

$$-tb_t = y_t - c_t - i_t - g_t$$

$$\text{Ratio}_t = \frac{tb_t}{y_t} = 1 - \frac{c_t}{y_t} - \frac{i_t}{y_t} - \frac{g_t}{y_t}$$

log-linearize this side

$$\therefore \text{Ratio}_t = 1 - \frac{\bar{c}}{\bar{y}} (1 + \tilde{c}_t - \tilde{y}_t) - \frac{\bar{i}}{\bar{y}} (1 + \tilde{i}_t - \tilde{y}_t) - \frac{\bar{g}}{\bar{y}} (1 + \tilde{g}_t - \tilde{y}_t)$$

$$= 1 - \frac{\bar{c}}{\bar{y}} - \frac{\bar{i}}{\bar{y}} - \frac{\bar{g}}{\bar{y}} - \frac{\bar{c}}{\bar{y}} \tilde{c}_t - \frac{\bar{i}}{\bar{y}} \tilde{i}_t - \frac{\bar{g}}{\bar{y}} \tilde{g}_t$$

$$+ \left(\frac{\bar{c}}{\bar{y}} + \frac{\bar{i}}{\bar{y}} + \frac{\bar{g}}{\bar{y}} \right) \tilde{y}_t \quad (\tilde{\cdot} \text{ means log-linearized})$$

steady state: $\overline{\text{Ratio}} = 1 - \frac{\bar{c}}{\bar{y}} - \frac{\bar{i}}{\bar{y}} - \frac{\bar{g}}{\bar{y}}$

$$\therefore \text{Ratio}_t - \overline{\text{Ratio}} = (1 - \overline{\text{Ratio}}) \tilde{y}_t - \frac{\bar{c}}{\bar{y}} \tilde{c}_t - \frac{\bar{i}}{\bar{y}} \tilde{i}_t - \frac{\bar{g}}{\bar{y}} \tilde{g}_t$$

$\overline{\text{Ratio}}$ is average of $\frac{\text{Trade balance}}{\text{Output}}$ ratio

model (Linear);

$$\frac{tb_t}{y_t} \text{ obs} = \text{Ratio}_t$$

↓
original $\frac{\text{Trade balance}}{\text{output}}$ data.