P_t}di + \Phi\right) \tag{B.17b}$$

Using the expression for the steady state markup, equation (B.16), the zero-profit condition (B.17b) implies that along the symmetric balanced growth path:

$$0 = \bar{\Pi}^p = \bar{y} - \frac{\bar{y}\int_0^1 \frac{P(i)}{P}di + \Phi}{1+\bar{\lambda}_p} = \bar{y} - \frac{\bar{y}+\Phi}{1+\bar{\lambda}_p} \quad \Rightarrow \frac{\Phi}{\bar{y}} = \bar{\lambda}_p. \tag{B.18}$$

B.1.3 Labor packers

Intermediate producers use a bundle of differentiated labor inputs, $\ell \in [0,1]$, purchased from labor packers. Labor packers aggregagte, or pack, differentiated labor, which they purchase from unions. They are perfectly competitive and face an analogous problem to final goods producers:

$$\max_{n_t, n_t(\ell)} W_t n_t - \int_0^1 W_t(\ell)n_t(\ell)d\ell \quad \text{s.t.} \quad \int_0^1 H\left(\frac{n_t(\ell)}{n_t}; \tilde{\epsilon}_t^{\lambda,w}\right)d\ell = 1, \tag{B.19}$$

where $H(\cdot)$ has the same properties as $G(\cdot)$: $H' > 0, H'' < 0, H(1) = 1$.

The FOC yield differentiated labor demand, analogous to intermediate goods demand (B.3):

$$n_t(\ell) = n_t H'^{-1}\left(\frac{W_t(\ell) n_t}{\Xi_t^n}\right) = n_t H'^{-1}\left(\frac{W_t(\ell)}{W_t}\int_0^1 H'\left(\frac{n_t(l)}{n_t};\tilde{\epsilon}_t^{\lambda,w}\right)\frac{n_t(l)}{n_t}dl\right). \tag{B.20}$$

Given the aggregate nominal wage $W_t = \int_0^1 \frac{n_t(\ell)}{n_t} w_t(\ell) d\ell$, labor packers are willing to supply any amount of packed labor n_t. Labor demand elasticity behaves analogously to the intermediate goods elasticity:

$$\eta_w(\mathrm{n}_t(\ell)) \equiv -\frac{W_t(\ell)}{n_t(\ell)}\frac{d\mathrm{n}_t(\ell)}{dW_t(\ell)}\Big|_{dn_t = d\Xi_t^l = 0} = -\frac{H'(\mathrm{n}_t(\ell))}{\mathrm{n}_t(\ell) H''(\mathrm{n}_t(\ell))} \tag{B.21}$$

$$\hat{\eta}_w(\mathrm{n}_t(\ell)) \equiv \frac{W_t(\ell)}{\eta_w(\mathrm{n}_t(\ell))}\frac{d\eta_w(\mathrm{n}_t(\ell))}{dW_t(\ell)} = 1 + \frac{1+\lambda^w(\mathrm{n}_t(\ell))}{\lambda^w(\mathrm{n}_t(\ell))}\left(\frac{1}{[1+\lambda^w(\mathrm{n}_t(\ell))]A_w(\mathrm{n}_t(\ell))} - 1\right), \tag{B.22}$$

where $\mathrm{n}_t(\ell) \equiv \frac{n_t(\ell)}{n_t}$ and the markup is defined as $\lambda_t^n(\mathrm{n}_t(\ell)) \equiv \frac{1}{\eta_w(\mathrm{n}_t(\ell))-1}$. $A_w(\mathrm{n}_t(\ell)) \equiv \frac{\lambda^w(\mathrm{n}_t(\ell))}{2+\frac{H'''(\mathrm{n}_t(\ell))}{H''(\mathrm{n}_t(\ell))}\mathrm{n}_t(\ell)}$ can be equivalently expressed as:

$$A_w(\mathrm{n}) = \frac{1}{\lambda^w(\mathrm{n})\hat{\eta}_w(\mathrm{n}) + 1}. \tag{B.23}$$

B.2 Households

There is a measure one of households in the economy, indexed by $j \in [0,1]$, endowed with a unit of labor each. Households are distributed uniformly over the real line (i.e., the measure of households is the Lebesgue measure Λ). We distinguish two types of households: intertemporally optimizing households $j \in [0, 1-\phi]$ and rule-of-thumb households $j \in (1-\phi, 1]$, so that they have measures $\Lambda([0,1-\phi]) = 1-\phi$ and $\Lambda([0,\phi]) = \phi$, respectively.

Households have preferences over streams of consumption and hours worked, $\{C_{t+s}(j), n_{t+s}(j)\}_{s=0}^{\infty}$, which are represented by the life-time utility function U_t:

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma}\big(C_{t+s}(j) - hC_{t+s-1}\big)^{1-\sigma}\right] \exp\left[\frac{\sigma-1}{1+\nu} n_{t+s}(j)^{1+\nu}\right]. \tag{B.24}$$

Here $h \in [0,1)$ captures external habit formation, σ denotes the inverse of the intertemporal elasticity of substitution, and ν equals the inverse of the labor supply elasticity. Households discount the future by $\beta \in (0,1)$, where β varies by household type.

The fraction $1-\phi$ of the labor force that is not credit-constrained maximizes its life-time utility subject to a lifetime budget constraint and a capital accumulation technology. The remainder of the labor force, i.e., a fraction ϕ, is credit constrained (or rule-of-thumb): they cannot save or borrow.

B.2.1 Intertemporally optimizing households

The intertemporally optimizing households choose consumption $\{C_{t+s}(j)\}$, investment in physical capital $\{X_{t+s}(j)\}$, physical capital $\{K^p_{t+s}(j)\}$, capacity utilization $\{u_{t+s}(j)\}$, nominal government bond holdings $B^n_{t+s}(j)$, and labor supply $\{n_{t+s}(j)\}$ to maximize (B.24) subject to a sequence of budget constraints (B.25), the law of motion for physical capital (B.26), and a no-Ponzi constraint. Households take prices $\{P_{t+s}\}$, nominal returns on government bonds $\{q^b_{t+s}R_{t+s}\}$, the nominal rental rate of capital $\{R^k_{t+s}\}$, and nominal wages $\{W_{t+s}\}$ as given.

The budget constraint for period $t+s$ is given by:

$$(1+\tau^c_{t+s})C_{t+s}(j) + X_{t+s}(j) + \frac{B^n_{t+s}(j)}{R^{gov}_{t+s}P_{t+s}} \leq$$
$$S_{t+s} + \frac{B^n_{t+s-1}(j)}{P_{t+s}} + (1-\tau^n_{t+s})\frac{[W^h_{t+s}n_{t+s}(j) + \lambda_{w,t+s}n_{t+s}W^h_{t+s}]}{P_{t+s}} +$$
$$+\left[(1-\tau^k_{t+s})\left(\frac{R^k_{t+s}u_{t+s}(j)}{P_{t+s}} - a(u_{t+s}(j))\right) + \delta\tau^k_{t+s}\right][(1-\omega^k_{t+s-1})K^p_{t+s-1}(j) + \omega^k_{t+s-1}K^{p,agg}_{t+s-1}] + \frac{\Pi^p_{t+s}\mu^{t+s}}{P_{t+s}}, \tag{B.25}$$

where $(\tau^c_{t+s}, \tau^k_{t+s}, \tau^n_{t+s})$ represent taxes on consumption expenditure, capital income, and labor income, respectively. The wage received by households differs from the one charged to labor packers because of union profits — union profits $\lambda_{w,t+s}n_{t+s}W^h_{t+s}$ are taken as given by households. Households also receive nominal lump-sum transfers $\{S_{t+s}\}$. $a(\cdot)$ represents the strictly increasing and strictly convex cost function of varying capacity utilization, whose first derivative in the case of unit capacity utilization is normalized as $a'(1) = \bar{r}^k$.[1] At unit capacity utilization, there is no additional cost: $a(1) = 0$. $\Pi^p_{t+s}\mu^{t+s}$ are nominal profits, which households also take as given.

There is a financial market friction present in the budget constraint; $\omega^k_{t+s} \neq 0$ represents a wedge between the returns on private and government bonds, and is a pure financial market friction — if $\omega^k_{t+s} > 0$ then households obtain less than one dollar for each dollar of after-tax capital income they receive, representing agency costs. Agency costs are reimbursed directly to unconstrained households, so that the friction has no effect on aggregate resources. This financial market friction is similar to a shock in **?**, who introduce it ad hoc in the investment Euler equation and motivate it as a short-cut to model informational frictions that disappear at the steady state.

[1] $\bar{r}^k$ represents the real steady state return on capital services.

Physical capital evolves according to the following law of motion:

$$K^p_{t+s}(j) = (1-\delta)K^p_{t+s-1}(j) + q^x_{t+s}\left[1 - S\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)}\right)\right] X_{t+s}(j), \qquad \text{(B.26)}$$

where new investment is subject to adjustment costs described by $S(\dot{})$. These costs satisfy $S(\mu) = S'(\mu) = 0, S'' > 0$. The relative price of investment changes over time, as captured by the exogenous $\{q^x_{t+s}\}$ process. Physical capital depreciates at rate δ.

For future reference, note that the effective capital stock is given by the product of capacity utilization and physical capital stock:

$$K^{eff}_{t+s}(j) = K^p_{t+s-1}(j)u_{t+s}(j). \qquad \text{(B.27)}$$

To obtain the aggregate capital stock, multiply the above quantity by $(1-\phi)$.

The solution to the household's problem is characterized completely by the law of motion for physical capital (B.26) and the following necessary and sufficient first order conditions. To derive these conditions, denote the Lagrange multipliers on the budget constraint (B.25) and the law of motion (B.26) by $\beta^t(\Xi_t, \Xi^k_t)$ – replacing the household index j by a superscript RA.

$$[C_t] \quad \Xi_t(1+\tau^c_t) = \exp\left(\frac{\sigma-1}{1+\nu}(n^{RA}_t)^{1+\nu}\right)[C^{RA}_t - hC^{RA}_{t-1}]^{-\sigma}$$

$$[n_t] \quad \Xi_t(1-\tau^n_t)\frac{W^h_t}{P_t} = \exp\left(\frac{\sigma-1}{1+\nu}(n^{RA}_t)^{1+\nu}\right)(n^{RA}_t)^{\nu}[C^{RA}_t - hC^{RA}_{t-1}]^{1-\sigma}$$

$$[B_t] \quad \Xi_t = \beta q^b_t R_t \mathbb{E}_t\left(\frac{\Xi_{t+1}}{P_{t+1}/P_t}\right)$$

$$[K^p_t] \quad \Xi^k_t = \beta\mathbb{E}_t\left(\Xi_{t+1}\left[\tilde{q}^k_t\left((1-\tau^k_{t+s})\left[\frac{R^k_{t+1}}{P_{t+1}}u_{t+1} - a(u_{t+1}) + \delta\tau^k_{t+1}\right] + (1-\delta)\frac{\Xi^k_{t+1}}{\Xi_{t+1}}\right]\right)$$

$$[X_t] \quad \Xi_t = \Xi^k_t q^x_t\left(1 - S\left(\frac{X^{RA}_t}{X^{RA}_{t-1}}\right) - S'\left(\frac{X^{RA}_t}{X^{RA}_{t-1}}\right)\left(\frac{X^{RA}_t}{X^{RA}_{t-1}}\right)\right)$$
$$+ \beta\mathbb{E}_t\left(\frac{\Xi^k_{t+1}}{\Xi_t}q^x_{t+1}S'\left(\frac{X^{RA}_{t+1}}{X^{RA}_t}\right)\left(\frac{X^{RA}_{t+1}}{X^{RA}_t}\right)^2\right)$$

$$[u_t] \quad \frac{R^k_{t+1}}{P_t} = a'(u^{RA}_{t+1}).$$

By setting $a'(1) \equiv \bar{r}^k$ we normalize steady state capacity utilization to unity: $\bar{u} \equiv 1$.

For what follows, it is useful to detrend these first order conditions and the law of motion for capital. To that end, use lower-case letters to denote detrended and real variables, as exemplified in the following definitions:

$$k_t^{RA} \equiv \frac{K_t^{RA}}{\mu^t}, w_t \equiv \frac{W_t}{P_t\mu^t}, w_t^h \equiv \frac{W_t^h}{P_t\mu^t}, r_t^k \equiv \frac{R_t^k}{P_t}, \xi_t \equiv \Xi_t\mu^{\sigma t}, Q_t \equiv \frac{\Xi_t^k}{\Xi_t}, \bar{\beta} = \beta\mu^{-\sigma}.$$

μ denotes the gross trend growth rate of the economy. For future reference, note that government expenditure is normalized differently: $g_t = \frac{G_t}{\bar{Y}\mu^t}$. Substituting in for the normalized variables yields:

$$\xi_t(1+\tau_t^c) = \exp\left(\frac{\sigma-1}{1+\nu}(n_t^{RA})^{1+\nu}\right)[c_t^{RA} - (h/\mu)c_{t-1}^{RA}]^{-\sigma} \tag{B.29a}$$

$$\xi_t(1-\tau_t^n)w_t^h = \exp\left(\frac{\sigma-1}{1+\nu}(n_t^{RA})^{1+\nu}\right)(n_t^{RA})^{\nu}[c_t^{RA} - (h/\mu)c_{t-1}^{RA}]^{1-\sigma} \tag{B.29b}$$

$$\xi_t = \bar{\beta}R_t^{gov}\mathbb{E}_t\left(\frac{\xi_{t+1}}{P_{t+1}/P_t}\right) \tag{B.29c}$$

$$Q_t = \bar{\beta}\mathbb{E}_t\left(\frac{\xi_{t+1}}{\xi_t}\left[\tilde{q}_t^k\big((1-\tau_{t+1}^k)[r_{t+1}^k u_{t+1} - a(u_{t+1})] + \delta\tau_{t+1}^k\big) + (1-\delta)Q_{t+1}\right]\right) \tag{B.29d}$$

$$\begin{aligned} 1 = {} & Q_t q_t^x\left(1 - S\left(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\right) - S'\left(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\right)\left(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\right)\right) \\ & + \bar{\beta}\mathbb{E}_t\left(\frac{\xi_{t+1}}{\xi_t}Q_{t+1}q_{t+1}^x S'\left(\frac{x_{t+1}^{RA}\mu}{x_t^{RA}}\right)\left(\frac{x_{t+1}^{RA}\mu}{x_t^{RA}}\right)^2\right) \end{aligned} \tag{B.29e}$$

$$r_{t+1}^k = a'(u_{t+1}^{RA}). \tag{B.29f}$$

The detrended law of motion for physical capital is given by

$$k_t^{p,RA} = \frac{(1-\delta)}{\mu}k_{t-1}^{p,RA} + q_t^x\left[1 - S\left(\frac{x_t^{RA}}{x_{t-1}^{RA}}\mu\right)\right]x_t^{RA}. \tag{B.30}$$

Combining the FOC for consumption and hours worked gives the static optimality condition for households:

$$\frac{1-\tau_t^n}{1+\tau_t^c}w_t^h = (n_t^{RA})^{\nu}[c_t^{RA} - (h/\mu)c_{t-1}^{RA}]. \tag{B.31}$$

Combining (B.29a) for two consecutive periods and using (B.29c) gives the consumption Euler equation:

$$\mathbb{E}_t\left(\frac{\xi_{t+1}(1+\tau^c_{t+1})}{\xi_t(1+\tau^c_{t+1})}\right)=\mathbb{E}_t\left(\exp\left(\frac{\sigma-1}{1+\nu}\left(\frac{n^{RA}_{t+1}}{n^{RA}_t}\right)^{1+\nu}\right)\left[\frac{c^{RA}_{t+1}-(h/\mu)c^{RA}_t}{c^{RA}_t-(h/\mu)c^{RA}_{t-1}}\right]^{-\sigma}\right). \tag{B.32}$$

Equation (B.29d) is the investment Euler equation. The FOC for capital (B.29e) can be used to compute the shadow price of physical capital Q_t.

Using the investment Euler equation shows that along the deterministic balanced growth path the value of capital equals unity (since $S'(\mu)=S(\mu)=0$ and $\bar{q}^x=1$). From the consumption Euler equation and $\bar{q}^b=1$. we obtain the interest rate paid on government bonds under balanced growth. Finally, the pricing equation for capital and the investment Euler equation pin down the rental rate on capital. Summarizing:

$$\bar{Q}=1, \tag{B.33a}$$

$$\bar{R}=\bar{\beta}^{-1}\bar{\pi}, \tag{B.33b}$$

$$1=\bar{\beta}[(1-\bar{\tau}^k)\bar{r}^k+\delta\bar{\tau}^k+(1-\delta)],$$

$$\Leftrightarrow \quad \bar{r}^k=\frac{\bar{\beta}^{-1}-1+\delta(1-\bar{\tau}^k)}{1-\bar{\tau}^k}. \tag{B.33c}$$

The bond premium shock q^b_t differs from a discount factor shock, although it results in an observationally equivalent consumption Euler equation – if time preference were time-varying, the period utility function would become:

$$\left[\frac{1}{1-\sigma}\big(C_{t+s}(j)-hC_{t+s-1}\big)^{1-\sigma}\right]\exp\left[\frac{\sigma-1}{1+\nu}n_{t+s}(j)^{1+\nu}\right]\prod_{l=1}^{s}\check{q}^b_{t+l-1},$$

so that the ratio $\frac{\check{\xi}_{t+1}}{\xi_t}$ would be proportional to $\check{q}^b_t$, so that the consumption Euler equation is unchanged. The effects differ, however, insofar that the present formulation on basis of the government discount factor also affects the investment Euler equation and the government budget constraint.

For measurement purposes, it is useful to rewrite the linearized FOC for capital, after substituting out for the discount factor. It shows that the private bond shock represents the premium paid for private bonds over government bonds holding the rental rate on capital fixed:

$$\frac{\bar{r}^k(1-\bar{\tau}^k)\mathbb{E}_t(\hat{r}^k_{t+1})+(1-\delta)\mathbb{E}_t(\hat{Q}_{t+1})}{\bar{r}^k(1-\bar{\tau}^k)+\delta\bar{\tau}^k+1-\delta}-\hat{Q}_t=\left(\hat{R}_t-\mathbb{E}_t[\pi_t]\right)+\hat{q}^b_t+\hat{q}^k_t.$$

Note: The shock $\tilde{q}^k_t$ in the budget constraint has been rescaled here; $\hat{q}^k_t$ is the deviation of the rescaled shock from its steady state value.

B.2.2 Credit-constrained or rule-of-thumb households

A fraction $\phi \in (0, 0.5)$ of the households is assumed to be credit-constrained. As a justification, one may suppose that credit-constrained households discount the future substantially more steeply, and they are thus uninterested in accumulating government bonds or private capital, unless their returns are extraordinarily high. Conversely, these households find it easy to default on loans, and they are therefore not able to borrow. We hold the identity of credit-constrained households, and thereby their fraction of the total population, constant.

Rule-of-thumb households face a static budget constraint in each period and are assumed to supply the same amount of labor as intertemporally optimizing households. Given

$$n_{t+s}^{RoT}(j) = n_{t+s}^{RA} = n_{t+s},$$

consumption follows from the budget constraint in each period:

$$(1+\tau_{t+s}^{c})C_{t+s}^{RoT}(j) \leq S_{t+s}^{RoT} + (1-\tau_{t+s}^{n})\frac{W_{t+s}^{h} n_{t+s}^{RoT}(j) + \lambda_{w,t+s} W_{t+s}^{h} n_{t+s}}{P_{t+s}} + \Pi_{t+s}^{p} \mu^{t+s}. \tag{B.34}$$

Rule-of-thumb households receive transfers, labor income including union profits, and profits made by intermediate goods-producing firms.

Removing the trend from the budget constraint (B.34), omitting the j index, and solving for (detrended) consumption:

$$c_{t+s}^{RoT} = \frac{1}{(1+\tau_{t+s}^{c})} \left(s_{t+s}^{RoT} + (1-\tau_{t+s}^{n})[w_{t+s}^{h} n_{t+s}^{RoT} + \lambda_{w,t+s} w_{t+s}^{h} n_{t+s}] + \Pi_{t+s}^{p} \right). \tag{B.35}$$

From the budget constraint (B.34), the following steady state relationship holds:

$$\bar{c}^{RoT} = \frac{\bar{s}^{RoT} + (1-\bar{\tau}_{t}^{n})\bar{w}\bar{n}}{1+\bar{\tau}^{c}}. \tag{B.36}$$

We assume that:

$$\bar{s}^{RoT} = \bar{s}. \tag{B.37}$$

B.2.3 Households: labor supply, wage setting

Households supply homogeneous labor to unions, which differentiate labor into varieties indexed by $\ell \in [0, 1]$ and sell it to labor packers. In doing so, unions take aggregate quantities (i.e., households' cost of supplying labor and aggregate labor demand and wages) as given. Unions maximize the expected present discounted value of net-of-tax wage income earned in excess of the cost of supplying labor. In the presence of rule-of-thumb households, unions act as if they were maximizing surplus for the intertemporally optimizing households only. If the mass of rule-of-thumb households is less than the mass of intertemporally optimizing households, i.e., $\phi < 0.5$, which is satisfied in the parameterizations used, a median-voter decision rule justifies this assumption.

The labor unions problem is analogous to that of price-setting firms, with the marginal rate of substitution between consumption and leisure in the representative household taking the role of marginal costs in firms' problems. From the FOC $[C_t]$ and $[n_t]$, the marginal rate of substitution is given by $\frac{U_{n,t+s}}{\Xi_{t+s}} = (n_t^{RA})^\nu [C_t^{RA} - hC_{t-1}^{RA}](1+\tau_t^c)$. Whenever a union has the chance to reset the wage it charges, it chooses $W_t^*(\ell)$:

$$W_t^*(\ell) = \arg\max_{\tilde{W}_t(\ell)} \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w)^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t} \left[(1-\tau_{t+s}^n) \frac{W_{t+s}(\ell)}{P_{t+s}} + \frac{U_{n,t+s}}{\Xi_{t+s}} \right] n_{t+s}(\ell), \tag{B.38}$$

subject to the labor demand equation (B.20). $1-\zeta_w$ denotes the probability that a union can reset its wage. If it cannot adjust, wages are adjusted according to a moving average of past and steady state inflation and labor productivity growth:

$$W_{t+s}(\ell) = W^*{}_t(\ell) \prod_{v=1}^{s} \mu (\pi_{t+v-1})^{\iota_w} \bar{\pi}^{1-\iota_w} \equiv W_t^*(\ell) \chi_{t,t+s}^w.$$

Using that $n_t = n_t^{RA}$, the first order condition is given by

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathrm{n}_{t,t+s}(\ell))} \frac{n_{t+s}(\ell)}{W_t^*(\ell)} \Bigg((1-\tau_{t+s}^n) \frac{W_t^*(\ell)\chi_{t,t+s}^w(\ell)}{P_{t+s}} \\ - [1+\lambda^w(\mathrm{n}_{t+s}(\ell))](1+\tau_{t+s}^c) n_{t+s}^\nu [C_{t+s}^{RA} - hC_{t+s-1}^{RA}] \Bigg) \tag{B.39}$$

and can be equivalently expressed as

$$\frac{W_t^*(\ell)}{P_t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathrm{n}_{t,t+s}(\ell))} n_{t+s}(\ell)[1+\lambda^w(\mathrm{n}_{t+s}(\ell))](1+\tau_{t+s}^c) n_{t+s}^\nu [C_{t+s}^{RA} - hC_{t+s-1}^{RA}]}{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathrm{n}_{t,t+s}(\ell))} n_{t+s}(\ell)(1-\tau_{t+s}^n) \frac{\chi_{t,t+s}^w(\ell)}{P_{t+s}/P_t}}. \tag{B.40}$$

Aggregate wages evolve as:

$$W_t = (1-\zeta_w) W_t^* H'^{-1}\left(\frac{W_t^* n_t}{\Xi_t^n}\right) + \zeta_w \pi_{t-1}^{\iota_w}\bar{\pi}^{1-\iota_w} W_{t-1} H'^{-1}\left(\frac{\pi_{t-1}^{\iota_w}\bar{\pi}^{1-\iota_w} W_{t-1} n_t}{\Xi_t^n}\right). \tag{B.41}$$

Along the deterministic balanced growth path, the detrended desired real wage is given by a constant markup over the marginal rate of substitution. Given constant inflation, the symmetric deterministic growth path also implies, from equation (B.41), that the desired real wage equals the actual real wage:

$$\bar{w} = \bar{w}^* = (1+\bar{\lambda}_w)\bar{w}^h = (1+\bar{\lambda}_w)\frac{1+\bar{\tau}^c}{1-\bar{\tau}^n}\bar{n}^\nu \bar{c}^{RA}[1-h/\mu], \tag{B.42}$$

where the second equality uses (B.31).

B.3 Government

The government sets nominal interest R_t according to an interest rate rule, purchases goods and services for government consumption G_t, pays transfers S_t to households, and provides public capital for the production of intermediate goods, K_t^g. It finances its expenditures by levying taxes on capital and labor income, a tax on consumption expenditure, and a tax on one period nominal bond issues. We consider a setup in which monetary policy is active in the neighborhood of the balanced growth path.

B.3.1 Fiscal policy

In modeling the government sector, we take as given the tax structure along the balanced growth path as in **?**, who used NIPA data to compute the capital and labor income and consumption expenditure tax rates for the US. Off the balanced growth path, we follow **?** in assuming that labor tax rates adjust gradually to balance the budget in the long run, whereas in the short run much of any additional government expenditure is tax financed.

