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## Monetary policy and the exchange rate: Evaluation of VAR models

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### A B S T R A C T

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This paper examines the ability of vector autoregressive (VAR) models to properly identify the transmission of monetary policy in a controlled experiment. Simulating data from a small open economy DSGE model estimated for Australia, we find that sign-restricted VAR models do reasonably well at estimating the responses of macroeconomic variables to monetary policy shocks. This is in contrast to models that use recursive zero-type restrictions, for which inflation can rise following an unexpected interest rate increase while the exchange rate can appreciate or depreciate depending on the ordering of the variables. Sign-restricted VAR models seem to be able to overcome puzzles related to the real exchange rate, provided that a sufficient number of different types of shocks are identified. Despite delivering the correct sign of the impulse responses, central tendency measures of sign-restricted VAR models can, however, be misleading and hardly ever coincide with the true impulses. This finding casts doubt on the common notion that the median impulses are the 'most probable' description of the true data-generating process.

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### 1. Introduction

Vector autoregressive models (VARs) are widely used for understanding the effects of monetary policy on the economy. While the results of these models are generally consistent with economic theory, they tend to suffer from various puzzles. One of these anomalies is the *price puzzle*, a term coined by

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Eichenbaum (1992), which refers to a situation in which an unexpected tightening in monetary policy leads to an increase in inflation. Other puzzles have been found regarding the behaviour of the real exchange rate in response to a monetary policy shock. Standard theory suggests that an unexpected tightening in monetary policy leads to an immediate appreciation of the currency and a future depreciation in line with uncovered interest rate parity (UIP).<sup>1</sup> However, many empirical studies, particularly those based on VAR models, find that following such a shock, the real exchange rate either depreciates, or if it appreciates, it does so over an extended period. In the literature, these phenomena have been referred to as the *exchange rate puzzle* and the *delayed overshooting puzzle*, respectively.

VAR studies have typically placed recursive, contemporaneous 'zero restrictions' on the interaction between monetary policy and the exchange rate (for instance, see Eichenbaum and Evans, 1995; Kim and Roubini, 2000 for G7 countries and Mojon and Peersman, 2001 and Peersman and Smets, 2003 for the euro area). Sign restrictions are an attractive alternative to recursive VARs as they avoid the use of strong restrictions on contemporaneous relationships for identification. An increasing number of VAR studies have employed sign restrictions to identify monetary policy shocks (see, for instance, Canova and De Nicoló, 2002 and Uhlig, 2005), and in particular the effects of monetary policy shocks on exchange rates. Using this approach, Faust and Rogers (2003) find no robust results regarding the timing of the peak response of the exchange rate. Scholl and Uhlig (2008) impose sign restrictions on a minimal set of variables but do not restrict the response of the exchange rate when identifying the monetary policy shock. While their findings confirm the exchange rate puzzles, their 'agnostic' sign restriction approach is open to criticism because it identifies only one shock and ignores all others.<sup>2</sup> The problem with such an approach is that the identification scheme is not unique – there are possibly other shocks which would also satisfy the minimal restrictions placed on the monetary policy shock. This raises the question of whether the use of a minimal set of sign restrictions is sufficient to identify a 'true' response of the exchange rate. This question is particularly pertinent, given that Bjørnland (2009) – using long-run restrictions on the effect of monetary policy shocks on the exchange rate – finds no evidence of exchange rate puzzles in four small open economies.<sup>3</sup>

This paper examines the consequences of using recursive and sign-restricted VAR models to identify monetary policy shocks when the data-generating process is an estimated small open economy DSGE model for Australia (in the spirit of Galí and Monacelli, 2005). In particular, it tests whether estimates of these models can replicate the true impulse responses from the DSGE model.<sup>4</sup> It finds that sign restriction models do reasonably well at estimating the responses of macroeconomic variables to monetary policy shocks, particularly compared to VAR models which use a recursive identification structure, which are generally inconsistent with the responses of the DSGE model. Using an identification procedure that is agnostic regarding the direction of the exchange rate response, the paper examines the ability of sign-restricted VAR models to overcome puzzles related to the real exchange rate.<sup>5</sup> It finds that the sign restriction approach recovers the impulse responses reasonably well, provided that a sufficient number of shocks are uniquely identified; if we only identify the monetary policy shocks, in line with Scholl and Uhlig (2008), the exchange rate puzzle remains. In addition, it shows that central tendency measures of sign-restricted VAR models can be misleading since they hardly ever coincide with the true impulses. This casts doubt on the common notion that the median impulses are 'most probable'.

