# Two agents, two sectors

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## Model

### Household A

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( log(c_{AA,t}) + a_{j,t} log(c_{AB,t}) - \frac{n_{A,t}^{1+\nu_A}}{1+\nu_A} \right) \tag{1}$$

$$P_{A,t}c_{AA,t} + P_{B,t}c_{AB,t} + P_{A,t}b_t = P_{A,t-1}R_{t-1}b_{t-1} + P_{A,t}w_{A,t}\, n_{A,t} \tag{2}$$

In real terms:

$$c_{AA,t} + p_{B,t}c_{AB,t} + b_t = \frac{R_{t-1}}{\Pi_{A,t}} b_{t-1} + w_{A,t}\, n_{A,t} \tag{3}$$

Euler equation:

$$\frac{1}{c_{AA,t}} = \frac{\beta R_t}{\Pi_{A,t+1}} E_t \frac{1}{c_{AA,t+1}} \tag{4}$$

Intratemporal condition:

$$\frac{a_{j,t}}{c_{AB,t}} = \frac{1}{c_{AA,t}} p_{B,t} \tag{5}$$

Labor supply:

$$n_{A,t}^{\nu_A} = w_{A,t} \frac{1}{c_{AA,t}} \tag{6}$$

Bonds are in zero supply:

$$b_t = 0 \tag{7}$$

### Household B

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( log(c_{BA,t}) + a_{j,t} log(c_{BB,t}) - \frac{n_{B,t}^{1+\nu_B}}{1+\nu_B} \right) \tag{8}$$

$$P_{A,t}c_{BA,t} + P_{B,t}c_{BB,t} + P_{A,t}b'_t = P_{A,t-1}R_{t-1}b'_{t-1} + P_{B,t}w_{B,t}\, n_{B,t} \tag{9}$$

In real terms:

$$c_{BA,t} + p_{B,t} c_{BB,t} + b'_t = \frac{R_{t-1}}{\Pi_{A,t}} b'_{t-1} + p_{B,t} w_{B,t} n_{B,t} \tag{10}$$

Euler equation:

$$\frac{1}{c_{BA,t}} = \frac{\beta R_t}{\Pi_{A,t+1}} E_t \frac{1}{c_{BA,t+1}} \tag{11}$$

Intratemporal condition:

$$\frac{a_{j,t}}{c_{BB,t}} = \frac{1}{c_{BA,t}} p_{B,t} \tag{12}$$

Labor supply:

$$n_{B,t}^{\nu_B} = w_{B,t} \frac{a_{j,t}}{c_{BB,t}} \tag{13}$$

Bonds are in zero supply:

$$b'_t = 0 \tag{14}$$

## Production

Technology:

$$Y_{A,t} = n_{A,t} \tag{15}$$

$$Y_{B,t} = n_{B,t} \tag{16}$$

Labor demand:

$$\frac{Y_{A,t}}{n_{A,t}} = w_{A,t} \tag{17}$$

$$\frac{Y_{B,t}}{n_{B,t}} = w_{B,t} \tag{18}$$

## Equilibrium

Taylor rule:

$$R_t = R_{t-1}^{r_R} \Pi_t{}^{(1-r_R)(r_P)} \left(\frac{1}{\beta}\right)^{(1-r_R)} \varepsilon_{e,t}$$

(19)

Aggregate inflation:

$$\Pi_t = \Pi_{A,t}^{\frac{Y_{A,t}}{Y_{A,t}+p_{B,t}Y_{B,t}}} \Pi_{B,t}^{\frac{p_{B,t}Y_{B,t}}{Y_{A,t}+p_{B,t}Y_{B,t}}} \tag{20}$$

Relative price evolution:

$$\frac{p_{B,t}}{p_{B,t-1}} = \frac{\Pi_{B,t}}{\Pi_{A,t}} \tag{21}$$

Market clearing:

$$c_{AA,t} + c_{BA,t} = Y_{A,t} \tag{22}$$

$$c_{AB,t} + c_{BB,t} = Y_{B,t} \tag{23}$$