The government flow budget constraint is given by:

$$G_t + X_t^g + S_t + \frac{B_{t-1}}{P_t} \leq \frac{B_t}{R_t^{gov} P_t} + \tau_t^c C_t + \tau_t^n n_t \frac{W_t}{P_t} + \tau_t^k \left[u_t \frac{R_t^k}{P_t} - a(u_t) - \delta\right] K_{t-1}^p. \tag{B.43}$$

Detrended, the government budget constraint is given by:

$$\bar{y}g_t + x_t^g + s_t + \frac{b_{t-1}}{\mu\pi_t} \le \frac{b_t}{R_t^{gov}} + \tau_t^c c_t + \tau_t^n n_t w_t + \tau_t^k k_t^s r_t^k - \tau_t^k[a(u_t)+\delta]\frac{k_{t-1}^p}{\mu}. \tag{B.44}$$

Government consumption $g_t = \frac{G_t}{\bar{y}\mu^t}$ is given exogenously and is stochastic, driven by genuine spending shocks as well as by technology shocks.

By introducing a wedge between the FFR and government bonds, we capture both short-term liquidity premia as well as changes in the term structure of government debt. Since the latter is absent with only one-period bonds, in the estimation, the bond premium may also reflect differences in the borrowing cost due to a more complex maturity structure.[2]

Labor tax rates have both a stochastic and a deterministic component. They adjust deterministically to ensure long-run budget balance at a speed governed by the parameter $\psi_\tau \in [\underline{\psi}_\tau, 1]$, where $\underline{\psi}_\tau$ is some positive number large enough to guarantee stability. To simplify notation denote the remaining detrended deficit prior to new debt and changes in labor tax rates as d_t:

$$d_t \equiv \bar{y}g_t + x_t^g + \bar{s} + s_t^{exo} + \frac{b_{t-1}}{\mu\pi_t} - \bar{\tau}^c c_t - \bar{\tau}^n w_t n_t - \bar{\tau}^k k_t^s r_t^k + \bar{\tau}^k \delta \frac{k_{t-1}^p}{\mu}.$$

In the baseline case, labor tax rates are adjusted according to the following rule:

$$(\tau_t^n - \bar{\tau}^n)w_t n_t + \epsilon_t^\tau = \psi_\tau(d_t - \bar{d}), \tag{B.45}$$

where ϵ_t^τ is an exogenous shock to the tax rate.

In general:

$$\psi_\tau(d_t - \bar{d}) - \epsilon_t^\tau = \begin{cases} (\tau_t^n - \bar{\tau}^n)w_t n_t & \text{Baseline}, \tau_t^c = \tau_t^k = s_t^{endo} = 0, \\ (\tau_t^c - \bar{\tau}^c)c_t & \text{Alternative 1}, \tau_t^n = \tau_t^k = s_t^{endo} = 0, \\ (\tau_t^k - \bar{\tau}^k)k_t^s(r_t^k - \delta) & \text{Alternative 2}, \tau_t^n = \tau_t^c = s_t^{endo} = 0, \\ -(s_t^{endo} - \bar{s}) & \text{Alternative 3}, \tau_t^n = \tau_t^c = \tau_t^k = 0. \end{cases} \tag{B.46}$$

Debt issues are then given by the budget constraint or equivalently as the residual from (B.45): $\frac{b_t}{R_t^{gov}} = (1-\psi_\tau)(d_t - \bar{d}) + \epsilon_t^\tau$.

Government investment is chosen optimally for a given tax structure. Given the congestion effect of production on public infrastructure, a tax on production would be optimal (**?**). Similarly, we neglect the potential cost of financing

[2] Historical data from the Board of Governors of the Federal Reserve System implies a maturity between 10 and 22 quarters with an average between 16 and 20 quarters (The Federal Reserve Board Bulletin, 1999, Figure 4).

of productive government expenditure via distortionary taxes. To motivate this assumption, note that along the balanced growth path, government capital can be completely debt-financed or privatized and financed through government bond issues, whereas other government expenditures, such as transfers, that are not backed by real assets have to backed by the government's power to levy taxes.

Formally, the government chooses investment and capital stock to maximize the present discounted value of output net of investment expenditure along the balanced growth path:

$$\max_{\{K^g_{t+s},X^g_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Xi_{t+s}}{\Xi_t}[Y_{t+s} - X^g_{t+s}],$$

given K^g_{t-1} and subject to the aggregate production function (B.8) and to the capital accumulation equation:

$$K^g_{t+s} = (1-\delta)K^g_{t+s-1} + q^{x,g}_{t+s}\left[1 - S_g\Big(\frac{[X^g_{t+s} + \tilde{u}^{x,g}_{t+s}]}{[X^g_{t+s-1} + \tilde{u}^{x,g}_{t+s-1}]}\Big)\right](X^g_{t+s} + \tilde{u}^{x,g}_{t+s}). \tag{B.47}$$

The government is subject to similar adjustment costs as the private sector is $S_g(\mu) = S'_g(\mu) = 0, S''_g > 0$, and investment is subject to shocks to its relative efficiency $q^{x,g}_{t+s}$. We assume that government capital depreciates at the same rate as private physical capital. $\tilde{u}^{x,g}$ represents exogenous shocks to government investment spending, such as stimulus spending.

Denote the Lagrange multiplier on (B.47) at time $t+s$ as $\beta^s \frac{\Xi^g_{t+s}}{\Xi_t}$. Then the first order conditions are:

$$\begin{aligned}
[X^g_t] \quad 1 = {} & \frac{\Xi^g_t}{\Xi_t} q^x_t \left(1 - S_g\Big(\frac{[\tilde{u}^{x,g}_t + X^g_t]}{[\tilde{\epsilon}^{x,g}_{t-1} + X^g_{t-1}]}\Big) - S'_g\Big(\frac{[\tilde{\epsilon}^{x,g}_t + X^g_t]}{[\tilde{\epsilon}^{x}_{t-1} + X^g_{t-1}]}\Big)\Big(\frac{[\tilde{u}^{x,g}_t + X^g_t]}{[\tilde{\epsilon}^{x,g}_{t-1} + X^g_{t-1}]}\Big)\right) \\
& + \beta \mathbb{E}_t \left(\frac{\Xi^g_{t+1}}{\Xi_t} q^x_{t+1} S'_g\Big(\frac{[\tilde{\epsilon}^{x,g}_{t+1} + X^g_{t+1}]}{[\tilde{u}^{x,g}_t + X^g_t]}\Big)\Big(\frac{[\tilde{\epsilon}^{x,g}_{t+1} + X^g_{t+1}]}{[\tilde{u}^{x,g}_t + X^g_t]}\Big)^2\right) \\
[K^g_t] \quad \frac{\Xi^g_t}{\Xi_t} = {} & \beta \mathbb{E}_t \Big(\frac{\Xi_{t+1}}{\Xi_t} \zeta \frac{Y_t + \mu^t \Phi}{K^g_{t-1}} + (1-\delta)\frac{\Xi^g_{t+1}}{\Xi_t}\Big).
\end{aligned}$$

Defining the shadow price of government capital as $Q^g_t \equiv \frac{\Xi^g_t}{\Xi_t}$ and detrending, the first order conditions can be equivalently written as:

$$1 = Q^g_t q^x_t \left(1 - S_g\Big(\frac{[\epsilon^{x,g}_t + ^g_t]\mu}{[\epsilon^{x,g}_{t-1} + x^g_{t-1}]}\Big) - S'_g\Big(\frac{[\epsilon^{x,g}_t + x^g_t]\mu}{[\tilde{\epsilon}^{x}_{t-1} + x^g_{t-1}]}\Big)\Big(\frac{[\epsilon^{x,g}_t + x^g_t]\mu}{[\epsilon^{x,g}_{t-1} + x^g_{t-1}]}\Big)\right)$$

$$+ \bar{\beta}\mathbb{E}_t \left(Q^g_{t+1} \frac{\xi_{t+1}}{\xi_t} q^x_{t+1} S'_g \Big(\frac{[\epsilon^{x,g}_{t+1} + x^g_{t+1}]\mu}{[\epsilon^{x,g}_t + x^g_t]} \Big) \Big(\frac{[\tilde{\epsilon}^{x,g}_{t+1} + x^g_{t+1}]\mu}{[\epsilon^{x,g}_t + x^g_t]} \Big)^2 \right) \tag{B.48a}$$

$$Q^g_t = \bar{\beta}\mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} \zeta \frac{y_t + \Phi}{k^g_{t-1}/\mu} + \frac{\xi_{t+1}}{\xi_t}(1-\delta) Q^g_{t+1} \right), \tag{B.48b}$$

where $\epsilon^{x,g}_t \equiv \frac{1}{\mu}\tilde{\epsilon}^{x,g}_t$ denotes the detrended investment spending shock.

Along the balanced growth path, $S_g(\mu) = S'_g(\mu) = 0, \bar{q}^{x,g} = 1, \bar{\epsilon}^{x,g} = 0$ ensure that the shadow price of capital equals unity. Introduce r^g_t as shorthand for the implied rental rate on government capital:

$$r^g_t = \zeta \frac{y_t + \Phi}{k^g_t/\mu}. \tag{B.49}$$

In the steady state, from (B.48b):

$$\bar{r}^g = \bar{\beta}^{-1} - (1-\delta). \tag{B.50}$$

Equation (B.48b) determines the optimal ratio of government capital to gross output. Importantly, the law of motion for government capital (B.47) and (B.48b) evaluated at the balanced growth path allow us to back out the share of government capital in the aggregate production function, for any given government investment to net output ratio $\frac{\bar{x}^g}{\bar{y}}$. From the law of motion along the balanced growth path:

$$\bar{x}^g = \left(1 - \frac{1-\delta}{\mu}\right)\bar{k}^g \quad \Leftrightarrow \quad \frac{\bar{x}^g}{\bar{y}} = [\mu - (1-\delta)\frac{\bar{k}^g}{\mu\bar{y}},$$

From the equation for r^g_t, we have that $\frac{\bar{k}^g}{\mu\bar{y}} = \zeta\frac{\bar{y}+\Phi}{\bar{y}}\frac{1}{\bar{r}^g}$. Combined with the previous equation this allows us to solve for the government capital share ζ:

$$\zeta = \frac{\bar{y}}{\bar{y}+\Phi} \frac{\bar{r}^g}{1-(1-\delta)} \frac{\bar{x}}{\bar{y}}. \tag{B.51}$$

B.3.2 Monetary policy

The specification of the interest rate rule follows **?**. The Federal Reserve sets interest rates according to the following rule:

$$\frac{R^{FFR}_t}{\bar{R}} = \left(\frac{R^{FFR}_{t-1}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi_1} \left(\frac{Y_t}{Y^f_t}\right)^{\psi_2} \right]^{1-\rho_R} \left(\frac{Y_t/Y_{t-1}}{Y^f_t/Y^f_{t-1}}\right)^{\psi_3} \epsilon^r_t, \tag{B.52}$$

where ρ_R determines the degree of interest rate smoothing and Y_t^f denotes the level of output that would prevail in the economy in the absence of nominal frictions and with constant markups (i.e., the flexible output level). $\psi_1 > 1$ determines the reaction to deviations of inflation from its long-run average, and $\psi_2, \psi_3 > 0$ determines the reaction to the deviation of actual output from the flexible economy output and to the change in the gap between actual and flexible output.

Due to financial market frictions, the return on government bonds differs from the FFR:

$$R_t^{gov} = R_t^{FFR}(1 + \omega_t^b).$$

The flexible economy is the limit point of the economy characterized above with $\zeta_p = \zeta_w = 0$ and no markup shocks: $\epsilon_t^{\lambda,p} = \epsilon_t^{\lambda,w} = 0$. From the pricing and wage-setting rules, this limiting solution implies:

$$\frac{P_t^f(i)}{P_t^f} = [1 + \lambda_p(\mathrm{y}_t^f(i))] mc_t^f(i), \tag{B.53}$$

$$\frac{W_t^f(\ell)}{P_t^f} = [1 + \lambda_w(\mathrm{n}_t^f(\ell))] \frac{1 + \tau_t^c}{1 - \tau_t^{n,f}} n_t^{f\,\nu} [C_t^f - hC_{t-1}^f], \tag{B.54}$$

where the superscript f denotes variables in the flexible economy. Given that final goods are the numeraire and given that firms are symmetric and can freely set their prices:

$$1 = P_t^f = P_t^f(i) = [1 + \lambda_p(1)] mc_t^f(i) \quad \forall t, \tag{B.55}$$

implying that marginal costs are constant for all firms.

Similarly, since all unions face a symmetric problem and can freely reset wages we have that, using that the numeraire equals unity and dividing by trend growth:

$$\frac{W_t^f(\ell)}{\mu} = \frac{W_t^f}{\mu} = w_t^f = [1 + \lambda_w(1)] \frac{1 + \tau_t^c}{1 - \tau_t^{n,f}} n_t^{f\,\nu} [c_t^f - (h/\mu) c_{t-1}^f]. \tag{B.56}$$

Money does not enter explicitly in the economy: the Federal Reserve supplies the amount of money demanded at interest rate R_t.

B.4 Exogenous processes

The exogenous processes are assumed to be log-normally distributed and, with the exception of government spending shocks, independent. Government spending shocks are correlated with technology shocks. Shocks to the two markup processes follow an ARMA(1,1) process, whereas the other shocks are AR(1) processes.

$$\log \epsilon_t^a = \rho_a \log \epsilon_{t-1}^a + u_t^a, \qquad u_t^a \overset{iid}{\sim} \mathcal{N}(0, \sigma_a^2) \tag{B.57a}$$

$$\log \epsilon_t^r = \rho_r \log \epsilon_{t-1}^r + u_t^r, \qquad u_t^r \overset{iid}{\sim} \mathcal{N}(0, \sigma_r^2) \tag{B.57b}$$

$$\log g_t = \log g_t^a + \tilde{u}_t^g, \tag{B.57c}$$

$$\log g_t^a = (1-\rho_g) \log \bar{g} + \rho_g \log g_{t-1}^a + \sigma_{ga} u_t^a + u_t^g, \qquad u_t^a \overset{iid}{\sim} \mathcal{N}(0, \sigma_a^2) \tag{B.57d}$$

$$\log s_t^{exo} = \tilde{u}_t^s, \tag{B.57e}$$

$$\log \epsilon_t^\tau = \rho_\tau \log \epsilon_{t-1}^\tau + u_t^\tau, \qquad u_t^\tau \overset{iid}{\sim} \mathcal{N}(0, \sigma_\tau^2) \tag{B.57f}$$

$$\log \tilde{\epsilon}_t^{\lambda,p} = \rho_{\lambda,p} \log \tilde{\epsilon}_{t-1}^{\lambda,p} + u_t^{\lambda,p} - \theta_{\lambda,p} u_{t-1}^{\lambda,p}, \qquad u_t^{\lambda,p} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\lambda,p}^2) \tag{B.57g}$$

$$\log \tilde{\epsilon}_t^{\lambda,w} = \rho_{\lambda,w} \log \tilde{\epsilon}_{t-1}^{\lambda,w} + u_t^{\lambda,w} - \theta_{\lambda,w} u_{t-1}^{\lambda,w}, \qquad u_t^{\lambda,w} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\lambda,w}^2) \tag{B.57h}$$

$$\log(1+\omega_t^b) \equiv \log q_t^b = \rho_b \log q_{t-1}^b + u_t^b, \qquad u_t^b \overset{iid}{\sim} \mathcal{N}(0, \sigma_b^2) \tag{B.57i}$$

$$\log(1-\omega_t^k) \equiv \log q_t^k = \rho_k \log q_{t-1}^k + u_t^k, \qquad u_t^k \overset{iid}{\sim} \mathcal{N}(0, \sigma_k^2) \tag{B.57j}$$

$$\log q_t^x = \rho_x \log q_{t-1}^x + u_t^x, \qquad u_t^x \overset{iid}{\sim} \mathcal{N}(0, \sigma_x^2) \tag{B.57k}$$

$$\log q_t^{x,g} = \rho_{x,g} \log q_{t-1}^{x,g} + u_t^{x,g}, \qquad u_t^{x,g} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{x,g}^2) \tag{B.57l}$$

Three shocks are deterministic and used for policy counterfactuals only:

$$\tilde{u}_t^s, \tilde{u}_t^g, \tilde{u}_t^{x,g}.$$

B.5 Equilibrium conditions

B.5.1 Aggregation

From the final goods producers' problem (B.1) and using the zero-profit condition in the competitive market, net output in nominal and real terms is given by:

$$P_tY_t = \int_0^1 P_t(i)Y_t(i)di \quad \Leftrightarrow \quad Y_t = \int_0^1 \frac{P_t(i)}{P_t}Y_t(i)di.$$

Outside the flexible economy, relative prices differ from unity, so that output is not simply the average production of intermediates. To a first order, however, price dispersion is irrelevant because $y_t(i) \approx y_t - \eta_p(1)y_t\left(\frac{P_t(i)}{P_t} - 1\right)$, so that the dispersion term averages out in the aggregate $\int_0^1 y_t(i)di \approx y_t$.

In the presence of heterogeneous labor, the measurement of labor supply faces similar issues because:

$$n_t = \int_0^1 \frac{W_t(\ell)}{W_t}n_t(\ell)d\ell,$$

which, by analogy to the above argument for output, generally differs from average hours. However, to a first order:

$$\int_0^1 n_t(\ell)d\ell \approx n_t \tag{B.58}$$

Noncredit constrained households are indexed by $j \in [0, 1-\phi]$, and there is measure $1-\phi$ of these households in the economy. Each noncredit-constrained household supplies $K_t(j) = K_t^{RA}$ units of capital services, so that total holdings of capital capital and government bonds per intertemporally optimizing household are given by $\frac{1}{1-\phi}$ times the aggregate quantity. Similarly, household investment is a multiple of aggregate investment. To see this, note that aggregate quantities of bond holdings B_t, investment X_t, physical capital K_t^p, and capital services K_t are computed as:

$$K_t = \int_0^{1-\phi} K_t(j)\Lambda(dj) = K_t(1-\phi)^{-1}\Lambda([0, 1-\phi]) = K_t.$$

Aggregate consumption is given by:

$$C_t = \int_0^1 C_t(j)\Lambda(dj) = \int_0^{1-\phi} C_t^{RA}\Lambda(dj) + \int_{1-\phi}^1 C_t^{RoT}\Lambda(dj) = (1-\phi)C^RA_t + \phi C_t^{RoT}. \tag{B.59}$$

Given the consumption of rule-of-thumb agents (B.36), that of intertemporally optimizing agents is given by:

$$\bar{c}^{RA} = \frac{\bar{c} - \phi\bar{c}^{RoT}}{1-\phi}. \tag{B.60}$$

Similarly, aggregate transfers are given by

$$S_t = (1-\phi)S_t^{RA} + \phi S_t^{RoT}, \tag{B.61}$$

where equation (B.37) implies that:

$$\bar{s} = \bar{s}^{RA} + \bar{s}^{RoT}.$$

Aggregate labor supply coincides with individual labor supply of either type of household.

B.5.2 Market clearing

Labor market clearing requires that labor demanded by intermediaries equals labor supplied by labor packers:

$$\int_0^1 n_t(i)di = n_t = n_t \int_0^1 \frac{W_t(\ell)}{W_t} n_t(\ell)d\ell,$$

where $n_t(\ell)$ is measured in units of the differentiated labor supplies and n_t is measured in units which differs from those supplied by households.

Adding the government and the budget constraints of the two types of households, integrated over $[0, 1-\phi]$ and $(1-\phi, 1]$, respectively, and substituting $\int_0^1 n_t(j)W_t^h(1+\lambda_{t,w})dj = W_t n_t$, which results from combining the labor packers' zero-profit condition with the union problem into the household budget constraint, yields the following equation:

$$\begin{aligned} C_{t+s} + X_{t+s}(j) + G_t + X_{t+s}^g = n_t \frac{W_{t+s}}{P_{t+s}} \\ + \left[\frac{R_{t+s}^k u_{t+s}}{P_{t+s}} - a(u_{t+s})\right] K_{t+s-1}^p + \frac{\Pi_{t+s}^p \mu^{t+s}}{P_{t+s}}, \end{aligned}$$

Detrending and substituting in for real profits from (B.17a) and using that $w_t \int_0^1 n_t(i)di = w_t n_t$ yields:

$$c_{t+s} + x_{t+s} + \bar{y}g_{t+s} + x_{t+s}^g = y_{t+s} - a(u_{t+s})\mu k_{t+s-1}^p, \tag{B.62}$$

which is the goods market clearing condition: Production is used for government and private consumption, government and private investment, as well as variations in capacity utilization.

B.6 Linearized equilibrium conditions

B.6.1 Firms

Log-linearizing the production function around the symmetric balanced growth path:

$$\hat{y}_t = \frac{\bar{y} + \Phi}{\bar{y}} \left(\hat{\epsilon}^a_t + \zeta \hat{k}^g_{t-1} + \alpha(1-\zeta)\hat{k}_t + (1-\alpha)(1-\zeta)\hat{n}_t \right). \tag{B.63}$$

The capital-labor ratio is approximated by (B.9):

$$\hat{k}_t = \hat{n}_t + \hat{w}_t - \hat{r}^k_t, \tag{B.64}$$

where symmetry around the balanced growth path was used.

Marginal costs in (B.65) are approximated by:

$$\widehat{mc}_t = (1-\alpha)\hat{w}_t + \alpha \hat{r}^k_t - \frac{1}{1-\zeta}\left(\zeta \hat{k}^g_t - \zeta \frac{\bar{y}}{\bar{y}+\Phi}\hat{y}_t + \hat{\epsilon}^a_t\right)\left(\frac{k^g_t}{y_t + \Phi}\right)^{\frac{\zeta}{1-\zeta}} \tilde{\epsilon}^a_t, \tag{B.65}$$

and in the flexible economy from (B.55):

$$\widehat{mc}^f_t = 0 \tag{B.66}$$

To log-linearize the pricing FOC (B.14), note that, to a first order, the common terms in numerator and denominator, i.e., $\frac{\xi_{t+s} y_{t,t+s}(i)}{\lambda_p(\mathrm{y}_{t+s}(\mathrm{i}))\xi_t}$, cancel out, using equation (B.16). As a preliminary step, notice that in the absence of markup shocks:

$$\begin{aligned}
\overline{mc} d\left(\frac{\eta_p(\mathrm{y}_{t+s}(i))}{1-\eta_p(\mathrm{y}_{t+s}(i))}\right)\Big|_{\mathrm{y}_{t+s}(i)=1} &= \overline{mc}\frac{\bar{\eta}_p}{1-\bar{\eta}_p}\frac{-1}{1-\bar{\eta}_p}\frac{d\eta_p(\mathrm{y}_{t+s}(i))|_{\mathrm{y}_{t+s}(i)=1}}{\bar{\eta}_p} \\
&= -\bar{\lambda}_p \hat{\eta}_p(1) d\left(\frac{P^*_t(i)}{P_{t+s}}\right)\Big|_{\frac{P_{t+s}(i)}{P_{t+s}}=1}, \\
d\left(\frac{P_{t+s}(i)}{P_{t+s}}\right)\Big|_{\frac{P^*_t(i)}{P_{t+s}}=1} &= d\left(\frac{\chi_{t,t+s}}{\prod_{l=1}^{s}\pi_{t+l}}\right) + d\left(\frac{P^*_t(i)}{P_t}\right).
\end{aligned}$$

Notice that from (B.5):

$$1 + \bar{\lambda}_p \hat{\eta}_p = \frac{1}{\bar{A}_p}.$$

To simplify notation and to address markup shocks, use $\bar{\epsilon}^{\lambda,p} = 1$ and define:

$$p^*_t(i) \equiv \frac{P^*_t(i)}{P_t},$$

$$\hat{\epsilon}^{\lambda,p}_{t+s} \equiv \frac{\partial}{\partial \epsilon^{\lambda,p}_{t+s}} \left(\frac{\eta_p(\mathrm{y}_{t+s}(i))}{1-\eta_p(\mathrm{y}_{t+s}(i))} \right)\Big|_{\mathrm{y}_{t+s}(i)=1} \hat{\hat{\epsilon}}^{\lambda,p}_{t+s} = \frac{\eta_p(1)}{[1-\eta_p(1)]^2} \left(\frac{G'_\epsilon(1)}{G'(1)} - \frac{G''_\epsilon(1)}{G''(1)} \right).$$

Now, taking a first-order approximation of (B.14) and using symmetry yields:

$$\begin{aligned}
0 &= \mathbb{E}_t \sum_{s=0}^{\infty} (\mu\bar{\beta}\zeta_p)^s \Big[\hat{p}^*_t(i) + \sum_{l=1}^{s} [\iota_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \Big] (1 + \bar{\lambda}_p \hat{\eta}(1)) \\
&\quad - [\widehat{mc}_{t+s} + \hat{\epsilon}^{\lambda,p}_{t+s}] \\
\Leftrightarrow \frac{1}{1-\bar{\beta}\zeta_p\mu} \frac{1}{\bar{A}_p} \hat{p}^*_t &= \mathbb{E}_t \sum_{s=0}^{\infty} (\mu\bar{\beta}\zeta_p)^s [\widehat{mc}_{t+s} + \hat{\epsilon}^{\lambda,p}_{t+s}] - \sum_{l=1}^{s} [\iota_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \frac{1}{\bar{A}_p} \\
&= \widehat{mc}_t + \hat{\epsilon}^{\lambda,p}_t - \frac{\bar{\beta}\mu\zeta_p}{1-\bar{\beta}\mu\zeta_p} \frac{1}{\bar{A}_p} [\iota_p \hat{\pi}_t - \mathbb{E}_t \hat{\pi}_{t+1}] \\
&\quad + \mu\bar{\beta}\zeta_p \mathbb{E}_t \mathbb{E}_{t+s} \sum_{s=0}^{\infty} (\mu\bar{\beta}\zeta_p)^s [\widehat{mc}_{t+1+s} + \hat{\epsilon}^{\lambda,p}_{t+1+s}] - \sum_{l=1}^{s} [\iota_p \hat{\pi}_{t+l} - \hat{\pi}_{t+1+l}] \frac{1}{\bar{A}_p} \\
&= \widehat{mc}_t + \hat{\epsilon}^{\lambda,p}_t - \frac{\bar{\beta}\mu\zeta_p}{1-\bar{\beta}\mu\zeta_p} \frac{1}{\bar{A}_p} [\iota_p \hat{\pi}_t - \mathbb{E}_t \hat{\pi}_{t+1}] + \mu\bar{\beta}\zeta_p \mathbb{E}_t \hat{p}^*_{t+1}.
\end{aligned}$$

