1 Model environment

We consider a two-country model, Home and Foreign. Variables and parameters without an asterisk (or with a subscript h) refers to the Home country, and those with an asterisk (or with a subscript f), refers to the Foreign country. Time is discrete, and a time period is denoted by $t = 0, 1, 2, ...$ Agents in each economy exist on a continuum $[0, 1]$ and have a common discount factor $\beta \in (0, 1)$. Each period in time is composed of two arbitrary subperiods, night and day. At night, agents trade anonymously in decentralized markets (DM). During the day, agents trade in Walrasian centralized markets (CM). The nature of consumption, production, and trade in each market will be explained in detail in Sections 2.X and 2.X.

1.1 Centralized Markets (CM)

In Home CM, a household consumes a general goods $X \in \boldsymbol{R}_{+}$that is produced by firms. Households also trade a home-country currency nominated non-state-contingent bond b and make an investment decision $i = k - (1-\delta)k_{-}$, where we denote k as a capital stock and δ as a depreciation rate. Household maximizes expected lifetime discouted utility subject to her budget constraint (**??**)
.

[utility]

Let $W(\cdot)$ be the value function of a household at CM. The individual state variables are real money balance m_{-}, bond holding b_{-}, capital stock k_{-} and credit l. Let s denote a vector of the aggregate state variables.

Given $(m_{-}, b_{-}, k_{-}, l, \boldsymbol{s})$, the household problem is to choose $(X, H, m, k) \geq 0$ and $b \geq -\infty$ to maximize expected lifetime utility.

$$W(m_{-}, b_{-}, k_{-}, l, \boldsymbol{s}) = \max_{X,H,m,k,b} \{U(X) - AH + \beta \mathbb{E} V(m, b, k, 0, \boldsymbol{s}_{+})\}$$

subject to

$$(1+\tau_X) X + k - (1-\delta) k_{-} + \frac{m}{p_x} + \frac{l}{p_x} + \frac{Qb}{p_x} = (1-\tau_H) wH + (1-\tau_k) rk_{-} + \frac{b_{-}}{p_x} + \frac{m_{-}}{p_x} + T$$

The first order conditions and the envelope condisions in the CM are

$$
\begin{aligned}
H: \quad & A && = \lambda \left(1 - \tau_H\right) w \\
X: \quad & U_X && = \left(1 + \tau_X\right) \lambda \\
m_: \quad & \tfrac{\lambda}{p_x} && = \beta \mathbb{E} V_m \left(m, b, k, 0, \boldsymbol{s}_+\right) \\
b: \quad & \tfrac{Q\lambda}{p_x} && = \beta \mathbb{E} V_b \left(m, b, k, 0, \boldsymbol{s}_+\right) \\
k: \quad & \lambda && = \beta \mathbb{E} V_k \left(m, b, k, 0, \boldsymbol{s}_+\right)
\end{aligned}
$$

Envelop conditions

$$
\begin{aligned}
m_-: \quad & W_m \left(m_-, b_-, k_-, l, \boldsymbol{s}\right) && = \frac{\lambda}{p_x} \\
b_-: \quad & W_b \left(m_-, b_-, k_-, l, \boldsymbol{s}\right) && = \frac{\lambda}{p_x} \\
k_-: \quad & W_k \left(m_-, b_-, k_-, l, \boldsymbol{s}\right) && = \lambda \left[\left(1 - \tau_K\right) r + (1 - \delta)\right] \\
l: \quad & W_l \left(m_-, b_-, k_-, l, \boldsymbol{s}\right) && = -\frac{\lambda}{p_x}
\end{aligned}
$$

Firms

Let P_h be the Home currency price of the Home produced intermediate good and P_f be that of the Foreign produced intermediate good use by the Home final-good firm.

The Home final-good producer solves

$$
\max_{y_h, y_f} \left\{ p_x \cdot G\left(Y_h, Y_f\right) - P_h Y_h - P_f Y_f \right\}.
$$

Profit maximizing implies

$$
\begin{aligned}
P_h &= p_x \cdot G_{Yh}\left(Y_h, Y_f\right) \\
P_f &= p_x \cdot G_{Yf}\left(Y_h, Y_f\right)
\end{aligned}
$$

Home intermediate-goods producer sloves

$$
\max_{H, K} \left\{ P_h \cdot zF\left(K, H\right) - p_x \cdot \left[wH + rK\right] \right\}.
$$