<sup>1</sup> The UIP condition is a key equation in structural open economy models; in its simplest formulation it suggests that the expected future change in the exchange rate equals the difference between domestic and foreign nominal interest rates.

<sup>2</sup> Farrant and Peersman (2006) also provide an open economy application, but they assume that the real exchange rate appreciates after a restrictive monetary policy shock.

<sup>3</sup> Bjørnland and Halvorsen (2008) combine sign and short-run (zero) restrictions. They find that following a contractionary monetary policy shock, the exchange rate appreciates on impact and then gradually depreciates back to baseline. However, as in Farrant and Peersman (2006), the appreciation of the real exchange rate after a monetary policy shock is imposed.

<sup>4</sup> The sign restriction approach is more natural than long-run restrictions in the context of this model; there are no permanent shocks in the model, so after a transitory shock the economy eventually returns to its steady state, making long-run restrictions irrelevant on simulated data.

<sup>5</sup> Canova and Paustian (2007) and Paustian (2007) assess the ability of sign restrictions to correctly identify monetary policy shocks in closed economy settings.

The rest of the paper is organised as follows. Section 2 outlines the small open economy DSGE model, which is used as a data-generating process in our controlled experiment. This model is estimated using data for Australia (and the United States as the 'large' economy) in Section 3, which also presents the theoretical impulse responses to a monetary policy shock generated from the model. Section 4 outlines the empirical VAR models and summarises the results based on estimates using simulated data. Section 5 concludes.

## 2. A small open economy DSGE model

This section presents the small open economy DSGE model. The model is based on a modified version of that proposed by Galí and Monacelli (2005) and is described in Jääskelä and Kulish (2010). All variables are expressed in log deviations from steady state and the key log-linear equations are given below.

### 2.1. The large economy

Variables with a *star subscript* correspond to the large, foreign economy, which can be described with a standard set of new Keynesian closed economy equations.

Firms operate under monopolistic competition in the goods market and Calvo-price stickiness. Factor markets are competitive and goods are produced with a constant returns to scale technology. The Phillips curve in the large economy is of the form:

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa \chi_t^* \quad (1)$$

where:  $\pi_t^*$  is the foreign inflation rate;  $\chi_t^*$  is the foreign output gap; the parameter  $\kappa$  is strictly positive and captures the degree of price rigidities; the household's discount factor,  $\beta$ , lies between zero and one; and  $E_t$  denotes expectations conditional on information at time  $t$ .

The IS curve implies that the current level of the foreign output gap depends on its expected future level ( $E_t \chi_{t+1}^*$ ) the *ex-ante* short-term real interest rate, foreign total factor productivity ( $a_t^*$ ) and a foreign aggregate demand disturbance ( $v_{x,t}^*$ ), as follows:

$$\chi_t^* = E_t \chi_{t+1}^* - \frac{1}{\sigma} (r_t^* - E_t \pi_{t+1}^*) - \phi_1 (1 - \rho_a^*) a_t^* + \frac{1 - \rho_x^*}{\sigma} v_{x,t}^* \quad (2)$$

where:  $r_t^*$  is the foreign nominal short-term interest rate;  $\sigma$  is strictly positive and governs intertemporal substitution;  $\rho_a^*$  is the persistence of  $a_t^*$ ;  $\rho_x^*$  is the persistence of  $v_{x,t}^*$ ; and  $\phi_1$  is equal to  $\frac{1 + \varphi}{\sigma + \varphi}$  with  $\varphi > 0$  governing the elasticity of labour supply.