Now, linearizing the evolution of the price index (B.15):

$$\hat{p}^*_t = \frac{\zeta_p}{1-\zeta_p} [\hat{\pi}_t - \iota_p \hat{\pi}_{t-1}] \quad \Leftrightarrow \quad \hat{\pi}_t = \frac{1-\zeta_p}{\zeta_p} \hat{p}^*_t + \iota_p \hat{\pi}_{t-1}.$$

Forwarding the equation once and substituting in and solving for $\hat{\pi}_t$ yields:

$$\hat{\pi}_t = \frac{\iota_p}{1+\iota_p\bar{\beta}\mu} \hat{\pi}_{t-1} + \frac{1-\zeta_p\bar{\beta}\mu}{1+\iota_p\bar{\beta}\mu} \frac{1-\zeta_p}{\zeta_p} \bar{A}_p (\widehat{mc}_t + \hat{\epsilon}^{\lambda,p}_t) + \frac{\bar{\beta}\mu}{1+\iota_p\bar{\beta}\mu} \mathbb{E}_t \hat{\pi}_{t+1}. \quad \text{(B.67)}$$

B.6.2 Households

The law of motion for capital (B.26) and the fact that individual capital holdings are proportional to aggregate capital holdings imply:

$$\hat{k}^p_t = \left(1 - \frac{\bar{x}}{\bar{k}^p}\right) \hat{k}^p_{t-1} + \frac{\bar{x}}{\bar{k}^p} (\hat{x}_t + \hat{q}^x_{t+s}). \quad \text{(B.68)}$$

From (B.27), capital services evolve as:

$$\hat{k}_t = \hat{u}_t + \hat{k}^p_{t-1}. \quad \text{(B.69)}$$

From the static optimality condition (B.31)

$$\hat{w}_t^h = \nu \hat{n}_t + \frac{\hat{c}_t^{RA} - (h/\mu)\hat{c}_{t-1}^{RA}}{1 - h/\mu} + \frac{d\tau_t^n}{1 - \bar{\tau}^n} + \frac{d\tau_t^c}{1 + \tau^c}. \tag{B.70}$$

In the flexible economy, given the absence of markup shocks equation (B.56) implies:

$$\hat{w}_t^f = \nu \hat{n}_t^f + \frac{\hat{c}_t^{RA,f} - (h/\mu)\hat{c}_{t-1}^{RA,f}}{1 - h/\mu} + \frac{d\tau_t^{n,f}}{1 - \bar{\tau}^n} + \frac{d\tau_t^{c,f}}{1 + \bar{\tau}^c}. \tag{B.71}$$

In the presence of rigidities, the dynamic wage-setting equation (B.40) can be linearized as in the derivation of (B.67), recognizing that the analogue to marginal costs is given by (B.70):[3]

$$\begin{aligned}\hat{w}_t = &\frac{\hat{w}_{t-1}}{1+\bar{\beta}\mu} + \frac{\bar{\beta}\mu \mathbb{E}_t[\hat{w}_{t+1}]}{1+\bar{\beta}\mu} \\ &+ \frac{(1-\zeta_w\bar{\beta}\mu)(1-\zeta_w)}{(1+\bar{\beta}\mu)\zeta_w}\bar{A}_w\left[\frac{1}{1-h/\mu}[\hat{c}_t - (h/\mu)\hat{c}_{t-1}] + \nu\hat{n}_t - \hat{w}_t + \frac{d\tau_t^n}{1-\tau_n} + \frac{d\tau_t^c}{1+\tau_c}]\right] \\ &- \frac{1+\bar{\beta}\mu\iota_w}{1+\bar{\beta}\mu}\hat{\pi}_t + \frac{\iota_w}{1+\bar{\beta}\mu}\hat{\pi}_{t-1} + \frac{\bar{\beta}\mu}{1+\bar{\beta}\mu}\mathbb{E}_t[\hat{\pi}_{t+1}] + \frac{\hat{\epsilon}_t^{\lambda,w}}{1+\bar{\beta}\mu}.\end{aligned} \tag{B.72}$$

From the consumption Euler equation (B.32):

$$\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t] + \mathbb{E}_t[d\tau_{t+1}^c - d\tau_t^c] =$$

[3] Here, the analogy with marginal costs holds only to a first order. Noting that common terms drop out the first order condition (B.39) and using (B.42) as well as $A_w \equiv [1 + \bar{\lambda}_w \hat{\eta}_w(1)]^{-1}$ linearizes as follows:

$$\begin{aligned}0 = &\mathbb{E}_t\left(\sum_{s=0}^{\infty}(\zeta_w\mu\bar{\beta})^s\frac{\bar{n}}{\bar{\lambda}_w}\bar{w}^*\left([\hat{w}_t^* + \sum_{l=1}^{s}(\iota_w\hat{\pi}_{t+l-1} - \hat{\pi}_{t+l})](1+\bar{\lambda}\hat{\eta}_w(1)) - \bar{\lambda}_w\hat{\eta}_w(1)\hat{w}_{t+s} + \hat{w}_{t+s}^h + \hat{\hat{\epsilon}}_{t+s}^{\lambda,w}\right)\right) \\ \propto &\frac{1}{1-\zeta_w\mu\bar{\beta}}A_w^{-1}[\hat{w}_t^* + \iota_w\hat{\pi}_t - \mathbb{E}_t(\hat{\pi}_{t+1})] \\ &+ \mathbb{E}_t\left(\sum_{s=0}^{\infty}(\zeta_w\mu\bar{\beta})^s\left([A_w^{-1}\sum_{l=1}^{s-1}(\iota_w\hat{\pi}_{t+l} - \hat{\pi}_{t+l+1})](1+\bar{\lambda}\hat{\eta}_w(1)) - [A_w^{-1} - 1]\hat{w}_{t+s} - \hat{w}_{t+s}^h - \hat{\hat{\epsilon}}_{t+s}^{\lambda,w}\right)\right) \\ \propto &\frac{1}{1-\zeta_w\mu\bar{\beta}}A_w^{-1}[\hat{w}_t^* + \iota_w\hat{\pi}_t - \mathbb{E}_t(\hat{\pi}_{t+1}) - \zeta_w\mu\bar{\beta}\mathbb{E}_t(w_{t+1}^*)] - \hat{w}_t^h - \hat{\hat{\epsilon}}_t^{\lambda,w} - (1 - A_w^{-1})\hat{w}_t\end{aligned}$$

Log-linearizing the law of motion for aggregate wages (B.41) around the symmetric balanced growth path yields:

$$\hat{w}_t^* = \frac{1}{1-\zeta_w}[\hat{w}_t - \zeta_w\hat{w}_{t-1} - \zeta\iota_w\hat{\pi}_{t-1} + \zeta_w\hat{\pi}_t.$$

Substituting this equation into the above for $\hat{w}_t^*, \hat{w}_{t+1}^*$ and re-arranging yields (B.72).

$$
\begin{aligned}
&= \mathbb{E}_t\left((\sigma-1)\bar{n}^{1+\nu}[\hat{n}_{t+1}-\hat{n}_t] - \frac{\sigma}{1-h/\mu}\left[\hat{c}^{RA}_{t+1} - \left(1+\frac{h}{\mu}\right)c^{RA}_t + \frac{h}{\mu}\hat{c}^{RA}_{t+1}\right]\right) \\
&= \frac{1}{1-h/\mu}\mathbb{E}_t\big((\sigma-1)\frac{\bar{n}^{1+\nu}[\bar{c}^{RA}-h/\mu\bar{c}^{RA}]}{\bar{c}^{RA}}[\hat{n}_{t+1}-\hat{n}_t] \\
&\qquad - \sigma\left[\hat{c}^{RA}_{t+1} - \left(1+\frac{h}{\mu}\right)c^{RA}_t + \frac{h}{\mu}\hat{c}^{RA}_{t+1}\right]\big) \\
&= \frac{1}{1-h/\mu}\mathbb{E}_t\big((\sigma-1)\frac{1}{1+\bar{\lambda}_w}\frac{1-\bar{\tau}^n}{1+\tau^c}\frac{\bar{w}\bar{n}}{\bar{c}^{RA}}[\hat{n}_{t+1}-\hat{n}_t] \\
&\qquad - \sigma\left[\hat{c}^{RA}_{t+1} - \left(1+\frac{h}{\mu}\right)c^{RA}_t + \frac{h}{\mu}\hat{c}^{RA}_{t+1}\right]\big),
\end{aligned}
$$

where the last equality uses (B.42). Solving for current consumption growth:

$$
\begin{aligned}
\hat{c}^{RA}_t = &\frac{1}{1+h/\mu}\mathbb{E}_t[\hat{c}^{RA}_{t+1}] + \frac{h/\mu}{1+h/\mu}\hat{c}^{RA}_{t-1} + \frac{1-h/\mu}{\sigma[1+h/\mu]}\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t + (d\tau^c_{t+1} - d\tau^c_t)] \\
&- \frac{[\sigma-1][\bar{w}\bar{n}/\bar{c}]}{\sigma[1+h/\mu]}\frac{1}{1+\lambda_w}\frac{1-\tau^n}{1+\tau^c}(\mathbb{E}_t[\hat{n}_{t+1}] - \hat{n}_t). \qquad \text{(B.73)}
\end{aligned}
$$

The remaining households' FOC linearize as:

$$
\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t] = -\hat{q}^b_t - \hat{R}_t + \mathbb{E}_t[\hat{\pi}_{t+1}], \qquad \text{(B.74a)}
$$

$$
\begin{aligned}
\hat{Q}_t = &-\hat{q}^b_t - (\hat{R}_t - \mathbb{E}_t[\pi_{t+1}]) + \frac{1}{\bar{r}^k(1-\tau^k)+\delta\tau^k+1-\delta}\times \\
&\times\Big[(\bar{r}^k(1-\tau^k)+\delta\tau^k)\hat{q}^k_t - (\bar{r}^k-\delta)d\tau^k_{t+1} + \qquad \text{(B.74b)} \\
&+ \bar{r}^k(1-\tau^k)\mathbb{E}_t(\hat{r}^k_{t+1}) + (1-\delta)\mathbb{E}_t(\hat{Q}_{t+1})\Big], \qquad \text{(B.74c)}
\end{aligned}
$$

$$
\hat{x}_t = \frac{1}{1+\bar{\beta}\mu}\left[\hat{x}_{t-1} + \bar{\beta}\mu\mathbb{E}_t(\hat{x}_{t+1}) + \frac{1}{\mu^2 S''(\mu)}[\hat{Q}_t + \hat{q}^x_t]\right], \qquad \text{(B.74d)}
$$

$$
\hat{u}_t = \frac{a'(1)}{a''(1)}\hat{r}^k_t \equiv \frac{1-\psi_u}{\psi_u}\hat{r}^k_t. \qquad \text{(B.74e)}
$$

For the credit-constrained households, (B.35) implies the following linear consumption process: consumption evolves as

$$
\hat{c}^{RoT}_t = \frac{1}{1+\tau^c}\left(\frac{\bar{s}^{RoT}}{\bar{c}^{RoT}}\hat{s}_t + \frac{\bar{w}\bar{n}}{\bar{c}^{RoT}}[(1-\tau^n)(\hat{w}_t+\hat{n}_t) - d\tau^n_t] - d\tau^c_t + \frac{\bar{y}}{\bar{c}^{RoT}}\frac{d\Pi^p_t}{\bar{y}}\right), \qquad \text{(B.75)}
$$

where the change in profits is given by:

$$\frac{d\Pi_t^p}{\bar{y}} = \frac{1}{1+\lambda_p}\hat{y}_t - \widehat{mc}_t.$$

B.6.3 Government

The financing need evolves as:

$$\frac{dd_t}{\bar{y}} = \frac{1}{\mu}\Bigg[\mu[\hat{g}_t^a + \hat{g}^s] + \mu\frac{\bar{s}}{\bar{y}}\hat{s}_t^{exog} + \frac{\bar{b}}{\bar{y}}\frac{\hat{b}_{t-1} - \hat{\pi}_t}{\bar{\pi}} - \mu\tau^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\Big(\hat{w}_t + \hat{n}_t\Big)$$
$$- \mu\tau_c\frac{\bar{c}}{\bar{y}}\hat{c}_t - \tau^k[\bar{r}^k r_t^k + (r_t^k - \delta)\hat{k}_{t-1}^p]\mu\frac{\bar{k}}{\bar{y}}\Bigg]. \tag{B.76}$$

In the benchmark case of distortionary labor taxes, labor tax rates evolve according to (B.45), which is linearized as:

$$\bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\left[\frac{d\tau_t^n}{\tau_n}\right] + \hat{\epsilon}_t^\tau = \psi_\tau\frac{dd_t}{\bar{y}}$$
$$= \frac{\psi_\tau}{\mu}\Bigg[\mu[\hat{g}_t^a + \hat{g}^s] + \mu\frac{\bar{s}}{\bar{y}}\hat{s}_t^{exog} + \frac{\bar{b}}{\bar{y}}\frac{\hat{b}_{t-1} - \hat{\pi}_t}{\bar{\pi}} - \mu\tau^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\Big(\hat{w}_t + \hat{n}_t\Big)$$
$$- \mu\tau_c\frac{\bar{c}}{\bar{y}}\hat{c}_t - \tau^k[\bar{r}^k r_t^k + (r_t^k - \delta)\hat{k}_{t-1}^p]\mu\frac{\bar{k}}{\bar{y}}\Bigg]. \tag{B.77}$$

In general, tax rates, or endogenous transfers, satisfy from (B.46):

$$\bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\left[\frac{d\tau_t^n}{\tau_n}\right] + \tau^c\frac{\bar{c}}{\bar{y}}\frac{d\tau_t^c}{\tau^c} + \tau^k\frac{[\bar{r}^k - \delta]\bar{k}}{\bar{y}}\frac{d\tau_t^k}{\tau^k} - \frac{\bar{s}}{\bar{y}}\hat{s}_t^{endog} + \hat{\epsilon}_t^\tau = \psi_\tau\frac{dd_t}{\bar{y}}. \tag{B.78}$$

Debt holdings are determined from the budget constraint (B.44):

$$\frac{1}{\bar{R}}\frac{\bar{b}}{\bar{y}}[\hat{b}_t - \hat{R}_t - \hat{q}_t^b] = (1-\psi_\tau)\frac{dd_t}{\bar{y}} - \bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\left[\frac{d\tau_t^n}{\tau_n}\right] - \tau^c\frac{\bar{c}}{\bar{y}}\frac{d\tau_t^c}{\tau^c}$$
$$- \tau^k\frac{[\bar{r}^k - \delta]\bar{k}}{\bar{y}}\frac{d\tau_t^k}{\tau^k} + \frac{\bar{s}}{\bar{y}}\hat{s}_t^{endog} - \hat{\epsilon}_t^\tau \tag{B.79}$$

The linearized counterpart to the law of motion for government capital (B.47) is given by:

$$\hat{k}^g = \left(1 - \frac{\bar{x}^g}{\bar{k}^g}\right)\hat{k}_{t-1}^g + \frac{\bar{x}^g}{\bar{k}^g}\hat{q}_t^{x,g} + \frac{\bar{x}^g}{\bar{k}^g}[\hat{x}_t^g + \hat{\epsilon}_t^{xg}], \tag{B.80}$$

where $u_t^{x,g} \equiv \frac{\tilde{u}_t^{x,g}}{\bar{x}^g}$.

The marginal product of government capital (B.49) is approximated by:

$$\hat{r}_t^g = \frac{\bar{y}}{\bar{y}+\Phi}\hat{y}_t - \hat{k}_{t-1}^g. \tag{B.81}$$

The shadow price of government capital (B.48b) has the following linear approximation:

$$\hat{Q}_t^g = -(\hat{R}_t + \hat{q}_t^b - \mathbb{E}_t[\pi_{t+1}]) + \frac{1}{\bar{r}^g + 1 - \delta}[\bar{r}^g \mathbb{E}_t(\hat{r}_{t+1}^g) + (1-\delta)\mathbb{E}_t(\hat{Q}_{t+1}^g)]. \tag{B.82}$$

The Euler equation for government investment (B.48a) is approximated as:

$$\hat{x}_t^g = \frac{1}{1+\bar{\beta}\mu}\left[\hat{x}_{t-1} + u_{t-1}^{xg} + \bar{\beta}\mu\mathbb{E}_t([\hat{x}_{t+1}^g + u_{t+1}^{xg}]) + \frac{1}{\mu^2 S_g''(\mu)}[\hat{Q}_t^g + \hat{q}_t^{x,g}]\right] - u_t^{xg}. \tag{B.83}$$

The monetary policy rule (B.52) is approximated by:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)[\psi_1\hat{\pi}_t + \psi_2(\hat{y}_t - \hat{y}_t^f)] + \psi_3\Delta(\hat{y}_t - \hat{y}_t^f) + \hat{\epsilon}_t^r \tag{B.84}$$

B.6.4 Exogenous processes

The shock processes (B.57) are linearized as

$$\hat{\epsilon}_t^a = \rho_a\hat{\epsilon}_{t-1}^a + u_t^a, \tag{B.85a}$$
$$\hat{\epsilon}_t^r = \rho_r\hat{\epsilon}_{t-1}^r + u_t^r, \tag{B.85b}$$
$$\hat{g}_t = \hat{g}_t^a + \tilde{u}_t^g, \tag{B.85c}$$
$$\hat{g}_t^a = \rho_g\hat{g}_{t-1}^a + \sigma_{ga}u_t^a + u_t^g, \tag{B.85d}$$
$$\hat{s}_t = \tilde{u}_t^s, \tag{B.85e}$$
$$\hat{\epsilon}_t^\tau = \rho_\tau\hat{\epsilon}_{t-1}^\tau + u_t^\tau, \tag{B.85f}$$
$$\hat{\hat{\epsilon}}_t^{\lambda,p} = \rho_{\lambda,p}\hat{\hat{\epsilon}}_{t-1}^{\lambda,p} + u_t^{\lambda,p} - \theta_{\lambda,p}u_{t-1}^{\lambda,p}, \tag{B.85g}$$
$$\hat{\hat{\epsilon}}_t^{\lambda,w} = \rho_{\lambda,w}\hat{\hat{\epsilon}}_{t-1}^{\lambda,w} + u_t^{\lambda,w} - \theta_{\lambda,w}u_{t-1}^{\lambda,w}, \tag{B.85h}$$
$$\hat{q}_t^b = \rho_b\hat{q}_{t-1}^b + u_t^b, \tag{B.85i}$$
$$\hat{q}_t^k = \rho_k\hat{q}_{t-1}^k + u_t^k, \tag{B.85j}$$
$$\hat{q}_t^x = \rho_x\hat{q}_{t-1}^x + u_t^x, \tag{B.85k}$$
$$\hat{q}_t^{x,g} = \rho_{x,g}\hat{q}_{t-1}^{x,g} + u_t^{x,g}. \tag{B.85l}$$

B.6.5 Aggregation

Aggregate consumption (B.59) and transfers (B.61) are linearized as:

$$\hat{c}_t = (1-\phi)\frac{\bar{c}^{RA}}{\bar{c}}\hat{c}_t^{RA} + \phi\frac{\bar{c}^{RoT}}{\bar{c}}\hat{c}_t^{RoT}, \tag{B.86}$$

$$\hat{s}_t = (1-\phi)\frac{\bar{s}^{RA}}{\bar{s}}\hat{s}_t^{RA} + \phi\frac{\bar{s}^{RoT}}{\bar{s}}\hat{s}_t^{RoT}. \tag{B.87}$$

B.6.6 Market clearing

Goods market clearing:

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{x}}{\bar{y}}\hat{x}_t + \frac{\bar{x}^g}{\bar{y}}\hat{x}_t^g + \hat{g}_t + \frac{\bar{r}^k\bar{k}}{\bar{y}}\hat{u}_t. \tag{B.88}$$

B.6.7 Solution

In addition to the exogenous processes in (B.85), the economy with frictions is reduced to 21 variables, whereas the flexible economy is characterized by 19 variables only, given perfectly flexible prices and wages. Table B.4 lists the remaining variables and the corresponding equations. For the flexible economy, all variables other than those with an "n/a" entry have an ^{f} superscript. The markup shock processes affect only the economy with frictions. Table B.5 lists the steady state relationships that enter the linearized equations.

B.7 Measurement equations

For the estimation of the model, the following measurement equations are appended to the model:

$$\Delta Y_t = 100(\hat{y}_t - \hat{y}_{t-1}) + 100(\mu - 1), \tag{B.89a}$$

$$\Delta C_t = 100(\hat{c}_t - \hat{c}_{t-1}) + 100(\mu - 1), \tag{B.89b}$$

$$\Delta X_t = 100(\hat{x}_t - \hat{x}_{t-1}) + 100(\mu - 1), \tag{B.89c}$$

$$\Delta X_t^g = 100(\hat{x}_t^g - \hat{x}_{t-1}^g) + 100(\mu - 1), \tag{B.89d}$$

$$\Delta\frac{W_t}{P_t} = 100(\hat{w}_t - \hat{w}_{t-1}) + 100(\mu - 1), \tag{B.89e}$$

$$\hat{\pi}_t^{obs} = 100\hat{\pi}_t + 100(\bar{\pi} - 1), \tag{B.89f}$$

$$\hat{R}_t^{obs} = 100\hat{R}_t + 100(\bar{R} - 1), \tag{B.89g}$$

$$\hat{q}_t^{k,obs} = 100\hat{q}_t^k + \bar{\hat{q}}^{k,obs}, \tag{B.89h}$$

$$\hat{n}_t^{obs} = 100\hat{n}_t + \bar{\hat{n}}^{obs}, \tag{B.89i}$$

Variable	Economy with frictions	Economy without frictions
$\hat{c}$	(B.86)	(B.86)
$\hat{c}^{RA}$	(B.73)	(B.73)
$\hat{c}^{RoT}$	(B.75)	(B.75)
$\hat{x}$	(B.74a) in (B.74d)	(B.74d), (B.74a)
$\hat{k}^p$	(B.68)	(B.68)
$\hat{k}$	(B.69)	(B.69)
$\hat{u}$	(B.74e)	(B.74e)
$\hat{Q}$	(B.74a) in (B.74c)	(B.74c), (B.74a)
$\hat{r}^k$	(B.64)	(B.64)
$\hat{x}^g$	(B.74a) in (B.83)	(B.83), (B.74a)
$\hat{k}^g$	(B.80)	(B.80)
$\hat{Q}^g$	(B.74a) in (B.82)	(B.82), (B.74a)
$\hat{r}^g$	(B.81)	(B.81)
$\begin{cases} d\tau^n, d\tau^c \\ d\tau^k, \hat{s}^{endo} \end{cases}$	one variable according to (B.78) & (B.76) other variables = 0	(B.78) & (B.76) other variables = 0
$\hat{b}$	(B.79)	(B.79)
$\hat{R}$	(B.84)	indirectly via (B.66)
$\hat{\pi}$	(B.67)	=0
$\widehat{mc}$	(B.65)	=0
$\hat{w}$	(B.72)	(B.71)
$\hat{y}$	(B.88)	(B.88)
$\hat{n}$	(B.63)	(B.63)

Table B.4: Unknowns and equations

Constant	Equation	Expression
$\frac{\bar{c}}{\bar{y}}$	(B.62)	$1-\frac{\bar{x}}{\bar{y}}-\frac{\bar{x}^g}{\bar{y}}-g$
$\frac{\bar{c}^{RA}}{\bar{y}}$	(B.60)	$\frac{\bar{c}-\phi\bar{c}^{RoT}}{\bar{y}(1-\phi)}$
$\frac{\bar{c}^{RoT}}{\bar{y}}$	(B.36)	$\frac{\bar{s}^{RoT}+(1-\tau^n)\bar{w}\bar{n}}{\bar{y}(1+\tau^c)}$
$\frac{\bar{x}}{\bar{k}^p}$	(B.30)	$1-\frac{1-\delta}{\mu}$
$\frac{\bar{x}}{\bar{k}}$	(B.30)	$\mu-(1-\delta)$
$\frac{\bar{k}}{\bar{y}}$	(B.8)	$\left(\frac{\bar{y}+\Phi}{\bar{y}}\right)^{\frac{1}{1-\zeta}}\left(\frac{\bar{k}^g}{\bar{y}}\right)^{\frac{-\zeta}{1-\zeta}}\left(\frac{\bar{k}}{\bar{n}}\right)^{1-\alpha}$
$\bar{u}$	normalization	$a'^{-1}(\bar{r}^k)$
$\bar{\beta}$	definition	$\beta\mu^{-1}$
$\bar{r}^k$	(B.33c)	$\frac{\bar{\beta}^{-1}-\delta\tau^k-(1-\delta)}{1-\tau^k}$
$\frac{\bar{k}^g}{\bar{y}}$	(B.47)	$\left(1-\frac{1-\delta}{\mu}\right)^{-1}\frac{\bar{x}^g}{\bar{y}}$
ζ	(B.51)	$\frac{\bar{y}}{\bar{y}+\Phi}\frac{\bar{r}^g}{1-(1-\delta)/\mu}\frac{\bar{x}}{\bar{y}}$
$\bar{r}^g$	(B.50)	$\bar{\beta}^{-1}-(1-\delta)$
$\bar{R}$	(B.33b)	$\bar{\beta}^{-1}\bar{\pi}$
$\overline{mc}$	(B.16)	$(1+\bar{\lambda}_p)^{-1}$
$\bar{\lambda}_p$	(B.18)	$\frac{\Phi}{\bar{y}}$
$\bar{w}$	(B.11)	$\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{(1+\lambda_w)^{\frac{1}{(1-\zeta)(1-\alpha)}}}\frac{\left(\frac{\bar{k}^g}{\bar{y}}\right)^{\frac{\zeta}{(1-\zeta)(1-\alpha)}}}{\bar{r^k}^{\frac{\alpha}{1-\alpha}}}$
$\frac{\bar{w}\bar{n}}{\bar{y}}$	$[n_t(i)]$, $[K_t(i)]$,(B.16),(B.18)	$1-\bar{r}^k\frac{\bar{k}}{\bar{y}}$
$\frac{\bar{k}}{\bar{n}}$	(B.9)	$\frac{\alpha}{1-\alpha}\frac{\bar{w}}{\bar{r}^k}$

Table B.5: Steady state relationships

$$\hat{b}_t^{obs} = 100\hat{b}_t + \bar{\hat{b}}^{obs}. \tag{B.89j}$$

The constants give the inflation rate $\bar{\pi}$ along the balanced growth path and the trend growth rates. $100(\mu - 1)$ represents the deterministic net trend growth imposed on the data. Note that apart from the trend growth rate and the constant nominal interest rate, the discount factor can be backed out of the constants:

$$\beta = \frac{\bar{\pi}}{\bar{R}}\mu^{\sigma}.$$

The constant terms in the measurement equation are necessary even if the data are demeaned for the particular observation sample because the allocation in the flexible economy cannot be attained in the economy with frictions. Given a nonzero output gap, other variables also will deviate from zero. To see why, notice that for the allocations to be the same in both the economy with frictions and in its frictionless counterpart required that the Calvo constraints on price- and wage-setting were slack – otherwise the equilibrium allocation would differ from that in the flexible economy. Slack Calvo constraints, in turn, required that aggregate prices and wages be constant, which implied a constant real wage. Finally, a constant real wage would be inconsistent with the allocation in the flexible economy.