The first-order conditions include

$$
\begin{aligned}
p_x \cdot r &= P_h \cdot zF_k(K, H) \\
p_x \cdot w &= P_h \cdot zF_H(K, H)
\end{aligned}
$$

1.2 Decentralized Markets with Competitive price taking

$$V(m_-, b_-, k_-, 0, \boldsymbol{s}) = \sigma V^B(m_-, b_-, k_-, 0, \boldsymbol{s}) + \sigma V^S(m_-, b_-, k_-, 0, \boldsymbol{s}) + (1 - 2\sigma) W(m_-, b_-, k_-, 0, \boldsymbol{s}),$$

where

$$
\begin{aligned}
V^B(m_-, b_-, k_-, 0, \boldsymbol{s}) &= \kappa \max_{q^b} \left[u\left(q^b\right) + W\left(m_- - pq^b, b_-, k_-, 0, s\right) \right] \\
&\quad + (1 - \kappa) \max_{\tilde{q}^b} \left[u\left(\tilde{q}^b\right) + W\left(m_-, b_-, k_-, l^b, s\right) \right], \\
V^S(m_-, b_-, k_-, 0, \boldsymbol{s}) &= \kappa \max_{q^s} \left[-c(q^s/z, k_-) + W(m_- + pq^s, b_-, k_-, 0, s) \right] \\
&\quad + (1 - \kappa) \max_{\tilde{q}^s} \left[-c(\tilde{q}^s/z, k_-) + W(m_-, b_-, k_-, -l^s, s) \right],
\end{aligned}
$$

The first-order conditions and the Envelope conditions for the DM are

$$
\begin{aligned}
\frac{A}{(1 - \tau_H) w} \frac{M}{p_x} &= \frac{1}{z} c_q(q/z, K_-) q \\
u_q(\tilde{q}) &= \frac{1}{z} c_q(\tilde{q}/z, K_-) \\
l &= u_q(\tilde{q}) \tilde{q} \cdot p_x \cdot \frac{(1 - \tau_H) w}{A}
\end{aligned}
$$

$$
\begin{aligned}
V_M(M_-, B_-, K_-, 0, \boldsymbol{s}) &= \frac{A}{(1 - \tau_H) w} \cdot \frac{1}{p_x} \left[(1 - \sigma\kappa) + \sigma\kappa \frac{z \cdot u_q(q)}{c_q(q/z, K_-)} \right] > 0 \\
V_B(M_-, B_-, K_-, 0, \boldsymbol{s}) &= W_B(M, B, K, 0, \boldsymbol{s}) = \frac{A}{(1 - \tau_H) w} \cdot \frac{1}{p_x} \\
V_K(M_-, B_-, K_-, 0, \boldsymbol{s}) &= -\sigma\kappa \cdot c_K(q/z, K_-) - \sigma(1 - \kappa) \cdot c_K(\tilde{q}/z, K_-) + W_K(M_-, B_-, K_-, 0, \boldsymbol{s})
\end{aligned}
$$

1.3 Government

Government follows a follwoing money supply rule,

$$M = \exp(\psi) M_-,$$

where $\exp(\psi) - 1$ is the one-period money supply growth rate.

Also she follows fiscal rules as in **?** which model the evolution of four policy instruments: government spending as a share of output, $\tilde{g}_t$, and tax rates on labor income,$\tau_{l,t}$, on capital income,$\tau_{k,t}$, and on consumption expenditure,$\tau_{c,t}$. For each instrument, the law of motion follows:

$$x_t - x = \rho_x (x_{t-1} - x) + \phi_{x,y} y_{t-1} + \phi_{x,b} \left(\frac{D_{t-1}}{Y_{t-1}} - \frac{D}{Y} \right) + exp(\sigma_x) \varepsilon_{x,t}$$

,where $\varepsilon_{x,t}$~N(0,1). for $x \in \{\tilde{g_t}, \tau_l, \tau_k, \tau_c\}$