Foreign monetary policy follows a Taylor rule of the form:

$$r_t^* = \rho_r^* r_{t-1}^* + \alpha_\pi^* \pi_t^* + \alpha_x^* \chi_t^* + \varepsilon_{r,t}^* \quad (3)$$

where  $\varepsilon_{r,t}^*$  is an independent and identically distributed (*iid*) foreign monetary policy shock, with zero mean and standard deviation  $\sigma_{\varepsilon_r^*}$ .  $\alpha_\pi^*$  and  $\alpha_x^*$  capture the reaction of the foreign interest rate to the deviation of foreign inflation from target (set to zero) and the foreign output gap.

The potential level of foreign output,  $\bar{y}_t^*$ , is the level that would prevail in the absence of nominal rigidities. For the large economy, it can be shown that the actual level of output,  $y_t^*$ , and the output gap,  $\chi_t^*$ , obey the following relationship:

$$\chi_t^* \equiv y_t^* - \bar{y}_t^* = y_t^* - \phi_1 a_t^* \quad (4)$$

Foreign exogenous processes evolve according to:

$$a_t^* \equiv \rho_a^* a_{t-1}^* + \varepsilon_{a,t}^* \quad (5)$$

$$v_{x,t}^* \equiv \rho_x^* v_{x,t-1}^* + \varepsilon_{x,t}^* \tag{6}$$

where: the shocks  $\varepsilon_{a,t}^*$  and  $\varepsilon_{x,t}^*$  are iid with zero mean and standard deviations  $\sigma_a^*$  and  $\sigma_x^*$ , respectively; and the autoregressive parameters,  $\rho_a^*$  and  $\rho_x^*$  are less than unity in absolute value.

### 2.2. The small open economy

In the small open economy, the IS curve links the output gap,  $x_t$ , to its expected future value, the *ex ante* real interest rate (where the nominal interest rate is deflated by the expected rate of domestically produced goods inflation), the expected growth rate of foreign output, foreign and domestic aggregate demand shocks and domestic total factor productivity. The open economy's IS curve takes the following form:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_\alpha} (r_t - E_t \pi_{h,t+1}) + \phi_3 E_t \Delta y_{t+1}^* + \frac{(1 - \rho_x)(1 - \phi_2)}{\sigma} v_{x,t} + \frac{1 - \rho_x^*}{\sigma} \phi_3 v_{x,t}^* - \phi_4 (1 - \rho_a) a_t \tag{7}$$

where:  $\rho_x$  and  $\rho_a$  are the persistence parameters of domestic aggregate demand and domestic productivity shocks, respectively; and the parameters  $\sigma_\alpha$ ,  $\phi_2$ ,  $\phi_3$  and  $\phi_4$  are functions of deep parameters. In particular, it can be shown that

$$\begin{aligned} \sigma_\alpha &\equiv \frac{\sigma}{(1 - \alpha) + \alpha\omega} \\ \omega &\equiv \sigma\tau + (1 - \alpha)(\sigma\iota - 1) \\ \phi_2 &\equiv \frac{\sigma_\alpha - \sigma}{\sigma_\alpha + \varphi} \\ \phi_3 &\equiv \alpha(\omega - 1) + \phi_2 \\ \phi_4 &\equiv \frac{1 + \varphi}{\sigma_\alpha + \varphi} \end{aligned}$$

where:  $\alpha \in [0, 1]$  captures the degree of openness;  $\tau$  is the intertemporal elasticity of substitution between foreign and domestically produced goods; and  $\iota$  is the elasticity of substitution across varieties of foreign-produced goods.

The dynamics of domestically produced goods inflation,  $\pi_{h,t}$ , are governed by a Phillips curve equation

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa_\alpha x_t + v_{\pi,t} \tag{8}$$

where:  $\kappa_\alpha \equiv \lambda(\sigma_\alpha + \varphi)$ ;  $\lambda \equiv \frac{(1 - \theta)(1 - \beta\theta)}{\theta}$ ;  $\theta$  governs the degree of price stickiness; and  $v_{\pi,t}$  is a cost-push shock.