B.8 Welfare implications

To evaluate welfare implications, we approximate the compensating variation in terms of quarterly consumption of each type of agent separately as well as the population-weighted average.

Independent of whether a household is constrained or not, equation (B.24) gives the preferences of the household. Using the log-linearized model solution around the deterministic balanced growth path, the lifetime utility of any time path of consumption and hours worked can be computed as:

$$\begin{aligned}
U_t(\{\hat{c}_{t+s}, \hat{n}_{t+s}\}) &= \sum_{s=0}^{\infty} \beta^s \left[\frac{(\mu^{1-\sigma})^{t+s}}{1-\sigma} \left(\bar{c}\exp[\hat{c}_{t+s}] - \frac{h}{\mu}\bar{c}\exp[\hat{c}_{t+s-1}] \right)^{1-\sigma} \right] \\
&\qquad \times \exp\left[\frac{\sigma - 1}{1+\nu} (\bar{n}\exp[\hat{n}_{t+s}])^{1+\nu} \right] \\
&= (\mu^{1-\sigma})^t \sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \left[\frac{\bar{c}^{1-\sigma}}{1-\sigma} \left(\exp[\hat{c}_{t+s}] - \frac{h}{\mu}\exp[\hat{c}_{t+s-1}] \right)^{1-\sigma} \right] \\
&\qquad \times \exp\left[-\frac{\bar{n}^{1+\nu}}{1+\nu} \exp[(1+\nu)\hat{n}_{t+s}] \right]^{1-\sigma}
\end{aligned}$$

$$= (\mu^{1-\sigma})^t \frac{\bar{c}^{1-\sigma}}{1-\sigma}$$
$$\times \sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) \exp\Big[- \frac{\bar{n}^{1+\nu}}{1+\nu} \exp[(1+\nu)\hat{n}_{t+s}] \Big] \right]^{1-\sigma}. \tag{B.90}$$

Now we can compute the compensating variation between to paths of consumption and leisure, with and without the fiscal stimulus, as:

$$\Gamma = \left[\frac{\sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \Big(e^{\hat{c}^{ARRA}_{t+s}} - \frac{h}{\mu} e^{\hat{c}^{ARRA}_{t+s-1}} \Big) \exp\Big[-\frac{\bar{n}^{1+\nu}}{1+\nu} \exp[(1+\nu)\hat{n}^{ARRA}_{t+s}] \Big] \Big)^{1-\sigma}}{\sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \Big(e^{\hat{c}^{wo}_{t+s}} - \frac{h}{\mu} e^{\hat{c}^{wo}_{t+s-1}} \Big) \exp\Big[-\frac{\bar{n}^{1+\nu}}{1+\nu} \exp[(1+\nu)\hat{n}^{wo}_{t+s}] \Big] \Big)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} -1. \tag{B.91}$$

An individual with discount factor β would be willing to give up a fraction Γ of consumption in each period to live in an otherwise identical world with the fiscal stimulus in place.

For large s, the deviations from the balanced growth path are numerically indistinguishable from zero. However, since $\beta\mu^{1-\sigma}$ is in practice close to unity, even for $s = 1,000$, the infinite sum has not converged. We therefore approximate:

$$\sum_{s=0}^{\infty} [\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) \exp\Big[- \frac{\bar{n}^{1+\nu}}{1+\nu} \big(\exp[(1+\nu)\hat{n}_{t+s}] \big) \Big] \right]^{1-\sigma}$$
$$\approx \sum_{s=0}^{T} [\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu} e^{\hat{c}_{t+s-1}} \right) \exp\Big[- \frac{\bar{n}^{1+\nu}}{1+\nu} \big(\exp[(1+\nu)\hat{n}_{t+s}] \big) \Big] \right]^{1-\sigma}$$
$$+ \frac{[\beta\mu^{1-\sigma}]^{T+1}}{1-\beta\mu^{1-\sigma}} (1 - h/\mu)^{1-\sigma},$$

for some large T. In practice, we use $T = 1,000$ but checked the results for $T = 5,000$.

To obtain $\bar{n}^{1+\nu}$, multiply equation (B.42) by $\bar{n}$ and divide by $\bar{y}$. This shows that $\bar{n}^{1+\nu} = \frac{\bar{w}\bar{n}}{\bar{y}} \frac{1}{(1+\lambda^w)} \frac{1}{\bar{c}^{RA}/\bar{y}} \frac{1}{1-\frac{h}{\mu}} \frac{1-\bar{\tau}^n}{1+\tau^c}$, which is in terms of the constants in Table B.5.

B.9 Simple New Keynesian Model

B.9.1 Setup

There is a unit mass of agents, a fraction ϕ of which are constrained to be rule-of-thumb (RoT) and consume their period labor income only. Intermediate goods are produced in monopolistic competition, while final goods are produced competitively as an aggregate of all intermediate goods $i \in [0, 1]$. The profits of intermediate goods producers go only to unconstrained agents. A competitive aggregate of differentiated labor is the only input into intermediate goods production. Differentiated labor of type $j \in [0, 1]$ is provided by trade unions, which differentiate households' homogeneous labor. Under the maintained assumption of $\phi < \frac{1}{2}$, wages are set by the union to maximize the intertemporal utility of unconstrained households. All agents provide the same amount of labor and receive an equal share of labor income inclusive of the markup.

Agents' flow utility is given by:

$$U(C(j), N(j)) = \log C(j) - \frac{N(j)^{1+\nu}}{1+\nu}. \tag{B.92}$$

In what follows, we drop the j index, unless needed, as we assume that agents insure completely against idiosyncratic labor income risk, and allocations are therefore independent of j.

A fraction $\phi \in [0, 1)$ of agents are rule-of-thumb, and they consume their period labor income net of taxes τ_t^n and transfers S_t, subject to a consumption tax of τ_t^c:

$$C_t^{RoT} = \frac{(1-\tau_t^n)N_t W_t + s_t P_t}{(1+\tau_t^c)P_t}. \tag{B.93}$$

Note that it is important that rule-of-thumb agents do not earn the same income as unconstrained agents. If they did earn real income equal to total production, in equilibrium, consumption of unconstrained and constrained agents would coincide.

The remaining $1 - \phi$ agents maximize expected lifetime utility, discounted at rate β:

$$\max_{\{C_t, N_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{B.94}$$

given initial nominal bond holdings B_{-1}. The maximization is subject to the budget constraint:

$$\frac{P_t C_t}{1+\tau_t^c} + B_t = R_{t-1}B_{t-1} + (1-\tau_t^n)N_t W_t + \Pi_t + s_t P_t, \tag{B.95}$$

where P_t is the price level and i_t is the nominal interest rate. Π_t denotes lump-sum transfers (e.g., from profit income). C_t and P_t are aggregates over individual varieties $j \in [0, 1]$.

A competitive final goods producer aggregates varieties according to a standard Dixit-Stiglitz aggregator with elasticity of substitution ϵ_p:

$$Y_t = \left(\int_0^1 y_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}.$$

Similarly, a competitive labor aggregator provides units of labor supply according to a Dixit-Stiglitz aggregator with elasticity of substitution ϵ_w:

$$N_t = \left(\int_0^1 y_t(j)^{1-\frac{1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}.$$

Producers of variety i have a constant returns to scale, labor-only production function: $Y_t(i) = N_t(i)$. They adjust prices subject to a Calvo friction. Opportunities for price adjustment arrive at rate $1 - \zeta$.

Market clearing implies that:

$$Y_t = \phi C_t^{RoT} + (1 - \phi) C_t^u + G_t. \tag{B.96}$$

The intertemporal equilibrium condition for unconstrained households implies that:

$$1 = \beta \mathbb{E}_t \left[\frac{e^{r_t}}{\pi_{t+1} \frac{1+\tau_{t+1}^c}{1+\tau_t^c}} \frac{C_t^u}{C_{t+1}^u} \right]. \tag{B.97}$$

The intratemporal equilibrium condition for unconstrained households implies:

$$MRS_t = N_t^\nu C_t^u. \tag{B.98}$$

Real marginal cost is given by:

$$MC_t = \frac{W_t}{P_t}. \tag{B.99}$$

A Taylor rule subject to the ZLB governs the interest rate:

$$R_t = (1 - 1_{ZLB,t}) e^{\gamma_\pi \pi_t}. \tag{B.100}$$

In general, there are both Calvo price-setting and wage-setting frictions, as specified by (B.101) and (B.102):

$$\text{Sticky prices: } \sum_{k=0}^{\infty} \zeta_p^k \mathbb{E}_t [SDF_{t,t+k} Y_{t+k}(i) (P_t^*(i) - \frac{\epsilon_p}{\epsilon_p - 1} P_{t+k} MC_t)] = 0, \tag{B.101a}$$

$$P_t^{1-\epsilon_p} = \zeta_p P_{t-1}^{1-\epsilon_p} + (1-\zeta_p) P_t^{*1-\epsilon_p}, \tag{B.101b}$$

Sticky wages: $$\sum_{k=0}^{\infty} \zeta_w^k \mathbb{E}_t \Big[SDF_{t,t+k} N_{t+k}(j) \Big(\frac{1-\tau_{t+k}^n}{1+\tau_{t+k}^c} \frac{W_t^*(j)}{P_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} MRS_{t+k} \Big) \Big] = 0, \tag{B.102a}$$

$$W_t^{1-\epsilon_w} = \zeta_w W_{t-1}^{1-\epsilon_w} + (1-\zeta_w) W_t^{*1-\epsilon_w}, \tag{B.102b}$$

where the stochastic discount factor is given by $SDF_{t,t+k} = \beta^k \frac{C_t}{C_{t+k}}$.

For the remainder of the analytical section we consider, however, the limit of either flexible prices $\zeta_p \to 0$ or flexible wages $\zeta_w \to 0$. In the limit of flexible prices, (B.101) implies that $P_t^*(i) = \frac{\epsilon_p}{\epsilon_p - 1} P_t MC_t \forall i$ and $P_t = P_t^*$: real wages are constant and equal the inverse of the markup: $MC_t = \frac{W_t}{P_t} = \frac{\epsilon_p - 1}{\epsilon_p}$. Similarly, for flexible wages (B.102) implies that the real net wage is a constant markup over the marginal rate of substitution: $\frac{W_t}{P_t} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{1+\tau_t^c}{1-\tau_t^n} MRS_t$.

The government financing requirement $D_t = P_t d_t$ evolves as follows:

$$P_t d_t = P_t(g_t + s_t^x) + B_{t-1} + \bar{\tau} W_t N_t, \tag{B.103}$$

where s_t^x are exogenous transfers as part of the stimulus. Together with the endogenous transfers s_t^e, which may be adjusted to finance deficits, they sum up to total transfers: $s_t = s_t^e + s_t^x$. We assume zero initial debt, $B_0 = 0$.

Taking government expenditure as exogenous, we consider policy rules of the following form for $\psi_\tau \in [0, 1]$:

$$\begin{aligned} &B_t = (1-\psi_\tau)\bar{R}^g P_t d_t && \Big((\tau_t - \bar{\tau}) W_t N_t + P_t c_t \tau_t^c - P_t s_t^e \Big) = \psi_\tau D_t && \text{if } G_t \neq 0 \\ &B_t = 0 && \Big((\tau_t - \bar{\tau}) W_t N_t + P_t c_t \tau_t^c - P_t s_t^e \Big) = D_t && \text{if } G_t = 0. \end{aligned} \tag{B.104}$$

B.9.2 Log-linearized equations

Log-linearize the model around a zero tax, zero government spending and transfers, and zero inflation steady state. In steady state, rule-of-thumb agents' consumption share equals their population share times the labor share in real income (i.e., $\phi_c \equiv \phi \frac{\epsilon - 1}{\epsilon}$; steady state profits amount to $\frac{1}{\epsilon}$ of real income).

Consumption of rule-of-thumb consumers from (B.93):

$$c_t^{RoT} = n_t + w_t - p_t - d\tau_t^n + \frac{\epsilon_p}{\epsilon_p - 1} s_t - d\tau_t^c, \tag{B.105}$$

where $d\tau_t^n, d\tau_t^c$ is in percentage of the steady state wage rate, while s_t is in percentage of steady state output.

The intratemporal marginal rate of substitution follows from (B.98):

$$mrs_t = c_t^u + \nu n_t. \tag{B.106}$$

The intertemporal Euler equation (B.97) implies:

$$0 = \mathbb{E}_t[c_t^u - c_{t+1}^u + r_t - \pi_{t+1} - (d\tau_{t+1}^c + d\tau_t^c)]. \tag{B.107}$$

Marginal costs (B.99) evolve as:

$$mc_t = w_t - p_t. \tag{B.108}$$

The resource constraint (B.96) implies:

$$y_t = \phi_c c_t^{RoT} + (1-\phi_c)c_t^u + g_t, \qquad g_t \equiv \frac{dg_t}{\bar{y}}. \tag{B.109}$$

Since price and wage dispersion do not matter to a first order, the production function implies:

$$y_t = n_t. \tag{B.110}$$

A piecewise linear approximation to the Taylor rule (B.100):

$$r_t = (1 - 1_{ZLB,t})\gamma_\pi \pi_t. \tag{B.111}$$

Under sticky prices and flexible wages (note that mc is in deviation from its steady state value $-\log \frac{\epsilon_p}{\epsilon_p - 1}$):

$$p_t^* = (1-\beta\zeta_w)\sum_{k=0}^{\infty}(\beta\zeta_p)\mathbb{E}_t[mc_{t+k} + p_{t+k}] \tag{B.112a}$$

$$\pi_t \equiv p_t - p_{t-1} = (1-\zeta_p)(p_t^* - p_{t-1}) \tag{B.112b}$$

$$mc_t = w_t - p_t = mrs_t + d\tau_t^n + d\tau_t^c \tag{B.112c}$$

Equations (B.112) imply:[4]

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p} mc_t \equiv \beta\mathbb{E}_t[\pi_{t+1}] + \lambda_p(mrs_t + d\tau_t^n + d\tau_t^c). \tag{B.113}$$

Under sticky wages and flexible prices:

$$w_t^* = (1-\beta\zeta_w)\sum_{k=0}^{\infty}(\beta\zeta_w)\mathbb{E}_t[mrs_{t+k} + p_{t+k} + d\tau_{t+k}^n + d\tau_{t+k}^c] \tag{B.114a}$$

$$w_t = \zeta_w w_{t-1} + (1-\zeta_w)w_t^* \tag{B.114b}$$

$$w_t = p_t. \tag{B.114c}$$

Similar to (B.113), the equations in (B.114) can be shown to imply:[5]

$$\pi_t = \mathbb{E}_t[\pi_{t+1}] + \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w}(mrs_t + d\tau_t^n + d\tau_t^c)$$

[4] To see this, note that the equations in (B.112) imply:

$$\begin{aligned}
p_t^* - p_{t-1} &= (1-\beta\zeta_w)\sum_{k=0}^{\infty}(\beta\zeta_p)\mathbb{E}_t[mc_{t+k} + p_{t+k} - p_{t-1}] \\
&= \beta\zeta_p\mathbb{E}_t[p_{t+1}^* - p_t] + (1-\beta\zeta_p)mc_t + \pi_t \\
\Leftrightarrow \pi_t = (1-\zeta_p)(p_t^* - p_{t-1}) &= \beta\zeta_p(1-\zeta_p)\mathbb{E}_t[p_{t+1}^* - p_t] + (1-\zeta_p)(1-\beta\zeta_p)(mrs_t + d\tau_t^n + d\tau_t^c) \\
&\quad + (1-\zeta_p)\pi_t \\
\Leftrightarrow \zeta_p\pi_t &= \beta\zeta_p\mathbb{E}_t[\pi_{t+1}] + (1-\zeta_p)(1-\beta\zeta_p)(mrs_t + d\tau_t^n + d\tau_t^c) \\
\Leftrightarrow \pi_t &= \beta\mathbb{E}_t[\pi_{t+1}] + \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}mc_t \equiv \beta\mathbb{E}_t[\pi_{t+1}] + \lambda_p(mrs_t + d\tau_t^n + d\tau_t^c).
\end{aligned}$$

[5] Use that $mrs_{t+k} = c_{t+k} + \nu n_{t+k}$ to rewrite recursively:

$$\begin{aligned}
w_t^* &= \beta\zeta_w\mathbb{E}_t[w_{t+1}^*] + (1-\beta\zeta_w)(w_t - (w_t - p_t - mrs_t) + d\tau_t) \\
\Leftrightarrow w_t^* - w_{t-1} &= \beta\zeta_w(\mathbb{E}_t[w_{t+1}^* - w_t] + w_t - w_{t-1}) + (1-\beta\zeta_w)(w_t - w_{t-1} - (w_t - p_t - mrs_t) + d\tau_t^n + d\tau_t^c) \\
\Leftrightarrow w_t - w_{t-1} &= \beta\zeta_w(\mathbb{E}_t[w_{t+1} - w_t] + (w_t - w_{t-1})(1-\zeta_w)) \\
&\quad + (1-\zeta_w)(1-\beta\zeta_w)(w_t - w_{t-1} - (w_t - p_t - mrs_t) + d\tau_t^n + d\tau_t^c) \\
&= \beta\zeta_w\mathbb{E}_t[w_{t+1} - w_t] + (1-\zeta_w)(w_t - w_{t-1}) \\
&\quad - (1-\zeta_w)(1-\beta\zeta_w)((w_t - p_t - mrs_t) - d\tau_t^n + d\tau_t^c) \\
\Leftrightarrow w_t - w_{t-1} &= \beta\mathbb{E}_t[w_{t+1} - w_t] + \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w}(mrs_t + d\tau_t) \\
&\equiv \mathbb{E}_t[w_{t+1} - w_t] + \lambda_w(mrs_t + d\tau_t^n + d\tau_t^c).
\end{aligned}$$

Because $w_t = p_t$, it follows trivially that $w_t - w_{t-1} = \pi_t$ for all t.

$$\equiv \mathbb{E}_t[\pi_{t+1}] + \lambda_w(mrs_t + d\tau_t^n + d\tau_t^c). \tag{B.115}$$

Using that $\bar{c} = \bar{y}$ in steady state, the government tax rule (B.104) becomes:

$$\begin{aligned}
b_t &= (1-\psi_\tau)\bar{R}^g d_t & \left(d\tau_t^n \frac{\bar{w}\bar{n}}{\bar{p}\bar{y}} + d\tau_t^c - s_t^e\right) &= \psi_\tau d_t & \text{if } G_t \neq 0 \\
b_t &= 0 & \left(d\tau_t^n \frac{\bar{w}\bar{n}}{\bar{p}\bar{y}} + d\tau_t^c - s_t^e\right) &= d_t & \text{if } G_t = 0,
\end{aligned} \tag{B.116}$$

where the financing requirement evolves as, given the assumption of zero steady state tax rates:

$$d_t = g_t + s_t^x + \bar{R}b_{t-1}, \qquad b_0 = 0. \tag{B.103'}$$

B.9.3 Simplified equilibrium conditions: sticky prices and flexible wages

First, solve this model under the assumption of sticky prices and flexible wages. Use the intratemporal equilibrium condition (B.112c) for $w_t - p_t - d\tau_t^n - d\tau_t^c = c_t^u + \nu n_t$ to substitute in the equation for RoT consumption (B.105) to get

$$c_t^{RoT} = c_t^u + (1+\nu)n_t + \frac{\epsilon_p}{\epsilon_p - 1}s_t. \tag{B.117}$$

Use equation (B.117) in the resource constraint (B.109) and the production function $y_t = n_t$. Then re-arrange to get

$$\begin{aligned}
c_t^u &= \frac{y_t - \phi_c c_t^u - \phi_c(1+\nu)y_t - g_t - \phi_c \frac{\epsilon_t}{\epsilon_t - 1}s_t}{(1-\phi_c)} \\
&\Leftrightarrow c_t^u = (1-\phi_c(1+\nu))y_t - g_t - \phi_c \frac{\epsilon_t}{\epsilon_t - 1}s_t.
\end{aligned} \tag{B.118}$$

Use this equation in the Euler equation to get the aggregate sticky price supply schedule:

$$\begin{aligned}
y_t = &\frac{g_t + \phi_c \frac{\epsilon_t}{\epsilon_t - 1}s_t}{1-\phi_c(1+\nu)} + \mathbb{E}_t[y_{t+1} - \frac{g_{t+1} + \phi_c \frac{\epsilon_t}{\epsilon_t - 1}s_{t+1}}{1-\phi_c(1+\nu)}] \\
&- \frac{1}{1-\phi_c(1+\nu)}(r_t - \mathbb{E}_t[\pi_{t+1} + d\tau_{t+1}^c - d\tau_t^c]).
\end{aligned} \tag{B.119}$$

Note that (B.119) is independent of the labor tax rate because workers are on their labor supply curve.

Use the intratemporal condition and market clearing in the expression for marginal cost:

$$mc_t = d\tau_t^n + d\tau_t^c + \Big(\nu + (1-\phi_c(1+\nu))\Big)y_t - g_t - \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_t. \tag{B.120}$$

Use the marginal cost equation (B.120) in the pricing equation to obtain the Phillips curve:

$$\pi_t = \beta \mathbb{E}_t[\pi_t] + \lambda \left(d\tau_t^n + d\tau_t^c + \Big(\nu + (1-\phi_c(1+\nu))\Big)y_t - g_t - \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_t \right). \tag{B.121}$$

B.9.4 Simplified equilibrium conditions: sticky wages and flexible prices

To solve the model under the assumption of flexible prices but sticky wages, note that the first step in the derivation for sticky prices and flexible wages does not apply. The marginal rate of substitution is not equalized to the real wage. Instead, the real wage is constant by (B.114c). Using $w_t = p_t$, substitute the RoT consumption function (B.105) into the resource constraint (B.109) after using again that $y_t = n_t$. Solve for the consumption of unconstrained agents:

$$\begin{aligned}
(1-\phi_c)c_t^u &= y_t - g_t - \phi_c(y_t - d\tau_t^n - d\tau_t^c - \frac{\epsilon_p}{\epsilon_p - 1} s_t) \\
&= (1-\phi_c)y_t - g_t + \phi_c(d\tau_t^n + d\tau_t^c - \frac{\epsilon_p}{\epsilon_p - 1} s_t) \\
\Leftrightarrow c_t^u &= y_t + \frac{-g_t + \phi_c(d\tau_t^n + d\tau_t^c - \frac{\epsilon_p}{\epsilon_p - 1} s_t)}{1-\phi_c}.
\end{aligned} \tag{B.122}$$

Substituting the MRS (B.106) and (B.122) in the sticky wage pricing equation (B.115) yields the sticky wage New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \lambda_w \left((1+\nu)y_t + \frac{-g_t - \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_t + \phi_c(d\tau_t^n + d\tau_t^c)}{1-\phi_c} + d\tau_t + d\tau_t^c) \right). \tag{B.123}$$

Using (B.122) in the Euler equation (B.107) to get the sticky wage version of the New Keynesian IS curve:

$$\begin{aligned}
y_t = &\frac{g_t + \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_t - \phi_c(d\tau_t^n + d\tau_t^c)}{1-\phi_c} - (r_t - \mathbb{E}_t[\pi_{t+1} + d\tau_{t+1}^c - d\tau_t^c]) \\
&+ \mathbb{E}_t[y_{t+1} + \frac{-g_{t+1} - \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_{t+1} + \phi_c(d\tau_{t+1}^n + d\tau_{t+1}^c)}{1-\phi_c}].
\end{aligned} \tag{B.124}$$

B.9.5 Persistent ZLB, immediate taxation

We now consider the case of a persistent ZLB and immediate taxation ($\psi_\tau = 1$) through labor taxes only, i.e., $s_t = d\tau_t^c = 0$.