Let us denote the tax revenue as Tax_t,

$$Tax_t \equiv \tau_x X + \tau_H w H + \tau_k r k.$$

Govenrment budget constraint is then given by

$$p_x \cdot G + D_{-1} = p_x \cdot Tax_t + Q \cdot D + (M - M_-),$$

1.4 Market clearings

Bond market clearing:

$$\frac{e p_x^*}{p_x} (B_- - D_-) + (B_-^* - D_-^*) = 0$$

Resource constraints:

$$G(Y_h, Y_f) = X + I + G$$
$$G\left(Y_f^*, Y_h^*\right) = X^* + I^* + G^*$$

Market clearing for the intermediate goods:

$$zF(K, H) = Y_h + Y_h^*$$
$$z^* F(K^*, H^*) = Y_f^* + Y_f$$

1.5 Other Variable Definitions

$$
\begin{aligned}
p_{DM} &:= \kappa p_q + (1-\kappa)\,\tilde{p}_q \\
q_{DM} &:= \kappa q + (1-\kappa)\,\tilde{q} \\
P_Y &:= \xi p_x + (1-\xi)\,p_{DM} \\
\xi &:= \frac{X}{X+\sigma q_{DM}} \\
RER &:= \frac{e\cdot P_Y^*}{P_Y} \\
\hat{p}^* &= \frac{p_x^*}{p_x}
\end{aligned}
$$

2 Characterization of Stationary Monetary Equilibrium

We starionize the nominal variables by $p_x$and p_x^*: $\hat{m} \equiv \frac{m}{p_x}, \hat{b} \equiv \frac{b}{p_x}, \hat{d} \equiv \frac{d}{p_x}, \hat{P}_h \equiv \frac{P_h}{p_x}, \hat{P}_f \equiv \frac{P_f}{p_x}$, $\hat{P}_f^* \equiv \frac{P_f^*}{p_x^*}, \hat{P}_h^* \equiv \frac{P_h^*}{p_x^*}$, $\pi_+ \equiv \frac{p_{x+}}{p_x}$, $\pi_+^* \equiv \frac{p_{x+}^*}{p_x^*}, \hat{p}^* = \frac{p_x^*}{p_x}$

Households: $\lambda, \lambda^*, X, X^*, \hat{m}_+ \hat{m}_+^*, b_+, b_+^*, k_+, k_+^*$ and another set for 6 foreign variables.

$$A = \lambda(1-\tau_H)w \tag{2.1}$$

$$A^* = \lambda^*(1-\tau_H^*)w^* \tag{2.2}$$

$$U_X = (1+\tau_X)\lambda \tag{2.3}$$

$$U_{X^*} = (1+\tau_X^*)\lambda^* \tag{2.4}$$

$$\lambda = \beta E\left[\lambda_+ \frac{1}{\pi_+}\left[(1-\sigma\kappa) + \sigma\kappa\frac{z_+ u_q\left(q_+\right)}{c_q\left(q_+/z_+, K_+\right)}\right]\right] \tag{2.5}$$

$$\lambda^* = \beta E\left[\lambda_+^* \frac{1}{\pi_+^*}\left[(1-\sigma\kappa) + \sigma\kappa\frac{z_+^* u_q\left(q_+^*\right)}{c_q\left(q_+^*/z_+^*, K_+^*\right)}\right]\right] \tag{2.6}$$

$$Q\cdot\lambda = \beta E\left[\lambda_+ \frac{1}{\pi_+}\right] \tag{2.7}$$

$$Q\cdot\lambda^* = \beta E\left[\lambda_+^* \frac{1}{\pi_+^*}\frac{e_+}{e}\right] \tag{2.8}$$

$$\lambda = \beta E\left[\lambda_+\left[1-\delta+(1-\tau_k)r_+\right] - \sigma\kappa c_K\left(q_+/z_+, K_+\right) - \sigma\left(1-\kappa\right)c_K\left(\tilde{q}_+/z_+, K_+\right)\right] \tag{2.9}$$

$$\lambda^* = \beta E\left[\lambda_+^*\left[1-\delta+(1-\tau_k^*)r_+^*\right] - \sigma\kappa c_K\left(q_+^*/z_+^*, K_+^*\right) - \sigma\left(1-\kappa\right)c_K\left(\tilde{q}_+^*/z_+^*, K_+^*\right)\right] \tag{2.10}$$

Firms: $Y, Y_h, Y_f, r, w, Y^*, Y_h^*, Y_f^*, r^*, w^*$(10 variables)