Monetary policy in the small economy is assumed to follow a Taylor rule that sets the nominal interest rate,  $r_t$ , in response to its own lagged value, the deviation of consumer price inflation,  $\pi_t$ , from its target (set to zero) and the output gap,  $x_t$ , as follows:

$$r_t = \rho_r r_{t-1} + \alpha_\pi \pi_t + \alpha_x x_t + \varepsilon_{r,t} \tag{9}$$

where  $\varepsilon_{r,t}$  is an iid monetary policy shock with zero mean and standard deviation  $\sigma_r$ .

The terms of trade,  $s_t$ , are defined as the price of foreign goods ( $p_{f,t}$ ) in terms of the price of home goods ( $p_{h,t}$ ). That is,  $s_t = p_{f,t} - p_{h,t}$ . The consumer price index is a weighted average of the price of foreign and domestically produced goods  $p_t = (1 - \alpha)p_{h,t} + \alpha p_{f,t}$ . It follows that consumer price inflation and domestically produced goods inflation are linked by the expression

$$\pi_t = \pi_{h,t} + \alpha \Delta s_t \tag{10}$$

The nominal exchange rate,  $e_t$ , is defined as the price of foreign currency in terms of the domestic currency, so positive values of  $\Delta e_t$  indicate a nominal depreciation of the domestic currency. The law of one price is assumed to hold, so  $p_{f,t} = e_t + p_t^*$ , which implies that the terms of trade can also be written

as  $s_t = e_t + p_t^* - p_{h,t}$ . Combining these expressions, it is easy to show that the real exchange rate,  $q_t$ , is proportional to the terms of trade:

$$\Delta q_t = (1 - \alpha)\Delta s_t \quad (11)$$

Complete international securities markets, together with the market clearing conditions, lead to the following relationship between the terms of trade, output and shocks to demand:

$$s_t = \sigma_\alpha (y_t - y_t^*) - \frac{\sigma_\alpha}{\sigma} (v_{x,t} - v_{x,t}^*). \quad (12)$$

The relationship between the actual level of output,  $y_t$ , and the output gap,  $x_t$ , satisfies the following equation:

$$x_t \equiv y_t - \bar{y}_t = y_t - \phi_t y_t^* - \frac{\phi_2}{\sigma} (v_{x,t} - v_{x,t}^*) - \phi_4 a_t \quad (13)$$

Finally, the domestic exogenous processes evolve according to

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \quad (14)$$

$$v_{\pi,t} = \rho_\pi v_{\pi,t-1} + \varepsilon_{\pi,t} \quad (15)$$

$$v_{x,t} = \rho_x v_{x,t-1} + \varepsilon_{x,t} \quad (16)$$

where: the shocks  $\varepsilon_{a,t}$ ,  $\varepsilon_{\pi,t}$ , and  $\varepsilon_{x,t}$  are *iid* with zero mean and standard deviations  $\sigma_a$ ,  $\sigma_\pi$ , and  $\sigma_x$ , respectively; and the autoregressive parameters,  $\rho_a$ ,  $\rho_\pi$  and  $\rho_x$  are less than unity in absolute value.