The effect of the ZLB is modeled as a persistent nonrecurrent Markov process, similar to **?**: The economy starts at the ZLB and remains in it with *iid* probability μ: $\Pr\{\mathbf{1}_{ZLB,t+1} = 1 | \mathbf{1}_{ZLB,t} = 1\} = \mu$. Government expenditure follows the same Markov process with $G_t = g\bar{Y}\mathbf{1}_{ZLB,t}$. As a consequence of the tax rule (B.116), it then follows that taxes are also Markov: $(\tau_t - \bar{\tau})\frac{\bar{w}\bar{N}}{\bar{p}} = g\bar{Y}\mathbf{1}_{ZLB,t}$ and $d\tau_t^n = \frac{\epsilon_p}{\epsilon_p - 1}g$.

Lemma 1 in Section B.9.9 implies that under the Taylor Principle, and the assumption that $\phi_c(1+\nu) < 1$ if prices are sticky, the economy jumps back to its steady state, after the ZLB becomes slack. Lemma 2 in Section B.9.9 then implies that for μ small enough, there is a unique Markov equilibrium. Assuming that the determinacy condition on μ in Lemma 2 is satisfied, the equilibrium conditions can be solved forward to solve for this Markov equilibrium.

Consider the case of sticky prices and flexible wages first. Take expectations in the sticky price Phillips curve (B.121) under the described Markov-structure to solve for inflation at the ZLB:

$$\pi_{ZLB}^{fw}(1 - \beta\mu) = \lambda_p \left(\frac{1}{\epsilon - 1} g + (1+\nu)(1-\phi_c)y \right), \tag{B.125}$$

using that $d\tau_t^n - g = \frac{1}{\epsilon_p - 1}g$.

Using the flexible wage ZLB inflation (B.125) in the sticky price and flexible wage Euler equation (B.119) and taking expectations under the Markov structure yields an expression for output with flexible wages during the ZLB:

$$y_{ZLB}^{fw} = \frac{\frac{1}{1-\phi_c(1+\nu)} + \frac{\lambda_p}{1-\phi_c(1+\nu)} \frac{\mu}{(1-\beta\mu)(1-\mu)} \left(\frac{1}{\epsilon - 1}\right)}{1 - \frac{\lambda_p}{1-\phi_c(1+\nu)} \frac{\mu}{(1-\beta\mu)(1-\mu)} (1+\nu)(1-\phi_c)}. \tag{B.126}$$

Now consider sticky wages. Taking expectations in the sticky wage Phillips curve (B.123) under the Markov structure yields the flexible price inflation rate at the ZLB:

$$\pi_{ZLB}^{fp}(1 - \beta\mu) = \lambda_w \left(y_{ZLB} - \frac{g}{1-\phi_c} + \sigma \frac{\phi_c}{1-\phi_c} \tau_{ZLB} + \nu y_{ZLB} + \tau_{ZLB}) \right). \tag{B.127}$$

Using the expression in the flexible price and sticky wage Euler equation

(B.124) yields flexible price output at the ZLB:

$$
\begin{aligned}
y_{ZLB}^{fp} &= \mu y_{ZLB}^{fp} + \frac{g(1-\mu) - \phi_c(1-\mu)\tau_{ZLB}}{1-\phi_c} + \mu\pi_{ZLB}^{fp} \\
&= \frac{(1-\mu)\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} - \frac{\mu}{1-\beta\mu}\lambda_w\frac{1}{1-\phi_c} + \frac{\mu}{1-\beta\mu}\lambda_w\left(\frac{\phi_c}{1-\phi_c}+1\right)\frac{\epsilon}{\epsilon-1}}{1-\mu-\frac{\mu}{1-\beta\mu}\lambda_w(1+\nu)} g \\
&= \frac{\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} + \frac{\mu}{(1-\beta\mu)(1-\mu)}\lambda_w\frac{1}{1-\phi_c}\frac{1}{\epsilon-1}}{1-\frac{\mu}{(1-\beta\mu)(1-\mu)}\lambda_w(1+\nu)} g. \qquad \text{(B.128)}
\end{aligned}
$$

B.9.6 One-period ZLB, slow taxation

In this section, we focus on the case of sticky wages and flexible prices. We consider the case of slow taxation when the ZLB binds for a single period. We also assume that government expenditures last only for one period. Throughout, we make use of the fact that when the ZLB is slack, Lemma 1 implies that, under the Taylor Principle, there is a unique locally bounded equilibrium from period two onward. Therefore, the expectations in the period-one Euler equation and the Phillips curve are pinned down, and we can solve for the unique equilibrium output and consumption in period one.

Since in period two the ZLB is slack, interest rates react to the inflation caused by higher taxes. Inflation follows from the NK Phillips curve (B.123) after using that $\pi_3 = 0$:

$$
\pi_2 = \lambda\left(y_2 + \nu y_2 + \frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p - 1}ds_2) + d\tau_2^n + d\tau_2^c\right). \qquad \text{(B.129)}
$$

This implies that the monetary authority raises interest rates in the second period in response, as prescribed by the Taylor rule (B.111).

Since a fraction ψ_τ of the period one deficit is repaid using taxes in period one and the remainder is repaid in period two:

$$
\left(\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}}d\tau_1^n + d\tau_1^c - s_1\right) = \psi_\tau d_1 (= \psi_\tau g) \qquad b_1 = (1-\psi_\tau)\bar{R}^g d_1 \qquad \text{(B.130a)}
$$

$$
\left(\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}}d\tau_2^n + d\tau_2^c - s_2\right) = d_2 (= (1-\psi_\tau)\bar{R}^g d_1 = (1-\psi_\tau)\bar{R}^g g) \quad b_2 = 0. \qquad \text{(B.130b)}
$$

Solve the model backward. The tax rate follows from (B.130). The inflation rate is still given by (B.129). The only modification in the derivation of period two output comes from the different tax rate and its aggregate demand and inflation effect.:

$$
\begin{aligned}
y_2 &= -\frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p - 1}s_2) - \gamma_\pi \pi_2 - d\tau_2^c \\
&= -\frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p - 1}s_2) - \gamma_\pi \lambda \left(y_2 + \frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p-1}s_2) + \nu y_2 + d\tau_2^n + d\tau_2^c) \right) \\
&\quad - d\tau_2^c \\
&= -\underbrace{\frac{\frac{\phi_c}{1-\phi_c} + \gamma_\pi \lambda \left(\frac{\phi_c}{1-\phi_c} + 1\right)}{1 + \gamma_\pi \lambda (1+\nu)}}_{\equiv m_2^n} \underbrace{\frac{\epsilon}{\epsilon - 1}(1-\psi_\tau)\bar{R}^g g}_{= d\tau_2^n} \quad \text{labor tax only} \qquad \text{(B.131a)} \\
&= -\underbrace{\frac{\frac{\phi_c}{1-\phi_c} + \gamma_\pi \lambda \left(\frac{\phi_c}{1-\phi_c} + 1\right) + 1}{1 + \gamma_\pi \lambda (1+\nu)}}_{\equiv m_2^c} \underbrace{(1-\psi_\tau)\bar{R}^g g}_{= d\tau_2^c} \quad \text{consumption tax only} \qquad \text{(B.131b)} \\
&= -\underbrace{\frac{\frac{\phi_c}{1-\phi_c} + \gamma_\pi \lambda \frac{\phi_c}{1-\phi_c}}{1 + \gamma_\pi \lambda (1+\nu)} \frac{\epsilon_p}{\epsilon_p - 1}}_{\equiv m_2^s} \underbrace{(1-\psi_\tau)\bar{R}^g g}_{= -s_2} \quad \text{transfers only} \qquad \text{(B.131c)}
\end{aligned}
$$

Now use the period one Euler equation (B.124):

$$
\begin{aligned}
y_1 &= \frac{g + \phi_c \frac{\epsilon_p}{\epsilon_p - 1}s_1 - \phi_c(d\tau_1^n + d\tau_1^c)}{1-\phi_c} + y_2 - \phi_c \frac{\frac{\epsilon_p}{\epsilon_p-1}s_2 - (d\tau_2^n + d\tau_2^c)}{1-\phi_c} + \pi_2 + d\tau_2^c - d\tau_1^c \\
&= \frac{g + \phi_c \frac{\epsilon_p}{\epsilon_p - 1}s_1 - \phi_c(d\tau_1^n + d\tau_1^c)}{1-\phi_c} + \left(\phi_c \frac{\frac{\epsilon_p}{\epsilon_p-1}s_2 - \phi_c(d\tau_2^n + d\tau_2^c)}{1-\phi_c} - \gamma_\pi \pi_2 - d\tau_2^c \right) \\
&- \phi_c \frac{\frac{\epsilon_p}{\epsilon_p-1}s_2 - (d\tau_2^n + d\tau_2^c)}{1-\phi_c} + \pi_2 + d\tau_2^c - d\tau_1^c \\
&= \frac{g + \phi_c \frac{\epsilon_p}{\epsilon_p - 1}s_1 - \phi_c(d\tau_1^n + d\tau_1^c)}{1-\phi_c}. - (\gamma_\pi - 1)\pi_2 - d\tau_1^c \qquad \text{(B.132)}
\end{aligned}
$$

Under the assumption of labor taxes only:

$$
\begin{aligned}
y_1^n &= \frac{g - \phi_c d\tau_1^n}{1-\phi_c} - (\gamma_\pi - 1)\lambda \Big(\Big(\frac{\phi_c}{1-\phi_c} d\tau_2^n + d\tau_2^n \Big) - (1+\nu) m_2^n d\tau_2^n \Big). \\
&= \frac{1 - \psi_\tau \phi_c \frac{\epsilon}{\epsilon - 1}}{1-\phi_c} g - (\gamma_\pi - 1)\lambda \Big(\frac{1}{1-\phi_c} - (1+\nu) m_2^\tau \Big) \frac{\epsilon_p}{\epsilon_p - 1}(1-\psi_\tau)\bar{R}^g g \\
&= \frac{1 - \psi_\tau \phi_c \frac{\epsilon}{\epsilon - 1}}{1-\phi_c} g - (\gamma_\pi - 1)\lambda \frac{\frac{1-\phi_c(1+\nu)}{1-\phi_c}}{1 + \gamma \lambda_w (1+\nu)} \frac{\epsilon_p}{\epsilon_p - 1}(1-\psi_\tau)\bar{R}^g g. \qquad \text{(B.133)}
\end{aligned}
$$

Now, assume consumption taxes only:

$$
\begin{aligned}
y_1^c &= \frac{g - \phi_c d\tau_1^c}{1-\phi_c} - (\gamma_\pi - 1)\lambda\Big(\Big(\frac{\phi_c}{1-\phi_c} d\tau_2^c + d\tau_2^c\Big) - (1+\nu) m_2^c d\tau_2^c\Big) - d\tau_1^c \\
&= \frac{1-\phi_c\psi}{1-\phi_c} g - (\gamma_\pi - 1)\lambda\Big(\frac{1}{1-\phi_c} - (1+\nu)m_2^c\Big)(1-\psi)g - \psi g \\
&= \frac{1-\phi_c\psi}{1-\phi_c} g + (\gamma_\pi - 1)\lambda \frac{\frac{\nu}{1-\phi_c}}{1+\gamma_\pi\lambda_w(1+\nu)}(1-\psi)g - \psi g \\
&= \frac{1-\psi}{1-\phi_c} g + (\gamma_\pi - 1)\lambda \frac{\frac{\nu}{1-\phi_c}}{1+\gamma_\pi\lambda_w(1+\nu)}(1-\psi)g. \qquad \text{(B.134)}
\end{aligned}
$$

Period one consumption tends to be higher with consumption tax when compared with labor taxes through the inflation channel, but it is lower through an aggregate demand channel when taxes are adjusted rapidly (i.e., when ψ is high).

$$
\frac{y_1^c - y_1^n}{g} = -\psi \frac{1-\phi_c\frac{\epsilon_p}{\epsilon_p - 1}}{1-\phi_c} + \frac{(\gamma_\pi - 1)\lambda_w}{1+\gamma_p i \lambda_w(1+\nu)} \frac{\nu + \frac{\epsilon_p}{\epsilon_p-1}(1-\phi_c(1+\nu))}{1-\phi_c}(1-\psi).
$$

Hence:

$$
\begin{aligned}
& y_1^c > y_1^n \\
\Leftrightarrow & \psi < \frac{(\gamma_\pi - 1)\lambda_w(\nu + \frac{\epsilon_p}{\epsilon_p-1}(1-\phi_c(1+\nu))}{(1-\phi_c\frac{\epsilon_p}{\epsilon_p-1})(1+\gamma_\pi\lambda_w(1+\nu)) + (\gamma_\pi - 1)\lambda_w(\nu + \frac{\epsilon_p}{\epsilon_p-1}(1-\phi_c(1+\nu))} \in (0,1). \\
& \qquad \text{(B.135)}
\end{aligned}
$$

Define $\bar{\psi}^n$ as the threshold in (B.135). For period two:

$$
\frac{y_2^c - y_2^n}{g} \propto -\left(1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1} - \gamma_\pi \lambda_w \frac{1}{\epsilon_p - 1}\right)(1-\psi). \qquad \text{(B.136)}
$$

Now, assume transfers only:

$$
\begin{aligned}
y_1^s &= \frac{g + \phi_c \frac{\epsilon_p}{\epsilon_p-1} s_1}{1-\phi_c} - (\gamma_\pi - 1)\lambda\Big(\Big(\frac{\phi_c}{1-\phi_c}\frac{\epsilon_p}{\epsilon_p - 1} s_2\Big) - (1+\nu)m_2^s s_2\Big) \\
&= \frac{1-\phi_c\psi\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c} - (\gamma_\pi - 1)\lambda\Big(\frac{\phi_c}{1-\phi_c}\frac{\epsilon_p}{\epsilon_p-1} - (1+\nu)m_2^s\Big)(1-\psi)g \\
&= \frac{1-\phi_c\psi\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c} + (\gamma_\pi - 1)\lambda\frac{\epsilon_p}{\epsilon_p-1}(1-\psi)\frac{\phi_c}{1-\phi_c}\frac{\nu}{1+\gamma_\pi\lambda_w(1+\nu)}(1-\psi)g \\
& \qquad \text{(B.137)}
\end{aligned}
$$

Thus:

$$\frac{y_1^s - y_1^n}{g} = (\gamma_\pi - 1)\lambda \frac{1}{1 + \gamma\lambda_w(1+\nu)} \frac{\epsilon_p}{\epsilon_p - 1}(1 - \psi_\tau)\bar{R}^g > 0. \tag{B.138}$$

The difference between transfer- and consumption-tax financed multipliers:

$$\frac{y_1^s - y_1^c}{g} = \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}}{1 - \phi_c}\psi - \frac{(\gamma_\pi - 1)\lambda_w \nu}{1 + \gamma_\pi \lambda_w (1+\nu)} \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}}{1 - \phi_c}(1 - \psi).$$

Thus:

$$y_1^s - y_1^c > 0 \quad \Leftrightarrow \quad \psi > \frac{(\gamma_\pi - 1)\lambda_w \nu}{(1 + \gamma_\pi \lambda_w(1+\nu) + (\gamma_\pi - 1)\lambda_w \nu} \in (0,1). \tag{B.139}$$

Define $\bar{\psi}^s$ as the threshold in (B.139). For period two:

$$\frac{y_2^s - y_2^c}{g} = \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}}{1 - \phi_c} \frac{1 + \gamma_\pi \lambda_w}{1 + \gamma_\pi \lambda_w(1+\nu)}(1 - \psi_\tau)\bar{R}^g. \tag{B.140}$$

Note that $\bar{\psi}^n > \bar{\psi}^s$. To see this, write:

$$\bar{\psi}^n = \frac{1}{1 + (1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}) \frac{1 + \gamma\lambda_w(1+\nu)}{(\gamma_\pi - 1)\lambda_w(\nu + \frac{\epsilon_p}{\epsilon_p - 1}(1 - \phi_c(1+\nu))}}$$

$$\bar{\psi}^s = \frac{1}{1 + \frac{1 + \gamma\lambda_w(1+\nu)}{(\gamma_\pi - 1)\lambda_w \nu}}.$$

Thus, $\bar{\psi}^n > \bar{\psi}^s$ if $\frac{(\nu + \frac{\epsilon_p}{\epsilon_p - 1}(1 - \phi_c(1+\nu))}{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}} > \nu$ or

$$\bar{\psi}^n > \bar{\psi}^s \quad \Leftrightarrow \quad \nu \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}}{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}} + \frac{\epsilon_p}{\epsilon_p - 1} \frac{1 - \phi_c}{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}} > \nu, \tag{B.141}$$

which is satisfied given $\phi = \phi_c \frac{\epsilon_p}{\epsilon_p - 1} < 1$.

Last, compare long-run consumption tax multipliers to long-run transfer multipliers by computing a weighted average of (B.139) and (B.140):

$$\frac{y_1^s + \beta y_2^s - (y_1^c + \beta y_2^c)}{g} = \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}}{1 - \phi_c} \times \left(\psi - (1 - \psi)\frac{(\gamma_\pi - 1)\lambda_w \nu}{1 + \gamma_\pi \lambda_w(1+\nu)} + \beta(1 - \psi)\frac{1 + \gamma_\pi \lambda_w}{1 + \gamma_\pi \lambda_w(1+\nu)} \right)$$

$$= \frac{1-\phi_c \frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c}\left(\psi + (1-\psi)\frac{\beta + \lambda_w(\beta\gamma_\pi - (\gamma_\pi - 1)\nu)}{1+\gamma_\pi\lambda_w(1+\nu)}\right). \tag{B.142}$$

B.9.7 One period ZLB, permanently higher taxes

We now consider a permanent rollover of the post-ZLB debt, as specified in (6.2b'). We only consider flexible prices and rigid wages. All taxes other than labor taxes are assumed to be zero.

With constant debt after the ZLB, from period $t = 2$ on, debt follows from the budget constraint as:

$$\hat{b} = (1-\psi)\bar{R}^g g. \qquad \left(1 - (\bar{R}^g)^{-1}\right)\hat{b} = \frac{\bar{n}\bar{w}}{\bar{y}\bar{p}}\hat{\tau}^n,$$

Since the economy is forward looking and there are no future shocks, every period is alike so that subscripts can be dropped. The hats to denote the new steady state (in deviations from the old steady state).

Notice that with constant labor taxes, constant output, and no other taxes, the NK IS curve (B.124) implies that:

$$\hat{\pi} = \hat{r}. \tag{B.143}$$

Consider first the Taylor rule analyzed above, which features no concern for output stabilization. This is the case analyzed before and the Taylor principle is necessary and sufficient for uniqueness. Then the only (non-ZLB) equilibrium inflation consistent with the Taylor rule is the zero inflation equilibrium. Output is such that it keeps households on their labor supply curve:

$$\hat{y} = -\frac{1}{1+\nu}\frac{1}{1-\phi_c}\hat{\tau}_n = -\frac{1}{1+\nu}\frac{\bar{R}^g - 1}{1-\phi_c}(1-\psi)\frac{\epsilon_p}{\epsilon_p - 1}g < 0.$$

Now consider the slightly more general Taylor rule:

$$r_t = (1 - \mathbf{1}_{ZLB,t})(\gamma_\pi \pi_t + \gamma_y y_t), \tag{B.111'}$$

which collapses to the previous case (B.111) if $\gamma_y = 0$.

Assume that $\gamma_\pi > 0$ and $\gamma_y > 0$. As in the textbook case (ch. 3 **?**), uniqueness of the non-ZLB equilibrium is guaranteed if:

$$\lambda_w(1+\nu)(\gamma - 1) + (1-\beta)\gamma_y > 0.$$

Thus the Taylor Principle continues to be sufficient for uniqueness.

The Taylor rule and the NK IS curve then require from (B.143):

$$\hat{y} = -\frac{\gamma_\pi - 1}{\gamma_y}\hat{\pi}$$

The NKPC (B.123) then implies that:

$$\frac{\hat{y}}{g} = -\frac{1}{(1+\nu) + \frac{1-\beta}{\gamma-1}\frac{\gamma_y}{\lambda_w}} \frac{\bar{R}^g - 1}{1-\phi_c}(1-\psi)\frac{\epsilon_p}{\epsilon_p - 1}$$

Uniqueness implies that $(1+\nu) + \frac{1-\beta}{\gamma-1}\frac{\gamma_y}{\lambda_w} > 0$ and if $\gamma_\pi > 1$, output is less negative with output stabilization than without.

If $\gamma_\pi - 1 > 0$ and $\gamma_y > 0$, then:

$$\frac{d\frac{\hat{y}}{g}}{d\lambda_w} < 0.$$

Inflation in the new higher tax steady state is given by:

$$\hat{\pi} = -\frac{\gamma_y}{\gamma_\pi - 1}\hat{y} = +\frac{\gamma_y}{\gamma_\pi - 1}\frac{1}{(1+\nu) + \frac{1-\beta}{\gamma-1}\frac{\gamma_y}{\lambda_w}} \frac{\bar{R}^g - 1}{1-\phi_c}(1-\psi)\frac{\epsilon_p}{\epsilon_p - 1} g > 0$$

Solving the period 1 NKIS equation for period 1 output gives:[6]

$$\begin{aligned}\frac{y_1}{g} =& \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}\psi}{1-\phi_c} - \frac{(\gamma-1)\lambda_w(1-(1+\nu)\phi_c)}{(1-\beta)\gamma_y + (1+\nu)\lambda_w(\gamma-1)}\frac{1}{1-\phi_c}(\bar{R}^g - 1)\frac{\epsilon_p}{\epsilon_p - 1} \\ &+ \gamma_y \frac{\lambda_w + (1-\beta)\phi_c}{(1-\beta)\gamma_y + (1+\nu)\lambda_w(\gamma-1)}\frac{1}{1-\phi_c}(\bar{R}^g - 1)\frac{\epsilon_p}{\epsilon_p - 1} \qquad \text{(B.144)}\end{aligned}$$

When the monetary authority puts more weight γ_y on output stabilization,

[6]Period 1 inflation, i.e. during the ZLB, follows as:

$$\begin{aligned}\pi_1 &= (1-\beta)\hat{\pi} + \lambda_w\left((1+\nu)y_1 + \frac{1}{1-\phi_c}d\tau_1^n\right) \\ &= -(1-\beta)\frac{\gamma_y}{\gamma - 1}\hat{y} + \lambda_w\left((1+\nu)y_1 + \frac{1}{1-\phi_c}d\tau_1^n\right).\end{aligned}$$

Period 1 inflation, however, does not enter the equilibrium conditions.

the period one multiplier rises:

$$\frac{d}{d\gamma_y}\frac{y_1}{g} = \frac{(\gamma-1)\lambda_w(1-\beta+(1+\nu)\lambda_w)}{((1-\beta)\gamma_y+(1+\nu)\lambda_w(\gamma-1))^2}\frac{1}{1-\phi_c}(\bar{R}^g-1)\frac{\epsilon_p}{\epsilon_p-1} > 0. \tag{B.145}$$

Taking derivatives of period 1 output also shows that, if $\gamma_y \geq 0$:

$$\frac{d}{d\lambda_w}\frac{y_1}{g} = -\frac{(1-\beta)(\gamma_\pi-1-\gamma_y)\gamma_y}{((1-\beta)\gamma_y+(1+\nu)\lambda_w(\gamma-1))^2}\frac{1}{1-\phi_c}(\bar{R}^g-1)\frac{\epsilon_p}{\epsilon_p-1}$$
$$\begin{cases} \leq 0 & \gamma_\pi \geq 1+\gamma_y, \gamma_y > 0, \\ > 0 & \gamma_\pi < 1+\gamma_y, \gamma_y > 0, \\ = 0 & \gamma_y = 0. \end{cases} \tag{B.146}$$

Equation (B.146) implies that when the concern for inflation stabilization dominates the concern for output stabilization sufficiently, then higher wage flexibility lowers the short-run multiplier. At the same time, however, equation (B.144) implies that the multiplier increases when the monetary authority puts more weight on output.