$$Y = G\left(Y_h, Y_f\right) \tag{2.11}$$

$$\hat{P}_h = G_1\left(Y_h, Y_f\right) \tag{2.12}$$

$$\hat{P}_f = G_2\left(Y_h, Y_f\right) \tag{2.13}$$

$$r = \hat{P}_h z F_1\left(K, H\right) \tag{2.14}$$

$$w = \hat{P}_h z F_2\left(K, H\right) \tag{2.15}$$

$$Y^* = G\left(Y_f^*, Y_h^*\right) \tag{2.16}$$

$$\hat{P}_f^* = G_1\left(Y_f^*, Y_h^*\right) \tag{2.17}$$

$$\hat{P}_h^* = G_2\left(Y_f^*, Y_h^*\right) \tag{2.18}$$

$$r^* = \hat{P}_f^* z^* F_1\left(K^*, H^*\right) \tag{2.19}$$

$$w^* = \hat{P}_f^* z^* F_2\left(K^*, H^*\right) \tag{2.20}$$

DM Competitive Pricing and Optinal Decisions q,$\tilde{q}$,l,p_q,$p_{\tilde{q}}$,q^*,$\tilde{q}^*$,l^*,p_q^*,$p_{\tilde{q}}^*$,(10 variables)

$$\frac{A}{\left(1-\tau_H\right)w}\hat{m} = \frac{1}{z}c_q\left(q/z, K\right)q \tag{2.21}$$

$$u_q(\tilde{q}) = \frac{1}{z}c_q\left(\tilde{q}/z, K\right) \tag{2.22}$$

$$p_q = \frac{\hat{m}}{q} \tag{2.23}$$

$$p_{\tilde{q}} = \frac{\hat{l}}{\tilde{q}} \tag{2.24}$$

$$\hat{l} = \frac{\left(1-\tau_H\right)w}{A}u_q\left(\tilde{q}\right)\tilde{q} \tag{2.25}$$

$$\frac{A^*}{\left(1-\tau_H^*\right)w}\hat{m}^* = \frac{1}{z^*}c_q\left(q^*/z^*, K^*\right)q^* \tag{2.26}$$

$$u_q(\tilde{q}^*) = \frac{1}{z^*}c_q\left(\tilde{q}^*/z^*, K^*\right) \tag{2.27}$$

$$p_q^* = \frac{\hat{m}^*}{q^*} \tag{2.28}$$

$$p_{\tilde{q}}^* = \frac{\hat{l}^*}{\tilde{q}^*} \tag{2.29}$$

$$\hat{l}^* = \frac{\left(1-\tau_H^*\right)w^*}{A^*}u_q\left(\tilde{q}^*\right)\tilde{q}^* \tag{2.30}$$

Resources $X, X^*, P_h, P_f^*, I, I^*$**(6 variables)**

$$Y = X + I + G \tag{2.31}$$

$$Y^* = X^* + I^* + G^* \tag{2.32}$$

$$K_+ = (1 - \delta) K + I \tag{2.33}$$

$$K_+^* = (1 - \delta) K^* + I^* \tag{2.34}$$

$$zF(K, H) = Y_h + Y_h^* \tag{2.35}$$

$$z^* F(K^*, H^*) = Y_f^* + Y_f \tag{2.36}$$

Law of one price P_f, P_h^*

$$\hat{P}_f = \hat{e} \cdot \hat{P}_f^* \tag{2.37}$$

$$\hat{P}_h = \hat{e} \cdot \hat{P}_h^* \tag{2.38}$$

Bonds market e

$$\left(\hat{b}_- - \hat{d}_-\right) + \hat{e} \cdot \left(\hat{b}_-^* - \hat{d}_-^*\right) = 0 \tag{2.39}$$

Governments $\hat{d}, \hat{d}^*, TAX, TAX^*$

$$\tilde{g} = G/Y \tag{2.40}$$

$$\tilde{g}^* = G^*/Y^* \tag{2.41}$$

$$\tilde{g}Y + \hat{d}_{-1} = TAX + Q \cdot \hat{d} \cdot \pi_+ + \hat{m} \cdot \pi_+ - \hat{m}_- \tag{2.42}$$

$$\tilde{g}^* Y^* + \hat{d}_{-1}^* = TAX^* + Q^* \cdot \hat{d}^* \cdot \pi_+^* + \hat{m}^* \cdot \pi_+^* - \hat{m}_-^* \tag{2.43}$$

$$TAX = \tau_X X + \tau_H wH + \tau_k rk \tag{2.44}$$

$$TAX^* = \tau_X^* X^* + \tau_H^* w^* H^* + \tau_k^* r^* k^* \tag{2.45}$$

other auxilary variables $\hat{p}_{DM}, q_{DM}, \hat{P}_Y, \xi, \hat{p}_{DM}^*, q_{DM}^*, \hat{P}_Y^*, \xi^*, RER$ (9 variables)