### 3. Estimating the small open economy model

#### 3.1. Parameter estimates

In order to derive parameter estimates for our controlled experiment, we estimate the DSGE model's parameters with Bayesian techniques (for a survey, see [An and Schorfheide, 2007](#)) using quarterly Australian and US data. For the large US economy, we use quarterly linearly-detrended log US real GDP ( $x_t^*$ ), demeaned US CPI inflation excluding food and energy ( $\pi_t^*$ ) and the demeaned US Federal Funds rate ( $r_t^*$ ) for the sample period 1984:Q1–2009:Q4. The sample period covers the post-float period for the Australian dollar. For the small open economy, Australia, we use quarterly linearly-detrended log real GDP ( $x_t$ ), demeaned trimmed-mean inflation excluding interest and taxes ( $\pi_t$ ), the demeaned RBA cash rate ( $r_t$ ) and linearly-detrended log of the bilateral real exchange rate ( $q_t$ ) for the same sample period. [Table 1](#) summarises the results of the estimation of this DSGE model. The posterior statistics are based on 1 million draws using the Markov Chains Monte Carlo (MCMC) methods with a 20 per cent burn-in period. We calibrate the discount factor  $\beta$  to be 0.99 (for both the large and small economies); the degree of openness,  $\alpha$ , is set at 0.25, consistent with the value of the share of foreign goods in the Australian consumption basket. Finally, for both economies we calibrate  $\sigma$ ,  $\tau$ ,  $\iota$  and  $\phi$  to be 1.5, 1.0, 1.0 and 3.0, respectively, in line with other studies. The persistence parameters  $\rho_a$  and  $\rho_x$  are calibrated to be 0.85 and 0.80, respectively. We choose to calibrate these two parameters as their estimates have a lot of probability mass around 1. This highlights the fact that the model has no endogenously generated persistence, thus the only way to match the level of persistence in the data is to opt for highly persistent shocks.

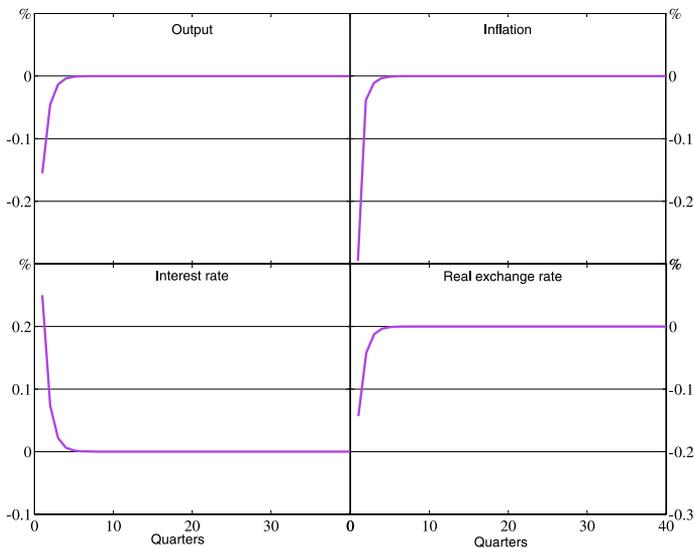
#### 3.2. True impulse responses

The 'true' impulse response functions (IRFs) generated by the DSGE model (based on the posterior mean of the estimated parameters) are presented in [Fig. 1](#). A contractionary monetary policy shock has a negative effect on the output gap and lowers inflation while the real exchange rate appreciates instantaneously (and depreciates thereafter consistent with the UIP condition). Most of the variables

**Table 1**

Parameter estimates of the DSGE model.