B.9.8 Three-period labor tax sticky wage model

We also consider an alternative "three-period" version of the labor tax rule:

$$\frac{W_1N_1}{P_1Y_1}(\tau_1^n-\bar{\tau}^n) = \psi_\tau d_1 \qquad b_1 = (1-\psi_\tau)\bar{R}^g d_1 \qquad d_1 = g, \tag{B.147a}$$

$$\frac{W_2N_2}{P_2Y_2}(\tau_2^n-\bar{\tau}^n) = \psi_\tau d_2 \qquad b_2 = (1-\psi_\tau)\bar{R}^g d_2 \qquad d_2 = b_1, \tag{B.147b}$$

$$\frac{W_3N_3}{P_3Y_3}(\tau_3^n-\bar{\tau}^n) = d_3(=(1-\psi_\tau)^2(\bar{R}^g)^2 g) \qquad b_3 = 0 \qquad d_3 = b_2. \tag{B.147c}$$

The corresponding linear 3-period tax rule (B.147) becomes:

$$\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}}d\tau_1 = \psi_\tau d_1(=\psi_\tau g) \qquad b_1 = (1-\psi_\tau)\bar{R}^g d_1 \qquad d_1 = g, \tag{B.148a}$$

$$\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}}d\tau_2 = \psi_\tau d_2(=\psi_\tau(1-\psi_\tau)\bar{R}^g g) \qquad b_2 = (1-\psi_\tau)\bar{R}^g d_2 \qquad d_2 = b_1, \tag{B.148b}$$

$$\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}}d\tau_3 = d_3(= (1-\psi_\tau)^2(\bar{R}^g)^2 g)d_3 \qquad b_3 = 0 \qquad\qquad d_3 = b_2. \tag{B.148c}$$

Using that the economy is in its steady state in period 4 ($y_4 = \pi_4 = 0$) and the equilibrium is unique under the Taylor Principle, the model can again be solved backward, iterating on the New Keynesian IS equation (B.124) and the NK Phillips curve (B.123). Solving for the output levels yields:

$$\frac{y_3}{g} = -\frac{\gamma\lambda\left(1+\sigma\frac{\phi_c}{1-\phi_c}\right)+\sigma\frac{\phi_c}{1-\phi_c}}{\sigma+\gamma\lambda(\sigma+\nu)}\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)^2(\bar{R}^g)^2 \tag{B.149a}$$

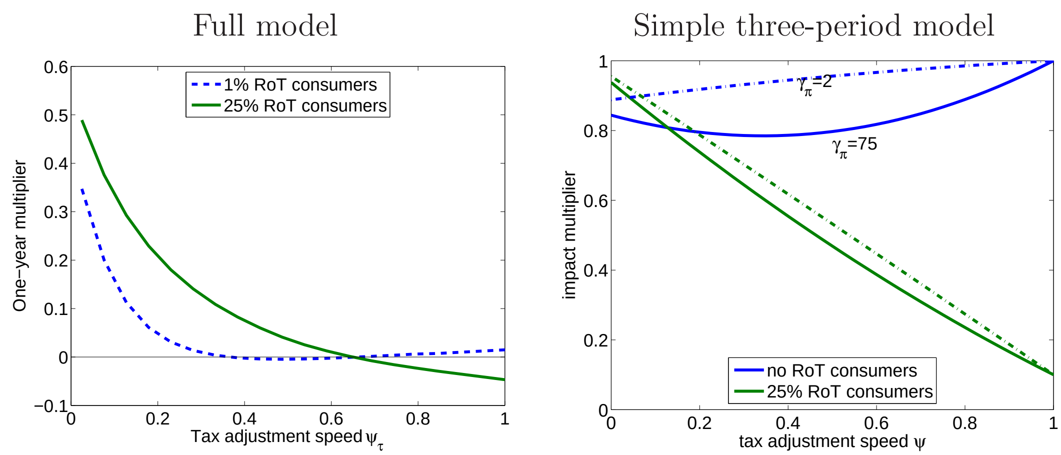
$$\begin{aligned}\frac{y_2}{g} &= -\frac{\lambda\sigma(\gamma(1+\beta)-1)\left(1-\frac{\phi_c\nu}{1-\phi_c}\right)}{(\sigma+\gamma\lambda(\sigma+\nu))^2}\times\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)\bar{R}^g\\ &\quad -\frac{\lambda\gamma\left(\frac{1}{1-\phi_c}-\beta\gamma\left(1-\frac{\phi_c\nu}{1-\phi_c}+\frac{\phi_c}{1-\phi_c}(2\gamma(\nu+\sigma)-(1+\nu))\right)\right)+\frac{\phi_c\sigma^2}{1-\phi_c}+(\gamma\lambda)^2\left(1+\frac{\phi_c\sigma}{1-\phi_c}\right)(\sigma+\nu)}{(\sigma+\gamma\lambda(\sigma+\nu))^2}\\ &\quad \times\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)\bar{R}^g\psi_\tau\end{aligned} \tag{B.149b}$$

$$\begin{aligned}\frac{y_1}{g} &= 1-(\gamma-1)\lambda\Big(1-\frac{\phi_c\nu}{1-\phi_c}\Big)\frac{((1+\beta+\lambda)\sigma+\lambda\nu)}{(\sigma+\gamma\lambda(\nu+\sigma))^2}\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)^2(\bar{R}^g)^2\\ &\quad -(\gamma-1)\lambda\Big(1-\frac{\phi_c\nu}{1-\phi_c}\Big)\frac{1}{\sigma+\gamma\lambda(\nu+\sigma)}\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)\bar{R}^g\psi_\tau-\frac{\phi_c}{1-\phi_c}\frac{\epsilon}{\epsilon-1}\psi_\tau\end{aligned} \tag{B.149c}$$

From equation (B.149c) it can be seen that the nonmonotonicity between the extremes of $\psi_\tau \in 0,1$ stems from the term involving $\psi_\tau(1-\psi_\tau)$, the term proportional to the tax increase in period 2, whose influence is maximized at the intermediate value of $\psi_\tau = \frac{1}{2}$.

$$-(\gamma-1)\lambda\Big(1-\frac{\phi_c\nu}{1-\phi_c}\Big)\frac{1}{\sigma+\gamma\lambda(\nu+\sigma)},$$

If $\phi_c(1+\nu) < 1$ and given the Taylor Principle $\gamma > 1$ this term is strictly decreasing in both γ and λ (the slope of the Phillips curve, which tends to increase in wage flexibility). Thus, an aggressive central bank or very flexible wages induce a nonmonotonicity in the impact multiplier in the absence of RoT agents. Economically, the accumulation of debt, and the corresponding tax increase, causes the most inflationary pressure in period two for intermediate values of ψ_τ. This leads to an aggressive response by the central bank, which actually causes consumption to fall in period two because of a negative substitution effect as real interest rates increase. This causes private consumption in the first period also to fall because agents desire to smooth consumption and increase their savings demand.



Note: The full model results set the habit parameter to $h = 0.5$ and otherwise uses the posterior mean for the simulation. The simple model uses $\beta = 1.01^{-1}, \nu = 1, \zeta = \frac{4}{5}, \epsilon_p = 3$ and $\gamma_\pi = 2$ for the two-period model and $\gamma_\pi = 25$ for the three period model.

Figure B.1: Multipliers as a function of tax adjustment speed and rule-of-thumb consumers

Figure B.1 provides a comparison of the simple three period model with the full empirical model. It shows that if the reaction of the monetary authority in the intermediate period two is strong enough, the simple three-period model can reproduce the qualitative feature of a nonmonotone reaction to the speed of tax adjustment.

B.9.9 Proofs

Lemma 1. *If the Taylor Principle is satisfied ($\gamma_\pi > 1$), outside of the ZLB the locally bounded equilibrium is unique under both sticky prices and flexible wages and under sticky wages and flexible prices.*

Proof: Consider the case of sticky prices and flexible wages. Since the ZLB is nonrecurrent, outside of the ZLB the equilibrium is characterized by an operational Taylor rule (B.111) *and by equations* (B.124) *and* (B.123)*. Writing in matrix form and substituting the Taylor rule, the system can be written as*

$$\underbrace{\begin{bmatrix} 1 & \frac{\gamma_\pi}{1-\phi_c(1+\nu)} \\ -\lambda_w(1-\phi_c(1+\nu)+\nu) & 1 \end{bmatrix}}_{\equiv A_p} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & \frac{1}{1-\phi_c(1+\nu)} \\ 0 & \beta \end{bmatrix}}_{\equiv B_p} \mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{1-\phi_c(1+\nu)} \\ \lambda & -1 \end{bmatrix} \begin{bmatrix} \tau_t \\ g_t \end{bmatrix}. \tag{B.150}$$

Uniqueness of the locally bounded equilibrium requires both eigenvalues $\Lambda_{1,2}$ of $C_p \equiv A_p^{-1}B_p$ to lie inside the unit circle. The characteristic equation for Λ can be written as $\Lambda^2 - \mathrm{tr}(C_p)\Lambda + \det(C_p) = 0$. Following standard textbook analysis of eigenvalues on the unit circle yields the sufficient and necessary conditions in **?**, *Appendix A for local uniqueness:*

$$|\det(C_p)| < 1 \qquad |\,\mathrm{tr}(C_p)| < \det(C_p) + 1. \tag{B.151}$$

Here, the determinant satisfies

$$\det(C_p) = \frac{\beta(1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi\lambda_p(\nu + (1-\phi_c(1+\nu)))} \in (0,1).$$

The trace is strictly positive under the assumption of $\phi_c(1+\nu) < 1$ and satisfies:

$$\begin{aligned}
\mathrm{tr}(C_p) &= \frac{(1+\beta)(1-\phi_c(1+\nu)) + \lambda_p(\nu + (1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi\lambda_p(\nu + (1-\phi_c(1+\nu))} \\
&< \frac{(1-\phi_c(1+\nu)) + \gamma_\pi\lambda_p(\nu + (1-\phi_c(1+\nu))) + \beta(1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi\lambda_p(\nu + (1-\phi_c(1+\nu)))} \\
&= 1 + \frac{\beta(1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi\lambda_p(\nu + (1-\phi_c(1+\nu)))} \\
&= 1 + \det(C_p),
\end{aligned}$$

where the inequality follows from the Taylor Principle.

Now consider the case of flexible prices and sticky wages. In this case, the matrices are given by:

$$A_w \equiv \begin{bmatrix} 1 & \gamma_\pi \\ -\lambda_w(1+\nu) & 1 \end{bmatrix}, \qquad B_W \equiv \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix}.$$

Proceeding analogously by defining $C_w \equiv A_w^{-1} B_w$ yields the following conditions for uniqueness:

$$\begin{aligned} \det(C_w) &= \frac{\beta}{1+\gamma_\pi \lambda_w(1+\nu)} \in (0,1) \\ \operatorname{tr}(C_w) &= \frac{\beta}{1+\gamma_\pi \lambda_w(1+\nu)} + \frac{1+\lambda_w(1+\nu)}{1+\gamma_\pi \lambda_w(1+\nu)} \\ &< \frac{\beta}{1+\gamma \lambda_w(1+\nu)} + \gamma_\pi \frac{1+\lambda_w(1+\nu)}{1+\gamma_\pi \lambda_w(1+\nu)} = \det(C_w) + 1, \end{aligned}$$

where the inequality holds again because of the Taylor principle.

Lemma 2. *Suppose the Taylor Principle is satisfied, the ZLB is nonrecurrent, and taxation is immediate ($\psi_\tau = 1$). Assume $\phi_c(1+\nu) < 1$ if wages are flexible and prices are sticky. If and only if the numerator of the Euler equations at the ZLB,* (B.126) *and* (B.128)*, are positive, there is a unique locally bounded Markov equilibrium in $\boldsymbol{1}_{ZLB,t}$ at the ZLB.*

Proof: Since the ZLB is nonrecurrent, by Lemma 1*, if $\boldsymbol{1}_{ZLB,t} = 0$, there is a unique bounded equilibrium. Moreover, since all exogenous variables are zero outside of the steady state with immediate taxation, $y_t = \pi_t = 0$ is the unique bounded solution conditional on exit from the ZLB. Conditional on $\boldsymbol{1}_{ZLB,t} = 1$, the dynamics can therefore by described by:*

$$\begin{aligned} A_{j|\gamma_\pi=0} \begin{bmatrix} y_{ZLB,t} \\ \pi_{ZLB,t} \end{bmatrix} &= B_j \mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} + D_j \begin{bmatrix} \tau_t \\ g_t \end{bmatrix}, \quad j = p, w. \\ &= B_j \mu \begin{bmatrix} y_{ZLB,t+1} \\ \pi_{ZLB,t+1} \end{bmatrix} + D_j \begin{bmatrix} \tau_t \\ g_t \end{bmatrix}, \quad j = p, w, \end{aligned}$$

where the second equality uses the Markov structure of the equilibrium. $j = p, w$ indexes the matrices defined in the proof of Lemma 1*.*

Define $C_{j,ZLB} = A_{j|\gamma_\pi=0}^{-1} B_j \mu$. Proceeding as in the proof of Lemma 1*, uniqueness of the locally bounded Markov equilibrium is equivalent to $|\det(C_{j,ZLB})| < 1$ and $|\operatorname{tr}(C_{j,ZLB})| < 1 + \det(C_{j,ZLB})$.*

Consider the case of flexible wages and sticky prices. Then $\det(C_{p,ZLB}) =$

$\beta\mu^2 \in (0,1)$. $|\operatorname{tr}(C_{j,ZLB})| < 1 + \det(C_{j,ZLB})$ *is equivalent to*

$$\mu(1+\beta) + \frac{\lambda_p \mu(\nu + 1 - \phi_c(1+\nu))}{1 - \phi_c(1+\nu))} < 1 + \beta\mu^2$$
$$\Leftrightarrow \frac{\mu(1+\beta) + \lambda_p \mu(\nu + 1 - \phi_c(1+\nu))}{1 - \phi_c(1+\nu))} < 1 + \beta\mu^2 - \mu(1+\beta) = (1-\beta\mu)(1-\mu)$$

Simplifying the left-hand side and dividing through by the right-hand side yields

$$\frac{\mu}{(1-\beta\mu)(1-\mu)} \frac{\lambda_p}{1-\phi_c(1+\nu)}(\nu+1)(1-\phi_c) < 1,$$

which is equivalent to the denominator in (B.126) *being positive.*

Now consider the case of flexible prices and sticky wages. Again, $\det(C_{w,ZLB}) = \beta\mu^2 \in (0,1)$. *The trace condition* $\operatorname{tr}(C_{j,ZLB})| < 1 + \det(C_{j,ZLB})$ *is equivalent to:*

$$\mu(1+\beta) + \mu\lambda_w(1+\nu) < 1 + \mu^2\beta$$
$$\Leftrightarrow \mu\lambda_w(1+\nu) < 1 + \mu^2\beta - \mu(1+\beta) = (1-\beta\mu)(1-\mu)$$

Dividing through by $(1-\beta\mu)(1-\mu)$ *yields:*

$$\frac{\mu}{(1-\beta\mu)(1-\mu)}\lambda_w(1+\nu) < 1,$$

which is equivalent to the denominator of (B.128) *being positive.*

Proposition 1. *Consider the model in Section* 6.1. *Assume immediate taxation* ($\psi_\tau = 1$)*, that the Taylor Principle holds and that the ZLB is a nonrecurrent Markov state which persists with probability* μ: $\Pr\{\mathbf{1}_{ZLB,t} = 1|\mathbf{1}_{ZLB,t-1} = 1\} = \mu, \Pr\{\mathbf{1}_{ZLB,t} = 1|\mathbf{1}_{ZLB,t-1} = 0\} = 0$. *In the case of sticky prices and flexible wages, also assume that* $0 < \phi < \frac{\frac{\epsilon_p}{\epsilon_p - 1}}{1+\nu}$. *Consider the case of financing through distortionary labor taxes:* $\hat{s}_t = d\tau_t^c = 0$.

(a) For sufficiently small persistence of the ZLB, μ*, the impact multiplier* $\frac{y_{ZLB}}{g}$ *is strictly smaller than one under flexible prices and sticky wages and strictly larger than one under flexible wages and sticky prices.*

(b) The multiplier increases monotonically in the expected duration of the ZLB with either sticky wages or sticky prices in the region of determinacy.

Proof: The Taylor Principle guarantees local uniqueness of the equilibrium around the steady state. For μ *small enough, the local uniqueness extends to the system at the ZLB and the derivation of* (B.152) *is valid. This allows to impose that the model returns to the steady state with probability* $1-\mu$ *in which case* *the unique Markov equilibrium at the ZLB is given by the following solution to the pairs of Euler equations and Phillips curves* (B.128) *and* (B.126)*:*

$$y_{ZLB}^{fw} = \frac{\frac{1}{1-\phi_c(1+\nu)} + \frac{\lambda_p}{1-\phi_c(1+\nu)}\frac{\mu}{(1-\beta\mu)(1-\mu)}\left(\frac{1}{\epsilon-1}\right)}{1 - \frac{\lambda_p}{1-\phi_c(1+\nu)}\frac{\mu}{(1-\beta\mu)(1-\mu)}(1+\nu)(1-\phi_c)} \quad \zeta_p > 0, \zeta_w = 0, \qquad \text{(B.126)}$$

$$\begin{aligned} y_{ZLB}^{fp} &= \frac{(1-\mu)\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} - \frac{\mu}{1-\beta\mu}\lambda_w\frac{1}{1-\phi_c} + \frac{\mu}{1-\beta\mu}\lambda_w\left(\frac{\phi_c}{1-\phi_c}+1\right)\frac{\epsilon}{\epsilon-1}}{1-\mu-\frac{\mu}{1-\beta\mu}\lambda_w(1+\nu)} g \\ &= \frac{\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} + \frac{\mu}{(1-\mu)(1-\beta\mu)}\lambda_w\frac{1}{1-\phi_c}\frac{1}{\epsilon-1}}{1-\frac{\mu}{(1-\mu)(1-\beta\mu)}\lambda_w(1+\nu)} g \quad \zeta_p = 0, \zeta_w > 0. \qquad \text{(B.128)} \end{aligned}$$

(a) Flexible wage case: The limit $\mu \searrow 0$ *in* (B.126) *yields*

$$\lim_{\mu\searrow 0} y_{ZLB}^{fw} = \frac{1}{1-\phi_c(1+\nu)} g > g,$$

given that $\phi_c \equiv \phi\frac{\epsilon_p - 1}{\epsilon_p} < (1+\nu)^{-1}$.

(a) Flexible price case: From equation (B.128) *it follows that*

$$\lim_{\mu\searrow 0} y_{ZLB}^{fp} = \frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} g < g,$$

given $\phi_c = \phi\frac{\epsilon-1}{\epsilon} \in [0,1)$*. The result follows by continuity for* μ *small enough.*

(b) Flexible price case: Note that $A(\mu) \equiv \frac{\mu}{(1-\mu)(1-\beta\mu)}$ *is strictly increasing in* $\mu \in [0,1)$*. Hence* $\frac{d}{d\mu}\frac{y_1}{g} = A'(\mu)\frac{d}{dA(\mu)}\frac{y_1}{g}$*. The latter expression is given by:*

$$\begin{aligned} &\frac{d}{dA(\mu)}\frac{y_1}{g} \\ &= \frac{\lambda_w\frac{1}{\epsilon_p-1}\big(1-A(\mu)\lambda_w(1+\nu)\big) + A(\mu)\lambda_w(1+\nu)\left(1-\frac{\epsilon_p}{\epsilon_p-1}\phi_c + A(\mu)\lambda_w\frac{1}{\epsilon_p-1}\right)}{\big(1-A(\mu)\lambda_w(1+\nu)\big)^2(1-\phi_c)} > 0 \end{aligned}$$

in the region of determinacy satisfying $1 > \frac{\mu}{(1-\mu)(1-\beta\mu)}\lambda_w(1+\nu)$ *from Lemma* 2*.*

(b) Flexible wage case: Define $A(\mu)$ *as above.*

$$\begin{aligned} &\frac{d}{dA(\mu)}\frac{y_1}{g} \\ &= \lambda_p\frac{\frac{1}{\epsilon_p-1}\left(1-\frac{1}{1-\phi_c(1+\nu)}\lambda_pA(\mu)(1+\nu)(1-\phi_c)\right) + \frac{1}{1-\phi_c(1+\nu)}\lambda_p(1+\nu)(1-\phi_c)(1+\lambda_pA(\mu)\frac{1}{\epsilon_p-1})}{\left(1-\frac{1}{1-\phi_c(1+\nu)}\lambda_pA(\mu)(1+\nu)(1-\phi_c)\right)^2(1-\phi_c(1+\nu))} > 0, \end{aligned}$$

in the region of determinacy satisfying $1 > \frac{1}{1-\phi_c(1+\nu)}\lambda_p A(\mu)(1+\nu)(1-\phi_c)$ *from Lemma* 2.

Proposition 2. *Consider the model in Section* 6.1 *with sticky wages, flexible prices, and a one period ZLB and stimulus. Assume the Taylor Principle is satisfied.*

(a) The impact multiplier is strictly lower when financed with labor taxes rather than lump-sum taxes if $\psi_\tau < 1$ *and equal otherwise. The long-run multiplier is lower with both labor taxes and consumption taxes than it is with lump-sum taxes if* $\psi_\tau < 1$ *and wages are sufficiently sticky* ($\zeta_w \nearrow 1$).
If taxes are adjusted sufficiently slowly, $\psi_\tau < \bar{\psi}^n < 1$, *the impact multiplier is higher with consumption taxes than it is with labor taxes. The impact multiplier is higher with consumption taxes than it is with transfer financing if* $\psi_\tau < \bar{\psi}^s$, *where* $\bar{\psi}^s < \bar{\psi}^n$.

The following results assume financing through labor taxes: $\hat{s}_t = d\tau_t^c = 0$.

(b) If wages are sufficiently sticky ($\zeta_w \nearrow 1$) *and* $\phi > 0$, *increasing the tax adjustment speed* ψ_τ *lowers the impact multiplier. Without RoT agents,* $\phi = 0$, *increasing the tax adjustment speed* ψ_τ *increases the impact multiplier.*

(c) If $\psi_\tau < 1$, *the baseline model with the tax rule* (6.2b) *yields a lower short-run multiplier than financing stimulus through permanently higher debt according to* (6.2b') *if* $\gamma_\pi > 1 + (\bar{R}^g - 1)\left(1 + \frac{1}{\lambda_w(1+\nu)}\right)$.

(d) Lowering the labor share $\frac{\epsilon_p - 1}{\epsilon_p}$ *lowers the multiplier if the impact multiplier is positive. A sufficient condition is that wages are sufficiently sticky* ($\zeta_w \nearrow 1$). *If the impact multiplier is nondecreasing in the labor share, the long-run multiplier is strictly lower for lower labor shares for all* $\psi_\tau < 1$.

Proof: Under the Taylor Principle, there is a locally unique bounded equilibrium outside of the ZLB due to Lemma 1. *The above backward induction is therefore valid.*

(a) Under the Taylor Principle, the inequality (B.138) *implies that the impact multiplier is higher with transfer financing than with labor taxes.*
The results for the consumption tax as compared with the labor tax and transfer financing follow from (B.135), *and* (B.139), *whereas the comparison of thresholds follows from* (B.141).
Since the long-run multiplier here is defined as $\frac{y_1+\beta y_2}{g}$, *the result for labor taxes compared with transfers follows immediately from comparing* (B.131c) *and* (B.131a), *which imply that period two output is higher with transfer financing. Since by* (B.138) *period one output is also higher, the result for the long-run multiplier is immediate.*
Comparing the long-run transfer financed multiplier to the consumption tax

multiplier implies from (B.142) *that their difference is proportional to:*

$$\psi + (1-\psi)\frac{\beta + \lambda_w(\beta\gamma_p i - (\gamma_\pi - 1)\nu)}{1 + \gamma_\pi \lambda_w (1+\nu)},$$

which is strictly positive for all $\psi \geq 0$ *if* $\beta + \lambda_w(\beta\gamma_p i - (\gamma_\pi - 1)\nu) > 0$. *Sufficient for this is that* $\zeta_w \nearrow 1$ *so that* $\lambda_w \searrow 0$. *By continuity, the result holds for a neighborhood of sticky wages.*

(b) For $\lambda_w \searrow 0$ *and* $\phi_c > 0$, (B.133) *yields* $\frac{y_1}{g} = \frac{1-\phi_c \frac{\epsilon_p}{\epsilon_p - 1}\psi_\tau}{1-\phi_c}$ *and hence*

$$\lim_{\lambda_w \searrow 0} \frac{d}{d\psi_\tau}\frac{y_1}{g} = -\frac{\phi_c \frac{\epsilon_p}{\epsilon_p - 1}}{1-\phi_c} < 0.$$

From (B.133) $\lim_{\phi_c \searrow 0} \frac{y_1}{g} = 1 - (\gamma_\pi - 1)\lambda \frac{\nu}{1+\gamma_\pi \lambda_w \left(1+\nu\right)} \frac{\epsilon_p}{\epsilon_p - 1}(1-\psi_\tau)\bar{R}^g$ *is strictly increasing in* ψ_τ *given* $\gamma_\pi > 1$.

(c) By assumption, $\psi_\tau < 1$ *and* $\gamma_y = 0$. *Under the baseline assumption* (6.2b)*, the multiplier is characterized by* (B.133). *Under the alternative of permanently higher debt financing* (6.2b')*, the multiplier is given by* (B.144).