$$\hat{p}_{DM} := \kappa\hat{p} + (1-\kappa)\,\hat{\tilde{p}} \tag{2.46}$$

$$q_{DM} := \kappa q + (1-\kappa)\,\tilde{q} \tag{2.47}$$

$$\xi := \frac{X}{X+\sigma q_{DM}} \tag{2.48}$$

$$\hat{P}_Y := \xi + (1-\xi)\,\hat{p}_{DM} \tag{2.49}$$

$$\hat{p}^*_{DM} := \kappa\hat{p}^* + (1-\kappa)\,\hat{\tilde{p}}^* \tag{2.50}$$

$$q^*_{DM} := \kappa q^* + (1-\kappa)\,\tilde{q}^* \tag{2.51}$$

$$\xi^* := \frac{X^*}{X^*+\sigma q^*_{DM}} \tag{2.52}$$

$$\hat{P}^*_Y := \xi + (1-\xi)\,\hat{p}^*_{DM} \tag{2.53}$$

$$RER :\equiv e \cdot \frac{\hat{P}^*_Y}{\hat{P}_Y} \cdot \hat{p}^* \tag{2.54}$$

Money supply:

$$\hat{m} \cdot \pi_+ = \exp(\psi)\,\hat{m}_-, \tag{2.55}$$

$$\hat{m}^* \cdot \pi^*_+ = \exp(\psi^*)\,\hat{m_-}^*, \tag{2.56}$$

change in variables $\tilde{d}, \tilde{d}^*, \tilde{y}, \tilde{y}^*$

$$\tilde{d} = d/Y \tag{2.57}$$

$$\tilde{d}^* = d^*/Y^* \tag{2.58}$$

$$\tilde{y} = y/\bar{y} \tag{2.59}$$

$$\tilde{y}^* = y^*/\bar{y}^* \tag{2.60}$$

Exogenous variables

$$\psi = \rho_\psi \psi_- + \sigma_\psi \varepsilon_\psi \tag{2.61}$$

$$\psi^* = \rho_\psi^* \psi_-^* + \sigma_\psi^* \varepsilon_\psi^* \tag{2.62}$$

$$\log z = \rho_z \log z_- + \sigma_z \varepsilon_z \tag{2.63}$$

$$\log z^* = \rho_z^* \log z_-^* + \sigma_z^* \varepsilon_z^* \tag{2.64}$$

$$\tilde{g} - \bar{\tilde{g}} = \rho_{\tilde{g}} \left(\tilde{g}_- - \bar{\tilde{g}}\right) + \phi_{\tilde{g},y} \cdot \tilde{y}_- + \phi_{\tilde{g},d} \left(\tilde{d}_- - \bar{\tilde{d}}\right) + \exp\left(\sigma_{\tilde{g}}\right) \varepsilon_{\tilde{g},t} \tag{2.65}$$

$$\tau_l - \bar{\tau}_l = \rho_{\tau_l} \left(\tau_{l-} - \bar{\tau}_l\right) + \phi_{\tau_l,y} \cdot \tilde{y}_- + \phi_{\tau_l,d} \left(\tilde{d}_- - \bar{\tilde{d}}\right) + \exp\left(\sigma_{\tau_l}\right) \varepsilon_{\tau_l,t} \tag{2.66}$$

$$\tau_k - \bar{\tau}_k = \rho_{\tau_k} \left(\tau_{k-} - \bar{\tau}_k\right) + \phi_{\tau_k,y} \cdot \tilde{y}_- + \phi_{\tau_k,d} \left(\tilde{d}_- - \bar{\tilde{d}}\right) + \exp\left(\sigma_{\tau_k}\right) \varepsilon_{\tau_k,t} \tag{2.67}$$

$$\tau_c - \bar{\tau}_c = \rho_{\tau_c} \left(\tau_{c-} - \bar{\tau}_c\right) + \phi_{\tau_c,y} \cdot \tilde{y}_- + \phi_{\tau_c,d} \left(\tilde{d}_- - \bar{\tilde{d}}\right) + \exp\left(\sigma_{\tau_c}\right) \varepsilon_{\tau_c,t} \tag{2.68}$$

$$\tilde{g}^* - \bar{\tilde{g}}^* = \rho_{\tilde{g}}^* \left(\tilde{g}_-^* - \bar{\tilde{g}}^*\right) + \phi_{\tilde{g},y}^* \cdot \tilde{y}_-^* + \phi_{\tilde{g},d}^* \left(\tilde{d}_-^* - \bar{\tilde{d}}^*\right) + \exp\left(\sigma_{\tilde{g}}^*\right) \varepsilon_{\tilde{g},t}^* \tag{2.69}$$