Parameters	Prior mean	Posterior mean	90 per cent probability intervals	Prior distribution	Prior std dev
<b>Calibrated parameters</b>					
$\beta$	0.99	0.99			
$\sigma$	1.50	1.50			
$\tau$	1.00	1.00			
$\iota$	1.00	1.00			
$\phi$	3.00	3.00			
<b>Calvo parameter</b>					
$\theta$	0.60	0.60	[0.44–0.77]	Beta	0.10
<b>Domestic monetary policy</b>					
$\rho_r$	0.80	0.86	[0.84–0.89]	Beta	0.02
$\alpha_x$	0.05	0.28	[0.22–0.34]	Normal	0.10
$\alpha_\pi$	0.40	0.60	[0.45–0.74]	Normal	0.10
<b>Foreign monetary policy</b>					
$\rho_r^*$	0.80	0.81	[0.76–0.84]	Beta	0.10
$\alpha_x^*$	0.05	0.15	[0.03–0.27]	Normal	0.10
$\alpha_\pi^*$	0.40	0.46	[0.32–0.59]	Normal	0.10
<b>Persistence of shocks</b>					
$\rho_\pi$	0.80	0.84	[0.81–0.87]	Beta	0.02
$\rho_a$	0.70	0.90	[0.88–0.93]	Beta	0.05
$\rho_x$	0.70	0.89	[0.86–0.92]	Beta	0.10
<b>Standard deviations of shocks (<math>\times 10^{-2}</math>)</b>					
$\sigma_a$	1.00	3.45	[2.96–3.91]	Inv gamma	2
$\sigma_x$	1.00	9.80	[8.68–10.92]	Inv gamma	2
$\sigma_\pi$	1.00	0.69	[0.52–0.86]	Inv gamma	2
$\sigma_r$	1.00	2.35	[1.90–2.80]	Inv gamma	2
$\sigma_a^*$	1.00	0.84	[0.74–0.94]	Inv gamma	2
$\sigma_x^*$	1.00	1.97	[1.59–2.34]	Inv gamma	2
$\sigma_r^*$	1.00	0.22	[0.19–0.25]	Inv gamma	2



**Fig. 1.** Structural Model – Impulse Responses to a Monetary Policy Shock.

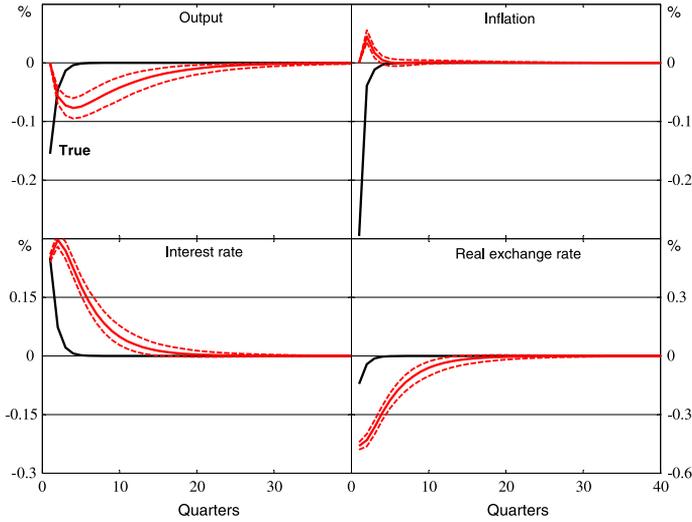


Fig. 2. Recursive VAR – Impulse Responses to a Monetary Policy Shock.

return to baseline relatively quickly. More generally, and consistent with other general equilibrium models, all variables respond to the monetary policy shock contemporaneously. This is inconsistent with the standard assumption used to estimate recursive VARs, suggesting that these models will encounter problems identifying monetary policy shocks using simulated data from this model.

**4. VAR models with simulated data**

In this section, we estimate a selection of VAR models using simulated data from the DSGE model. As our baseline experiment, we simulate 500 observations from the DSGE model for the following variables (using the posterior mean of the estimated parameters in Table 1):  $y_t^*$  (foreign output);  $\pi_t^*$

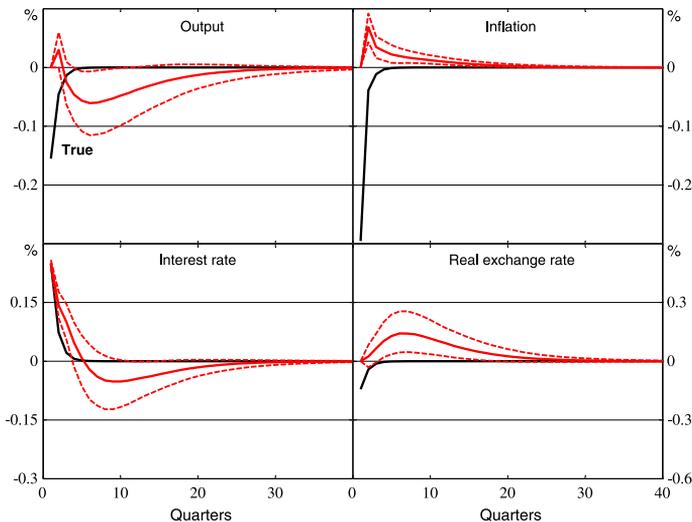


Fig. 3. Recursive VAR – Impulse Responses to a Monetary Policy Shock.