Subtracting (B.133) *from* (B.144) *and dividing through by the positive, common terms* $\bar{R}^g \frac{1-\psi_\tau}{1-\phi_c}(\gamma_\pi - 1)(1-(1+\nu)\phi_c)\frac{\epsilon_p}{\epsilon_p - 1}$ *yields the following difference in multipliers, denoted by* $\Delta\frac{y_1}{g}$*:*

$$\Delta\frac{y_1}{g} = -\frac{\bar{R}-1}{(1+\nu)(\gamma_\pi - 1)} + \frac{\lambda_w \bar{R}}{1+\lambda_w(1+\nu)\gamma_\pi} > 0$$
$$\Leftrightarrow \gamma_\pi > 1 + (\bar{R}-1)\left(1 + \frac{1}{(1+\nu)\lambda_w}\right)$$

(d) To see the effect of changing the labor share on the impact multiplier, it is useful to make the dependence of the consumption share of RoT agents $\phi_c = \phi\frac{\epsilon_p - 1}{\epsilon_p}$ *on the labor share explicit in* (B.133)*. Rewrite:*

$$\frac{y_1}{g} = \frac{1}{1-\phi\frac{\epsilon_p - 1}{\epsilon_p}}\left(1 - \psi_\tau\phi - (\gamma_\pi - 1)\lambda\frac{\left(\frac{\epsilon_p}{\epsilon_p - 1} - \phi(1+\nu)\right)}{1+\gamma\lambda_w(1+\nu)}(1-\psi_\tau)\bar{R}^g\right).$$

Define $\frac{y_1}{g} \equiv \frac{1}{1-\phi\frac{\epsilon_p - 1}{\epsilon_p}} \times P$. *Clearly, in the limit of perfectly sticky wages,* $P = 1 - \phi\psi_\tau > 0$. *Hence by continuity, for sufficiently sticky wages the multiplier is strictly positive.*

Now differentiate the impact multiplier with respect to the labor share:

$$\frac{d}{d\frac{\epsilon_p-1}{\epsilon_p}}\frac{y_1}{g}=\phi\left(\frac{1}{1-\phi\frac{\epsilon_p-1}{\epsilon_p}}\right)^2\times P+\frac{1}{1-\phi\frac{\epsilon_p-1}{\epsilon_p}}\times(\gamma_\pi-1)\lambda\frac{\left(\frac{\epsilon_p}{\epsilon_p-1}\right)^2}{1+\gamma\lambda_w(1+\nu)}(1-\psi_\tau)\bar{R}^g>0,$$

using that the Taylor Principle holds. Thus, the impact multiplier is increasing in the labor share.
If the impact multiplier is nondecreasing in the labor share, it is sufficient to show that the period 2 multiplier is increasing in the labor share for the long-run multiplier to increase in the labor share. To see the effect of the labor share on the period 2 multiplier $\frac{y_2}{g}$*, rewrite* (B.131a) *as:*

$$\frac{y_2}{g}=-\frac{1}{\frac{\epsilon_p-1}{\epsilon_p}\left(1-\phi\frac{\epsilon_p-1}{\epsilon_p}\right)}\underbrace{\frac{1+\gamma_\pi\lambda_w}{1+\gamma_\pi\lambda_w(1+\nu)}(1-\psi)}_{\geq 0}.$$

Since $\frac{d}{d\frac{\epsilon_p-1}{\epsilon_p}}-\frac{1}{\frac{\epsilon_p-1}{\epsilon_p}\left(1-\phi\frac{\epsilon_p-1}{\epsilon_p}\right)}=+\frac{1-2\phi_c}{\left(\frac{\epsilon_p-1}{\epsilon_p}\right)^2\left(-\phi\frac{\epsilon_p-1}{\epsilon_p}\right)^2}>0$, $\frac{y_2}{g}$ *is increasing in the labor share given* $\phi_c<\phi<\frac{1}{2}$.

Proposition 3. *Consider the model in Section* 6.1 *with sticky wages, flexible prices, and a one period ZLB and stimulus. Assume the Taylor Principle is satisfied. Consider the case of financing through distortionary labor taxes:* $\hat{s}^e_t=d\tau^c_t=0$.

(a) If taxes are adjusted sufficiently slowly ($\psi_\tau<\frac{\epsilon_p-1}{\epsilon_p}$*), the impact multiplier increases strictly in the share of RoT agents* ϕ.

(b) The multiplier on transfers is strictly increasing in the fraction of transfers RoT agents receive and weakly smaller than the government spending multiplier.

Proof: Under the Taylor Principle, the steady state is the locally unique bounded equilibrium outside of the ZLB. Thus, the above backward induction is valid.

(a) Since $\phi_c=\frac{\epsilon_p}{\epsilon_p-1}\phi$*, we can equivalently consider an increase in* ϕ *or* ϕ_c*. An increase in* ϕ_c *affects the multiplier as follows:*

$$\begin{aligned}\frac{d}{d\phi_c}\frac{y_1^\tau}{g}&=\frac{d}{d\phi_c}\frac{1-\psi_\tau\frac{\epsilon_p}{\epsilon_p-1}\phi_c}{1-\phi_c}-\frac{(\gamma_\pi-1)\lambda_w}{1+\gamma_\pi\lambda_w(1+\nu)}\frac{\epsilon_p}{\epsilon_p-1}(1-\psi_\tau)\bar{R}^g\frac{d}{d\phi_c}\frac{1-\phi_c(1+\nu)}{1-\phi_c}\\&=\frac{1-\psi_\tau\frac{\epsilon_p}{\epsilon_p-1}}{(1-\phi_c)^2}+\frac{(\gamma_\pi-1)\lambda_w}{1+\gamma_\pi\lambda_w(1+\nu)}\frac{\epsilon_p}{\epsilon_p-1}(1-\psi_\tau)\bar{R}^g\frac{\nu}{(1-\phi_c)^2}.\end{aligned}$$

Because the second term is strictly positive under the Taylor Principle, a suffi-

cient condition for the entire expression to be positive is for ψ_τ to be sufficiently small: $1 > \psi_\tau \frac{\epsilon_p}{\epsilon_p - 1}$.

(b) From the tax rule (B.130)*, the path of labor taxes is the same for equal spending on stimulus transfer* s_t^x *and government spending* g_t*. Thus from the Phillips curve* (B.123)*, period two inflation is the same with transfers or government spending. Given that period two output is unchanged, but only a fraction* $\phi = \phi_c \frac{\epsilon_p}{\epsilon_p - 1}$ *of transfers is spent during period one, it follows from the Euler equation* (B.124)*, that period one output and hence the impact multiplier is strictly lower with transfers. Since period two output is unchanged, the long-run multiplier is also strictly lower.*

Proposition 4. *Consider the model in Section* 6.1 *with sticky wages, flexible prices, and a one period ZLB and stimulus. Assume the Taylor Principle is satisfied. Consider the case of financing through distortionary labor taxes:* $\hat{s}_t = d\tau_t^c = 0$.

If $\psi_\tau < 1$ *and* $\phi < \frac{\epsilon_p}{\epsilon_p - 1}\frac{1}{1+\nu}$*, increasing wage flexibility* ($\zeta_w \searrow 0$) *lowers the impact multiplier.*

Proof: Under the Taylor Principle, the steady state is the locally unique bounded equilibrium outside of the ZLB. Thus, the above backward induction is valid.

Note that $\lambda_w = \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w}$ *is decreasing in* $\zeta_w \in (0, 1)$*. Equivalently, when wages become more flexible (*ζ_w *falls),* λ_w *increases. Then it suffices to characterize the impact of higher* λ_w *on the short-run multiplier* $\frac{y_1}{g}$*. From* (B.133) *and under the Taylor Principle,* $\frac{d}{d\lambda_w}\frac{y_1}{g} \propto -\frac{1-\phi_c(1+\nu)}{1-\phi_c}(1-\psi_\tau)\bar{R}^g$ *since* $\frac{d}{d\lambda_w}(\gamma_\pi - 1)\frac{\epsilon_p}{\epsilon_p - 1}(1-\psi_\tau)\bar{R}^g \frac{\lambda_w}{1+\lambda_w\gamma_\pi(1+\nu)} > 0$ *under the Taylor Principle if* $\psi_\tau < 1$*. Using* $\phi_c = \phi\frac{\epsilon_p - 1}{\epsilon_p}$*, it follows that* $\frac{d}{d\lambda_w}\frac{y_1}{g} < 0$ *if* $\phi < \frac{\epsilon_p}{\epsilon_p - 1}\frac{1}{1+\nu}$.

Proposition 5. *Consider the model in Section* 6.1 *with sticky wages, flexible prices, a one period ZLB and stimulus, and the modified Taylor rule with* $\gamma_y \geq 0$*. Assume the Taylor Principle is satisfied. Assume the stimulus is partially debt financed. Consider the case of financing permanently higher debt* $\hat{b} = (1-\psi)\bar{R}^g g > 0$ *through distortionary labor taxes:* $\hat{s}_t = d\tau_t^c = 0$.

(a) The short-run multiplier is independent of wage stickiness if $\gamma_y = 0$.

(b) The short-run multiplier is increasing in γ_y.

(c) If $\gamma_y > 0$ *and* $\gamma_\pi > 1 + \gamma_y$*, increasing wage flexibility* ($\zeta_w \searrow 0$) *lowers the impact multiplier.*

Proof: With $\gamma_y \geq 0$*, the Taylor Principle is sufficient to ensure that the steady state is the locally unique bounded equilibrium outside of the ZLB. Thus, the above backward induction is valid.*

(a) Set $\gamma_y = 0$ *in equation* (B.144).

(b) See equation (B.145).

(c) Since $\hat{b} > 0$, $\psi_\tau < 1$ follows. Note that $\lambda_w = \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w}$ is decreasing in $\zeta_w \in (0,1)$. Equivalently, when wages become more flexible (ζ_w falls), λ_w increases. Then it suffices to characterize the impact of higher λ_w on the short-run multiplier $\frac{y_1}{g}$. From equation (B.146) *it follows that $\frac{d}{d\lambda_w}\frac{y_1}{g} < 0$ if $\gamma_\pi > 1 + \gamma_y$.*

C Backing out the unemployment rate

To back out the model implications for the unemployment rate, we regress the time series for hours worked used for the model estimation on the average quarterly unemployment rate. Table C.6 shows the regression results. Figure C.2 displays the actual and fitted unemployment rate. Multiplying hours worked on the OLS regression coefficient gives the implied change in the unemployment rate.

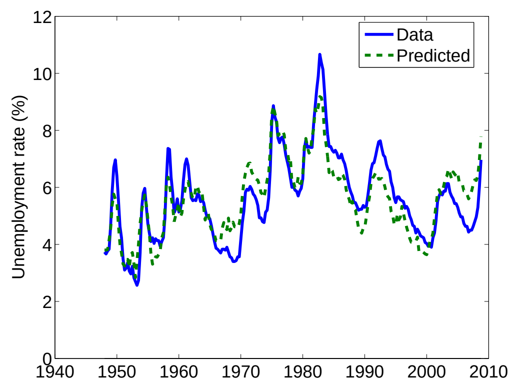


Figure C.2: Regression of quarterly unemployment rate on the model-implied employment measure: actual vs. predicted unemployment rate

Table C.6: OLS regression estimates of unemployment rate on the model-implied employment measure

	Constant	Employment (lab_t)	R^2
Unemployment Rate (UR_t)	5.60	-0.46	0.77
	(5.51, 5.69)	(-0.49, -0.43)	

Sample period: 1948:1 – 2008:4. Unemployment rate is the arithmetic mean over the quarter. Labor input in the model is measured as $\text{lab}_t \equiv \log \frac{\text{Avg. hours}_t \times \text{Employment}_t}{\text{Population}_t} - \text{mean}$; 95 percent OLS confidence intervals in parentheses.

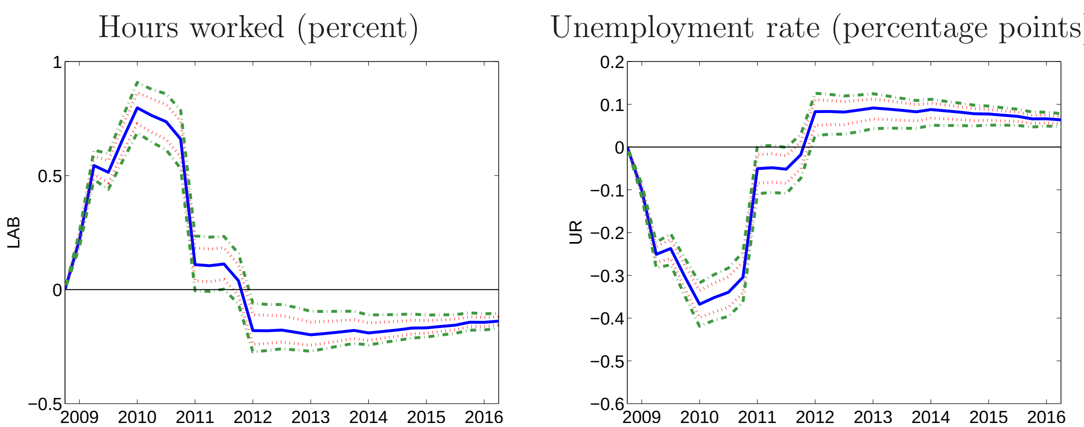


Figure C.3: *Employment and unemployment impact of ARRA (benchmark scenario)*

D Additional results

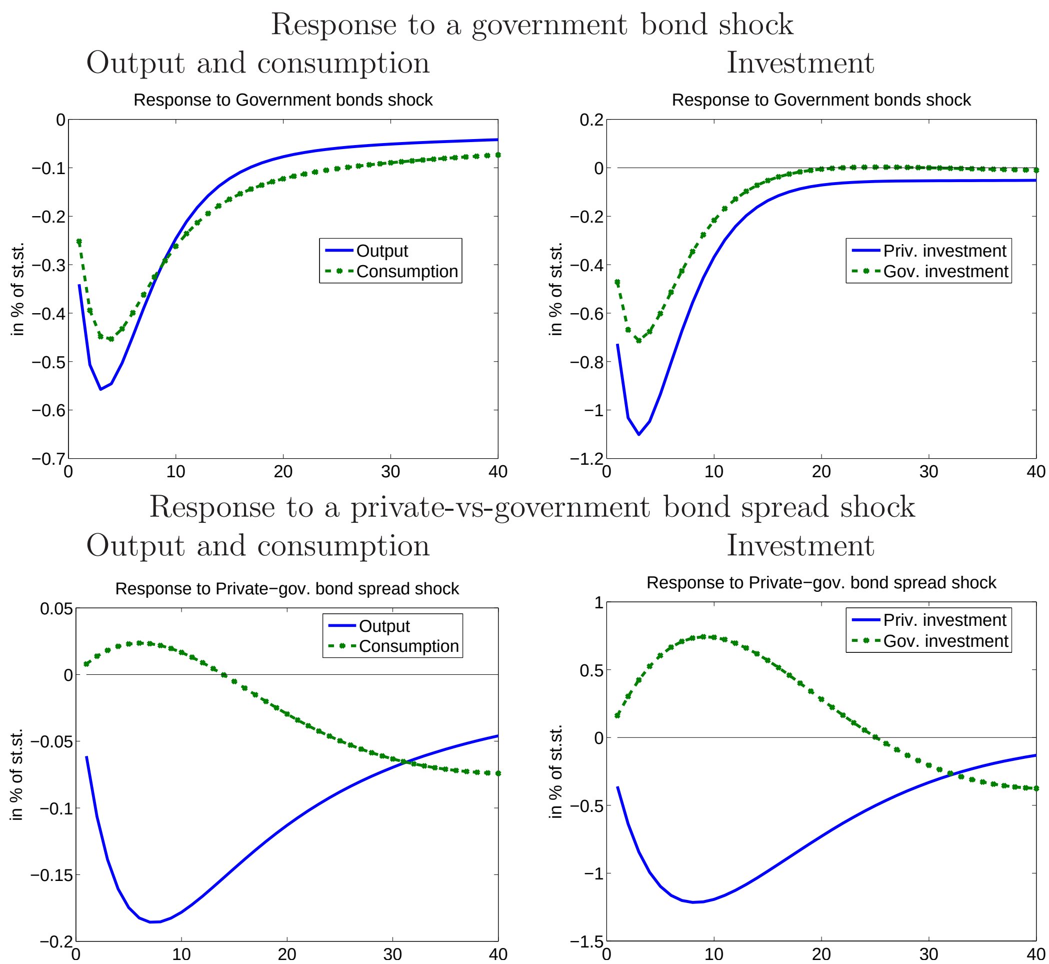


Figure D.4: Response to the bond shocks

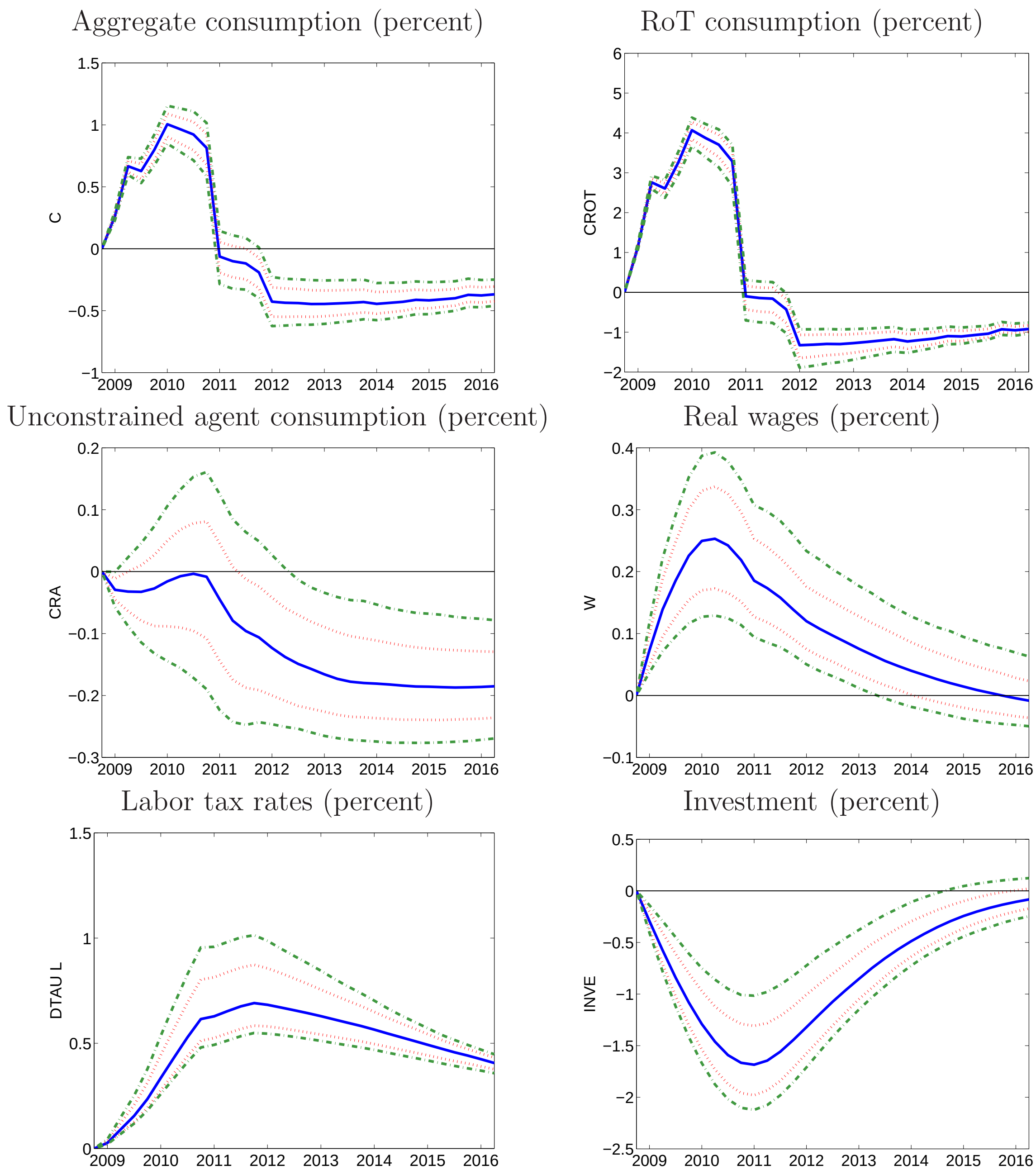


Figure D.5: Benchmark impact of ARRA: consumption, investment, tax rates, and real wages

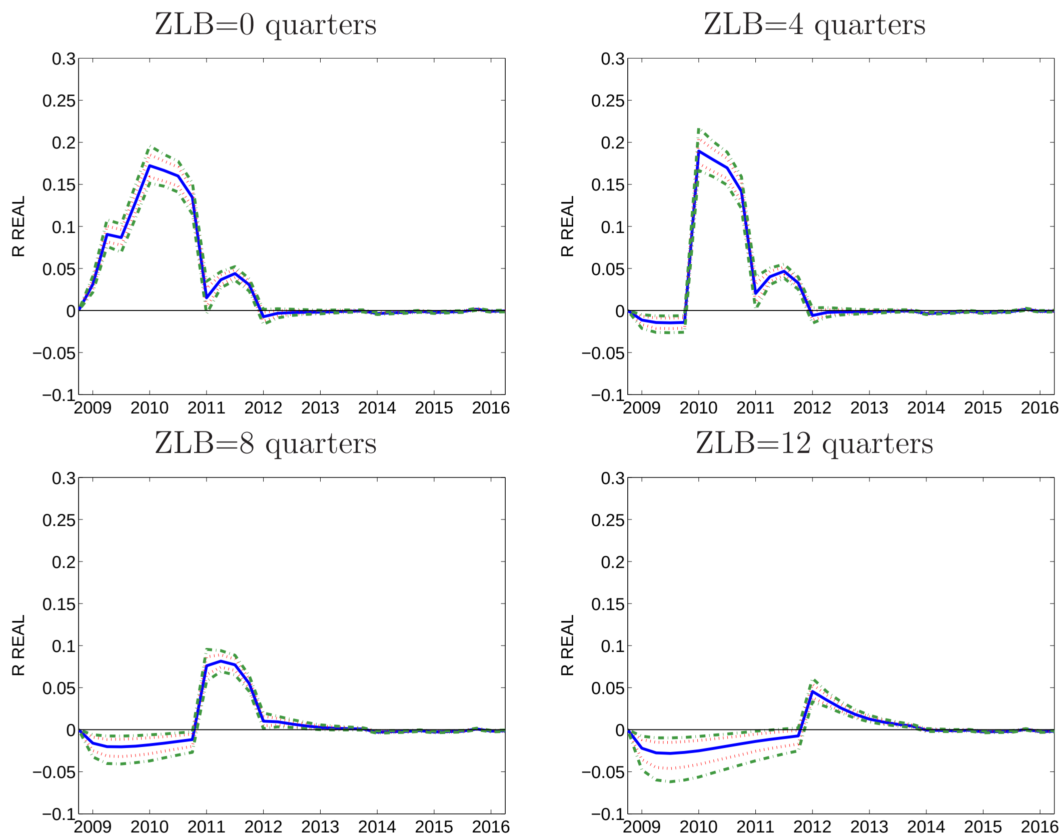


Figure D.6: Impact of ARRA on real interest rates for varying ZLB length

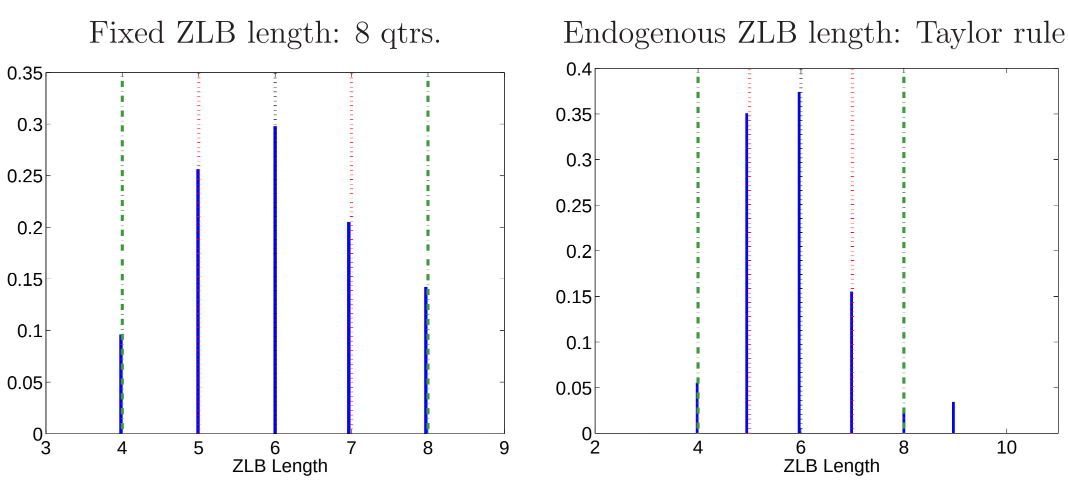
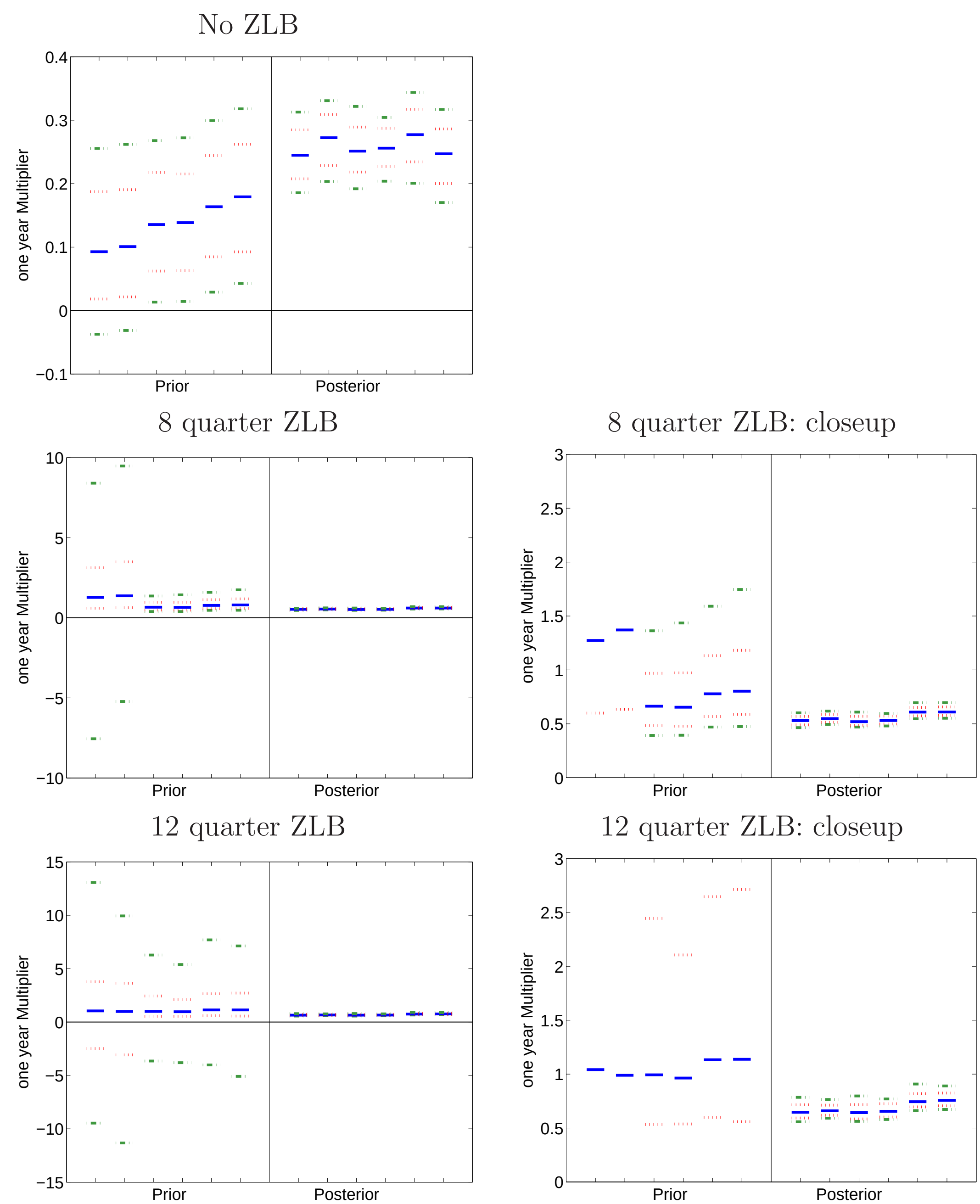


Figure D.7: ZLB duration implied by Taylor rule



Note: Priors and posterior multipliers correspond to six different models (from left to right): (1) Baseline point estimates: $\psi_\tau = 0.0375$, $\phi = 0.25$, free fixed cost Φ. (2) Slower tax adjustment: $\psi_\tau = 0.025$, $\phi = 0.25$, free fixed cost Φ. (3) Baseline point estimates: $\psi_\tau = 0.0375$, $\phi = 0.25, \Phi = 1.94$. (4) Slower tax adjustment: $\psi_\tau = 0.025$, $\phi = 0.25, \Phi = 1.94$. (5) Large fraction of RoT agents: $\psi_\tau = 0.0375$, $\phi = 0.40, \Phi = 1.94$. (6) Slower tax adjustment and large fraction of RoT agents: $\psi_\tau = 0.025$, $\phi = 0.40, \Phi = 1.94$. All other parameters are re-estimated. Starting point of the simulations is the steady state.