$$\tau_l^* - \bar{\tau}_l{}^* = \rho_{\tau_l}^* \left(\tau_l^*{}_- - \bar{\tau}_l^*\right) + \phi_{\tau_l,y}^* \cdot \tilde{y}_-^* + \phi_{\tau_l,d}^* \left(\tilde{d}_-^* - \bar{\tilde{d}}^*\right) + \exp\left(\sigma_{\tau_l}^*\right) \varepsilon_{\tau_l,t}^* \tag{2.70}$$

$$\tau_k^* - \bar{\tau}_k^* = \rho_{\tau_k}^* \left(\tau_k^*{}_- - \bar{\tau}_k^*\right) + \phi_{\tau_k,y}^* \cdot \tilde{y}_-^* + \phi_{\tau_k,d}^* \left(\tilde{d}_-^* - \bar{\tilde{d}}^*\right) + \exp\left(\sigma_{\tau_k}^*\right) \varepsilon_{\tau_k,t}^* \tag{2.71}$$

$$\tau_c^* - \bar{\tau}_c{}^* = \rho_{\tau_c}^* \left(\tau_c^*{}_- - \bar{\tau}_c{}^*\right) + \phi_{\tau_c,y}^* \cdot \tilde{y}_-^* + \phi_{\tau_c,d}^* \left(\tilde{d}_-^* - \bar{\tilde{d}}^*\right) + \exp\left(\sigma_{\tau_c}^*\right) \varepsilon_{\tau_c,t}^* \tag{2.72}$$

3 Numerical Exercise

Preference in CM

$$U(X) = B\frac{X^{1-\gamma} - 1}{1-\gamma}, U'(X) = B \cdot X^{-\gamma}$$

Preference in DM

$$u(q) = C\frac{(q + \underline{q})^{1-\eta} - b^{1-\eta}}{1-\eta}, u'(q) = C \cdot (q + \underline{q})^{-\eta}$$

Production Technology in CM

$$zF(K, H) = zK^\alpha H^{1-\alpha}$$

Final good production

$$G\left(y_h, y_f\right) = \left[\vartheta \left(y_h\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(y_f\right)^{\frac{1}{\epsilon}}\right]^\epsilon$$

$$G\left(y_f^*, y_h^*\right) = \left[\vartheta \left(y_f^*\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(y_h^*\right)^{\frac{1}{\epsilon}}\right]^\epsilon$$

Figure 1: Parameters

table1.pdf

Production in DM

$$c\left(q,K\right)=q^{\omega}K^{1-\omega},c_{q}\left(q,K\right)=\omega q^{\omega-1}K^{1-\omega},c_{K}\left(q,K\right)=\left(1-\omega\right)q^{\omega}K^{-\omega}.$$

4 Steady State

45 endogenous variables: $\bar{X},\bar{X}^{*},\bar{H},\bar{H}^{*},\bar{\hat{m}}_{+}//\bar{\hat{m}}_{+}^{*},\bar{b}_{+},\bar{b}_{+}^{*},\bar{k}_{+},\bar{k}_{+}^{*},//\bar{Y},\bar{Y}_{h},\bar{Y}_{f},\bar{r},\bar{w},//\bar{Y}^{*},\bar{Y}_{h}^{*},\bar{Y}_{f}^{*},\bar{r}^{*},\bar{w}^{*}/$ $\bar{\pi},\bar{\pi}^{*},\bar{P}_{h},\bar{P}_{f}^{*},\bar{P}_{f},//\bar{P}_{h}^{*},\bar{e},\bar{d},\bar{d}^{*},T\bar{A}X,//T\bar{A}X^{*},\bar{p}_{DM},\bar{q}_{DM},\bar{P}_{Y},\bar{\xi},//R\bar{E}R,\bar{\tilde{g}},\bar{\tilde{g}}^{*},\bar{\bar{d}},\bar{\bar{d}}^{*}//,\bar{\tilde{y}},\bar{\tilde{y}}^{*},q,\tilde{q},\tilde{l}$
12 exogenous variables: ψ, ψ^{*}, z, z^{*}, $\tilde{g}$, $\tilde{g}^{*}$, τ_{l}, τ_{l}^{*}, τ_{c}, τ_{c}^{*}, τ_{k}, τ_{k}^{*}

$$B\cdot X^{-\gamma}=\frac{A\left(1+\tau_{X}\right)}{\left(1-\tau_{H}\right)w} \tag{4.1}$$

$$B\cdot X^{*-\gamma}=\frac{A^{*}\left(1+\tau_{X}^{*}\right)}{\left(1-\tau_{H}^{*}\right)w^{*}} \tag{4.2}$$

$$1=\beta\left[\frac{1}{\pi}\left[\left(1-\sigma\kappa\right)+\sigma\kappa\frac{C\cdot\left(q+\underline{q}\right)^{-\eta}}{\omega\left(K/q\right)^{1-\omega}}\right]\right] \tag{4.3}$$

$$1=\beta\left[\frac{1}{\pi^{*}}\left[\left(1-\sigma\kappa\right)+\sigma\kappa\frac{C\cdot\left(q^{*}+\underline{q}\right)^{-\eta}}{\omega\left(K^{*}/q^{*}\right)^{1-\omega}}\right]\right] \tag{4.4}$$

$$Q=\beta\left[\frac{1}{\pi}\right] \tag{4.5}$$

$$Q=\beta\left[\frac{1}{\pi^{*}}\right] \tag{4.6}$$

$$U_{X}=\beta E\left[U_{X}\left[1+\left(1-\tau_{k}\right)\left(r-\delta\right)\right]-\sigma\kappa\left(1-\omega\right)\left(q/K\right)^{\omega}-\sigma\left(1-\kappa\right)\left(1-\omega\right)\left(\tilde{q}/K\right)^{\omega}\right] \tag{4.7}$$

$$U_{X}^{*}=\beta E\left[U_{X}^{*}\left[1+\left(1-\tau_{k}^{*}\right)\left(r^{*}-\delta\right)\right]-\sigma\kappa\left(1-\omega\right)\left(q^{*}/K^{*}\right)^{\omega}-\sigma\left(1-\kappa\right)\left(1-\omega\right)\left(\tilde{q}^{*}/K^{*}\right)^{\omega}\right] \tag{4.8}$$

$$Y = \left[\vartheta \left(Y_h\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(Y_f\right)^{\frac{1}{\epsilon}}\right]^{\epsilon} \tag{4.9}$$

$$\hat{P}_h = \vartheta \left[\vartheta \left(Y_h\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(Y_f\right)^{\frac{1}{\epsilon}}\right]^{\epsilon-1} \left(Y_h\right)^{\frac{1}{\epsilon}-1} \tag{4.10}$$

$$\hat{P}_f = (1-\vartheta) \left[\vartheta \left(Y_h\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(Y_f\right)^{\frac{1}{\epsilon}}\right]^{\epsilon-1} \left(Y_f\right)^{\frac{1}{\epsilon}-1} \tag{4.11}$$

$$r = \hat{P}_h z \alpha K^{\alpha-1} H^{1-\alpha} \tag{4.12}$$

$$w = \hat{P}_h z \left(1-\alpha\right) K^{\alpha} H^{-\alpha} \tag{4.13}$$

$$Y^* = \left[\vartheta \left(Y_f^*\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(Y_h^*\right)^{\frac{1}{\epsilon}}\right]^{\epsilon} \tag{4.14}$$

$$\hat{P}_f^* = \vartheta \left[\vartheta \left(Y_f^*\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(Y_h^*\right)^{\frac{1}{\epsilon}}\right]^{\epsilon-1} \left(Y_f^*\right)^{\frac{1}{\epsilon}-1} \tag{4.15}$$

$$\hat{P}_h^* = (1-\vartheta) \left[\vartheta \left(Y_f^*\right)^{\frac{1}{\epsilon}} + (1-\vartheta)\left(Y_h^*\right)^{\frac{1}{\epsilon}}\right]^{\epsilon-1} \left(Y_h^*\right)^{\frac{1}{\epsilon}-1} \tag{4.16}$$

$$r^* = \hat{P}_f^* z^* \alpha \left(K^*\right)^{\alpha-1} \left(H^*\right)^{1-\alpha} \tag{4.17}$$

$$w^* = \hat{P}_f^* z^* \left(1-\alpha\right) \left(K^*\right)^{\alpha} \left(H^*\right)^{-\alpha} \tag{4.18}$$

$$\frac{A\hat{m}}{\left(1-\tau_H\right) w} = \frac{1}{z}\omega \left(q/z\right)^{\omega-1} \left(K\right)^{1-\omega} q \tag{4.19}$$

$$\frac{A\tilde{l}}{\left(1-\tau_H\right) w} = \frac{1}{z}\omega \left(\tilde{q}/z\right)^{\omega-1} \left(K\right)^{1-\omega} \tilde{q} \tag{4.20}$$