Figure D.8: Priors and posteriors of short-run ARRA multipliers across models for different durations of the ZLB

Table D.7: Short-run fiscal multipliers: prior sensitivity

Prior multipliers						
Specification	Log-Likelihood	5 pc	16.5 pc	median	83.5 pc	95 pc
No ZLB: Baseline, fixed cost Φ estimated	-2413.9	-0.02	0.04	0.11	0.21	0.28
No ZLB: Slower tax adjustment, Φ estimated	-2413.5	-0.02	0.04	0.12	0.21	0.28
No ZLB: Baseline	-2400.6	0.02	0.07	0.15	0.23	0.28
No ZLB: Slower tax adjustment	-2397.4	0.02	0.07	0.15	0.23	0.28
No ZLB: Higher RoT fraction	-2447.8	0.04	0.10	0.18	0.26	0.31
No ZLB: Slower tax adj., higher RoT fr.	-2448.8	0.05	0.10	0.19	0.27	0.33
8 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	-8.24	0.67	1.41	3.46	9.43
8 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	-5.87	0.71	1.50	3.97	10.78
8 qtr ZLB: Baseline	-2400.6	0.42	0.51	0.70	1.03	1.45
8 qtr ZLB: Slower tax adjustment	-2397.4	0.42	0.50	0.69	1.03	1.54
8 qtr ZLB: Higher RoT fraction	-2447.8	0.50	0.60	0.82	1.21	1.71
8 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	0.51	0.62	0.85	1.25	1.87
12 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	-10.42	-2.66	1.18	4.26	13.89
12 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	-12.63	-3.46	1.10	4.01	11.41
12 qtr ZLB: Baseline	-2400.6	-3.84	0.56	1.05	2.58	6.63
12 qtr ZLB: Slower tax adjustment	-2397.4	-4.03	0.56	1.01	2.22	5.69
12 qtr ZLB: Higher RoT fraction	-2447.8	-4.29	0.64	1.19	2.79	8.18
12 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	-5.28	0.60	1.20	2.89	7.64
Posterior multipliers						
Specification	Log-Likelihood	5 pc	16.5 pc	median	83.5 pc	95 pc
No ZLB: Baseline, fixed cost Φ estimated	-2413.9	0.19	0.21	0.25	0.29	0.32
No ZLB: Slower tax adjustment, Φ estimated	-2413.5	0.20	0.23	0.27	0.31	0.33
No ZLB: Baseline	-2400.6	0.20	0.22	0.26	0.29	0.33
No ZLB: Slower tax adjustment	-2397.4	0.21	0.23	0.26	0.29	0.31
No ZLB: Higher RoT fraction	-2447.8	0.21	0.24	0.28	0.32	0.35
No ZLB: Slower tax adj., higher RoT fr.	-2448.8	0.17	0.20	0.25	0.29	0.32
8 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	0.47	0.50	0.54	0.58	0.61
8 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	0.50	0.52	0.55	0.59	0.62
8 qtr ZLB: Baseline	-2400.6	0.48	0.49	0.53	0.58	0.62
8 qtr ZLB: Slower tax adjustment	-2397.4	0.49	0.51	0.54	0.58	0.60
8 qtr ZLB: Higher RoT fraction	-2447.8	0.56	0.59	0.62	0.66	0.71
8 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	0.56	0.59	0.62	0.67	0.71
12 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	0.57	0.60	0.65	0.72	0.79
12 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	0.60	0.62	0.66	0.71	0.76
12 qtr ZLB: Baseline	-2400.6	0.57	0.59	0.65	0.72	0.80
12 qtr ZLB: Slower tax adjustment	-2397.4	0.58	0.61	0.66	0.73	0.77
12 qtr ZLB: Higher RoT fraction	-2447.8	0.67	0.71	0.75	0.83	0.91
12 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	0.68	0.71	0.76	0.83	0.90

Table D.8: Long-run fiscal multipliers as $t \to \infty$: prior sensitivity

Prior multipliers						
Specification	Log-Likelihood	5 pc	16.5 pc	median	83.5 pc	95 pc
No ZLB: Baseline, fixed cost Φ estimated	-2413.9	-1.18	-0.95	-0.70	-0.52	-0.43
No ZLB: Slower tax adjustment, Φ estimated	-2413.5	-1.13	-0.95	-0.72	-0.56	-0.48
No ZLB: Baseline	-2400.6	-1.61	-1.37	-1.09	-0.87	-0.72
No ZLB: Slower tax adjustment	-2397.4	-1.62	-1.38	-1.10	-0.84	-0.69
No ZLB: Higher RoT fraction	-2447.8	-1.31	-1.12	-0.87	-0.67	-0.55
No ZLB: Slower tax adj., higher RoT fr.	-2448.8	-1.31	-1.12	-0.89	-0.68	-0.55
8 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	-5.35	-0.59	0.09	1.42	4.45
8 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	-4.21	-0.47	0.20	1.66	5.60
8 qtr ZLB: Baseline	-2400.6	-1.15	-0.88	-0.53	-0.09	0.42
8 qtr ZLB: Slower tax adjustment	-2397.4	-1.16	-0.89	-0.52	-0.09	0.41
8 qtr ZLB: Higher RoT fraction	-2447.8	-0.85	-0.62	-0.28	0.16	0.64
8 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	-0.84	-0.60	-0.24	0.21	0.83
12 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	-8.24	-2.80	-0.14	1.92	8.24
12 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	-9.44	-3.52	-0.16	1.74	6.98
12 qtr ZLB: Baseline	-2400.6	-5.99	-0.85	-0.10	1.70	6.91
12 qtr ZLB: Slower tax adjustment	-2397.4	-5.92	-0.86	-0.12	1.36	5.10
12 qtr ZLB: Higher RoT fraction	-2447.8	-6.17	-0.63	0.15	1.91	7.53
12 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	-6.70	-0.73	0.17	2.07	7.44
Posterior multipliers						
Specification	Log-Likelihood	5 pc	16.5 pc	median	83.5 pc	95 pc
No ZLB: Baseline, fixed cost Φ estimated	-2413.9	-1.04	-0.88	-0.73	-0.60	-0.52
No ZLB: Slower tax adjustment, Φ estimated	-2413.5	-0.92	-0.82	-0.70	-0.59	-0.51
No ZLB: Baseline	-2400.6	-1.03	-0.91	-0.75	-0.62	-0.52
No ZLB: Slower tax adjustment	-2397.4	-0.89	-0.81	-0.68	-0.56	-0.47
No ZLB: Higher RoT fraction	-2447.8	-0.91	-0.78	-0.63	-0.53	-0.45
No ZLB: Slower tax adj., higher RoT fr.	-2448.8	-0.89	-0.79	-0.65	-0.55	-0.45
8 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	-0.75	-0.56	-0.39	-0.22	-0.08
8 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	-0.63	-0.51	-0.37	-0.23	-0.08
8 qtr ZLB: Baseline	-2400.6	-0.76	-0.64	-0.43	-0.25	-0.08
8 qtr ZLB: Slower tax adjustment	-2397.4	-0.61	-0.51	-0.35	-0.19	-0.04
8 qtr ZLB: Higher RoT fraction	-2447.8	-0.57	-0.42	-0.26	-0.12	0.00
8 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	-0.52	-0.41	-0.27	-0.14	-0.03
12 qtr ZLB: Baseline, fixed cost Φ estimated	-2413.9	-0.58	-0.36	-0.15	0.11	0.35
12 qtr ZLB: Slower tax adjustment, Φ estimated	-2413.5	-0.46	-0.33	-0.14	0.07	0.24
12 qtr ZLB: Baseline	-2400.6	-0.60	-0.45	-0.20	0.09	0.34
12 qtr ZLB: Slower tax adjustment	-2397.4	-0.43	-0.31	-0.11	0.13	0.34
12 qtr ZLB: Higher RoT fraction	-2447.8	-0.37	-0.21	-0.02	0.21	0.44
12 qtr ZLB: Slower tax adj., higher RoT fr.	-2448.8	-0.35	-0.20	-0.02	0.17	0.34

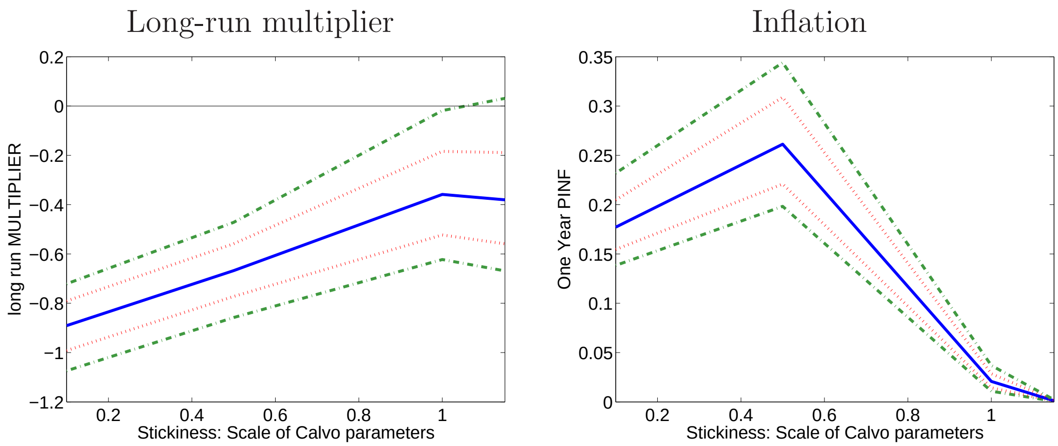


Figure D.9: Long-run multiplier and inflation response: sensitivity to price and wage stickiness

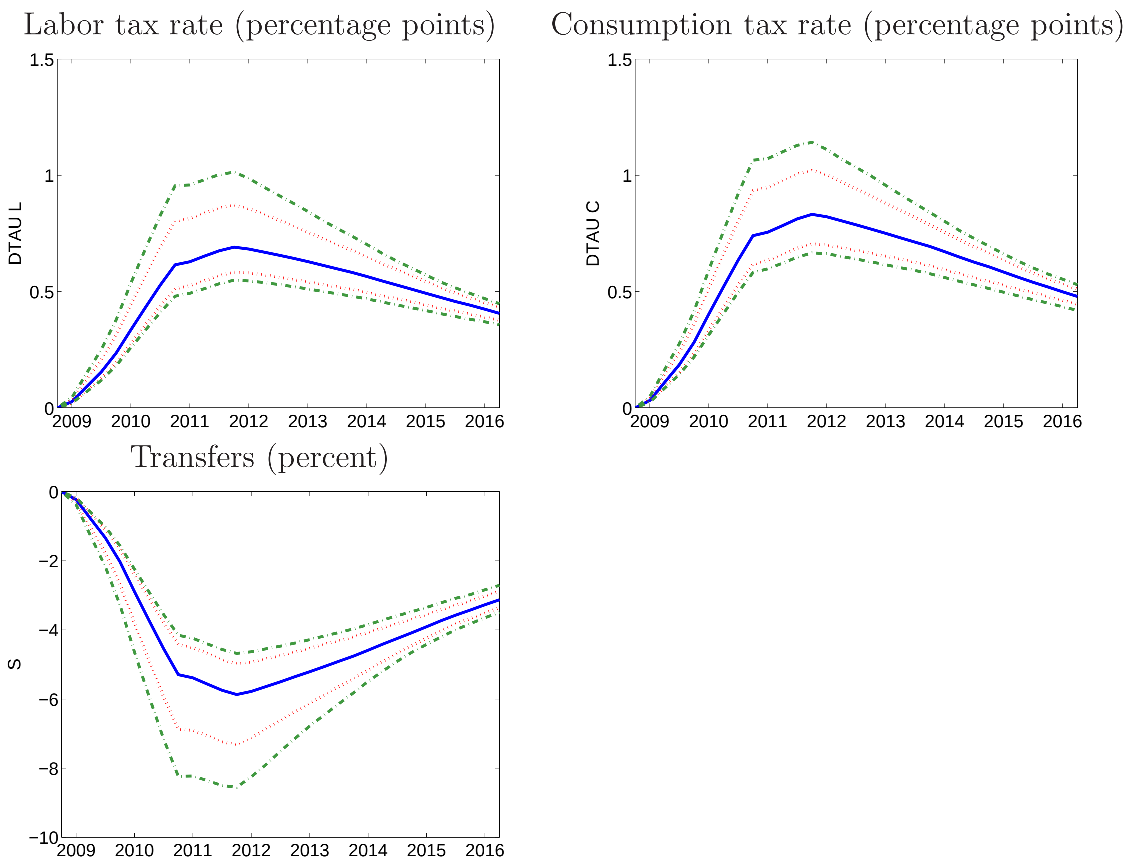


Figure D.10: Changes in tax rates and lump-sum transfers due to stimulus

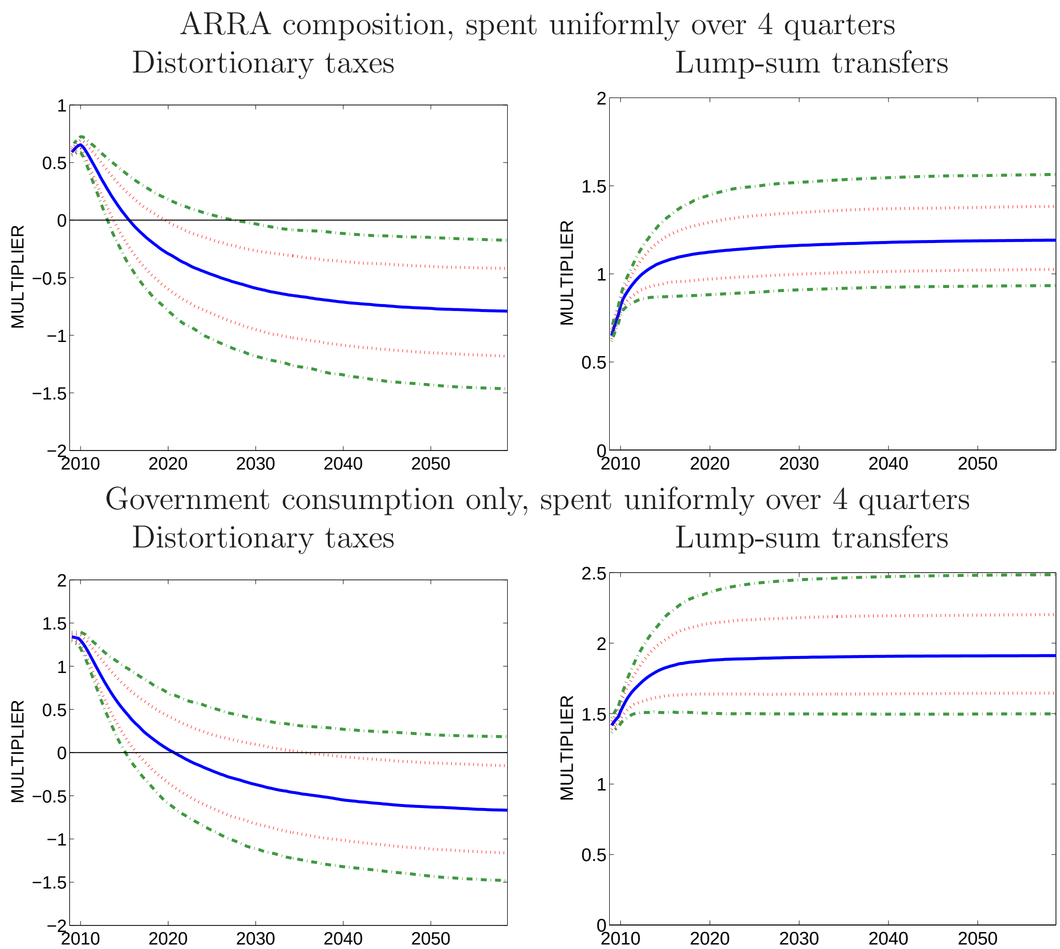


Figure D.11: Fiscal multipliers: Stimulus spent uniformly over first four quarters. Comparing labor taxes (benchmark) and lump-sum taxation

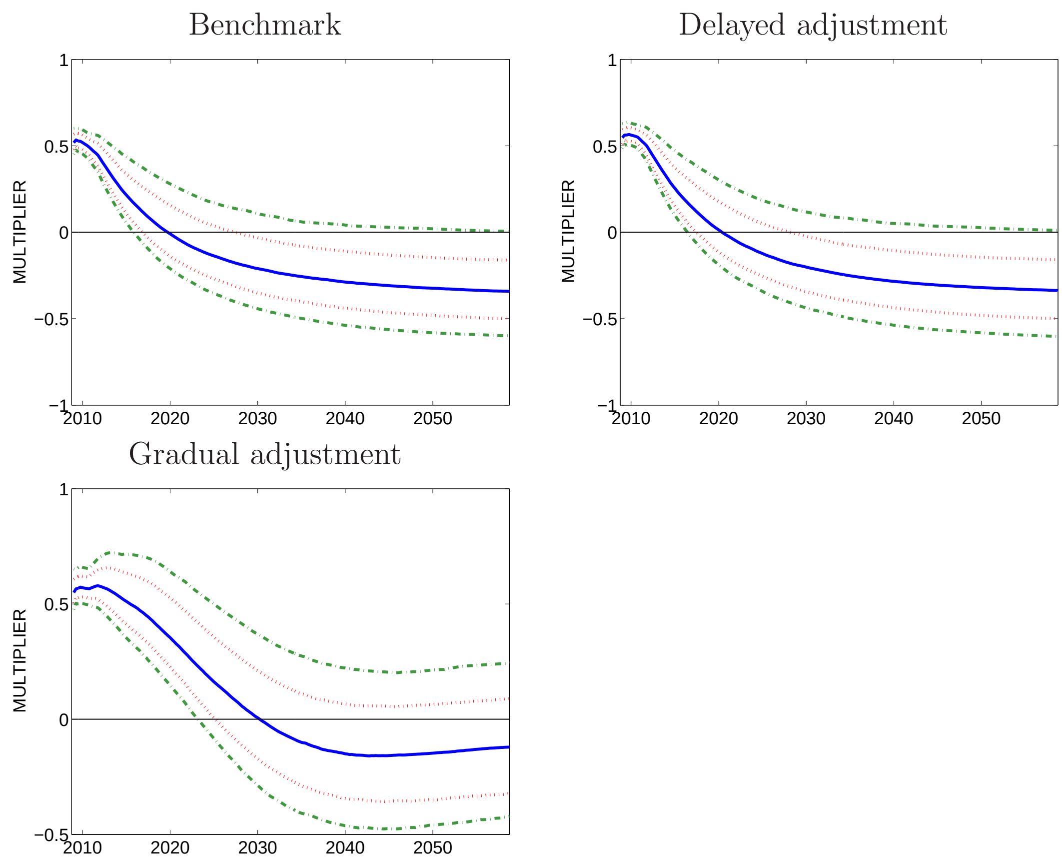


Figure D.12: Fiscal multipliers: Modified tax rules

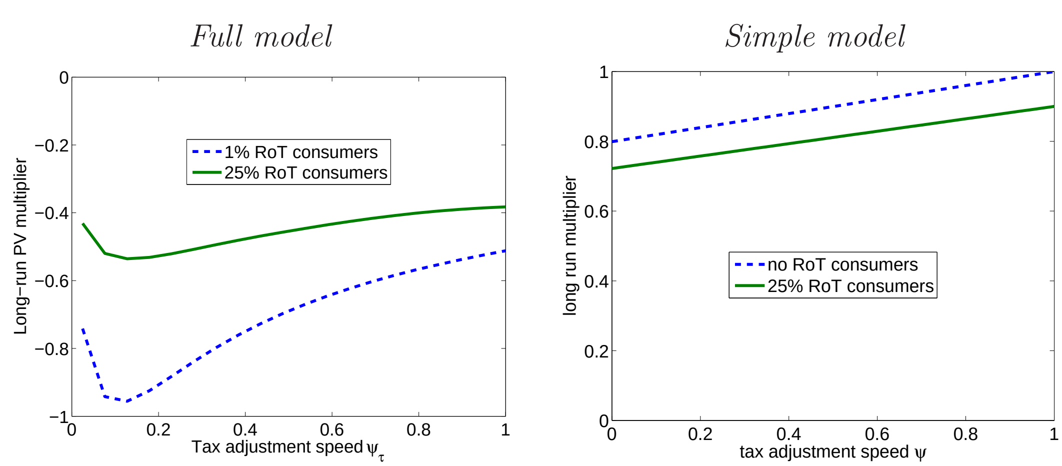


Figure D.13: Multipliers as a function of tax adjustment speed and rule-of-thumb consumers

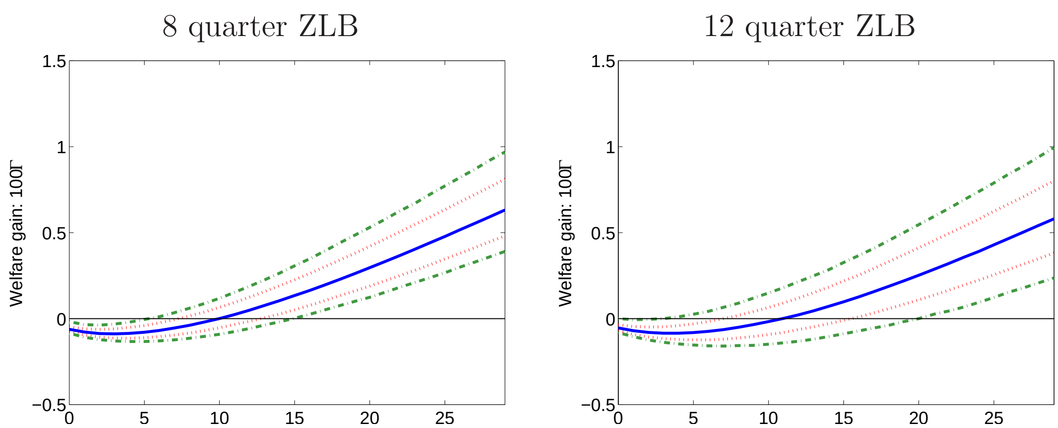


Figure D.14: Long-run welfare gains from stimulus: 8 and 12 quarter ZLB, varying annual rate of time preference as compared with unconstrained agents