$$\hat{p} = \hat{m}/q \tag{4.21}$$

$$\hat{\tilde{p}} = \hat{l}/\tilde{q} \tag{4.22}$$

$$\tilde{l} = \frac{\left(1-\tau_H\right) w}{A} C \cdot \left(\tilde{q} + \underline{q}\right)^{-\eta} \tilde{q} \tag{4.23}$$

$$\frac{A^*\hat{m}^*}{\left(1-\tau_H^*\right) w^*} = \frac{1}{z^*}\omega \left(q^*/z^*\right)^{\omega-1} \left(K^*\right)^{1-\omega} q^* \tag{4.24}$$

$$\frac{A\tilde{l}^*}{\left(1-\tau_H^*\right) w^*} = \frac{1}{z^*}\omega \left(\tilde{q}^*/z^*\right)^{\omega-1} \left(K^*\right)^{1-\omega} \tilde{q}^* \tag{4.25}$$

$$\hat{p}^* = \hat{m}^*/q^* \tag{4.26}$$

$$\hat{\tilde{p}}^* = \hat{l}^*/\tilde{q}^* \tag{4.27}$$

$$\tilde{l}^* = \frac{\left(1-\tau_H^*\right) w^*}{A^*} C \cdot \left(\tilde{q}^* + \underline{q}\right)^{-\eta} \tilde{q}^* \tag{4.28}$$

$$Y = X + I + G \tag{4.29}$$
$$Y^* = X^* + I^* + G^* \tag{4.30}$$
$$zK^\alpha H^{1-\alpha} = Y_h + Y_h^* \tag{4.31}$$
$$z^* (K^*)^\alpha (H^*)^{1-\alpha} = Y_f^* + Y_f \tag{4.32}$$

$$\hat{P}_f = \hat{P}_f^* \tag{4.33}$$
$$\hat{P}_h = \hat{P}_h^* \tag{4.34}$$

$$b_- + d_- + b_-^* + d_-^* = 0 \tag{4.35}$$

$$G + \hat{d}_{-1} = TAX + Q\hat{d}\pi + \hat{m} \cdot \pi - \hat{m}_- \tag{4.36}$$
$$G^* + \hat{d}_{-1}^* = TAX^* + Q^*\hat{d}^*\pi^* + +\hat{m}^* \cdot \pi^* - \hat{m}_-^* \tag{4.37}$$
$$TAX = \tau_X X + \tau_H wH + \tau_k rk \tag{4.38}$$
$$TAX^* = \tau_X^* X^* + \tau_H^* w^* H^* + \tau_k^* r^* k^* \tag{4.39}$$

$$\hat{p}_{DM} := \kappa\hat{p} + (1-\kappa)\,\hat{\tilde{p}} \tag{4.40}$$
$$q_{DM} := \kappa q + (1-\kappa)\,\tilde{q} \tag{4.41}$$
$$\xi := \frac{X}{X + \sigma q_{DM}} \tag{4.42}$$
$$\hat{P}_Y := \xi + (1-\xi)\,\hat{p}_{DM} \tag{4.43}$$
$$\hat{p}_{DM}^* := \kappa\hat{p}^* + (1-\kappa)\,\hat{\tilde{p}}^* \tag{4.44}$$
$$q_{DM}^* := \kappa q^* + (1-\kappa)\,\tilde{q}^* \tag{4.45}$$
$$\xi^* := \frac{X^*}{X^* + \sigma q_{DM}^*} \tag{4.46}$$
$$\hat{P}_Y^* := \xi + (1-\xi)\,\hat{p}_{DM}^* \tag{4.47}$$
$$RER := \frac{\hat{P}_Y^*}{\hat{P}_Y} \tag{4.48}$$

$$\hat{m}_+ \pi_+ = \exp(\psi)\, \hat{m}, \tag{4.49}$$

$$\hat{m}^*_+ \pi^*_+ = \exp(\psi^*)\, \hat{m}^*, \tag{4.50}$$

$$\tilde{g} = G/Y \tag{4.51}$$

$$\tilde{g}^* = G^*/Y^* \tag{4.52}$$

$$\tilde{d} = d/Y \tag{4.53}$$

$$\tilde{d}^* = d^*/Y^* \tag{4.54}$$

$$\tilde{y} = y/\bar{Y} \tag{4.55}$$

$$\tilde{y}^* = y^*/\bar{Y}^* \tag{4.56}$$

References