**Table 2**

VAR sign restrictions.

Shock to:	$y^*$	$\pi^*$	$r^*$	$y$	$\pi$	$r$	$q$
Foreign demand	↑	↑	↑	–	–	–	–
Foreign productivity	↑	↓	↓	–	–	–	–
Foreign monetary policy	↓	↓	↑	–	–	–	–
Demand	0	0	0	↑	↑	↑	–
Productivity	0	0	0	↑	↓	↓	–
Monetary policy	0	0	0	↓	↓	↑	–

Note: ↑(↓) means positive (negative) response of the variables in columns to shocks in rows. 0 means no response (as implied by the small open economy assumption). – means no restriction is imposed on the response.

(foreign inflation);  $r_t^*$  (foreign interest rate);  $y_t$  (domestic output);  $\pi_t$  (domestic inflation);  $r_t$  (domestic interest rate); and  $q_t$  (the real exchange rate). These variables are the ones that researchers typically use to estimate VAR models. Consistent with the small open economy assumption, we impose block exogeneity, with foreign variables unaffected by domestic shocks. We estimate VARs of order two, consistent with the VAR representation of the DSGE model.

The size of the monetary policy shock is normalised to 25 basis points. This ensures that the differences between the true IRF's and the estimated IRF's are not simply due to a bias in estimating the size of the policy shock.

#### 4.1. Recursive VARs

Using our simulated data, we estimate a recursive VAR based on the ordering given above – that is,  $y_t^*$ ,  $\pi_t^*$ ,  $r_t^*$ ,  $y_t$ ,  $\pi_t$ ,  $r_t$  and  $q_t$  – with the real exchange rate being the most endogenous variable (that is, it responds contemporaneously to all of the other variables). We call this Ordering (1). Some studies identify monetary policy by restricting the exchange rate from reacting immediately to a monetary policy shock (see Mojon and Peersman, 2001; Peersman and Smets, 2003). Thus, we also swap the ordering of the last two variables to make the domestic interest rate the most endogenous variable (we call this Ordering 2). Figs. 2 and 3 compare the impulse responses from the recursive models with the true responses from the DSGE model (the solid lines plot the median impulse responses and the dashed lines represent the 14th and 86th percentiles of the responses). Similar to Carlstrom et al. (2009), the magnitudes and shapes of the impulse responses are at odds with the results from the DSGE model.<sup>6</sup> Ordering (2) (Fig. 3) exhibits the exchange rate puzzle, with the exchange rate depreciating following the contraction in monetary policy; moreover, output rises at first in response to the monetary policy contraction. Ordering (1) (Fig. 2) produces a real exchange rate appreciation, but the size of the appreciation is much larger than the theoretical response. In the DSGE model, monetary policy and the exchange rate interact contemporaneously, so it seems likely that the puzzles relating to the real exchange rate follow from the 'zero-type' restrictions which prevent this (see also (Faust and Rogers, 2003)). Both VAR models produce the price puzzle, with inflation rising following the monetary policy shock. Figs. 2 and 3 highlight the fact that the estimates of the impulse responses are sensitive to identifying assumptions (that is, Orderings 1 and 2 are quite different). Overall, the results suggest that care should be used when using VAR models of this type to identify the monetary transmission mechanism.

#### 4.2. Sign-restricted VARs

We now examine how well sign-restricted VARs can identify monetary policy shocks. These models achieve identification by imposing the direction that key variables will move (over a given horizon) in

<sup>6</sup> Carlstrom et al. (2009) simulate data from a three-equation DSGE model that is consistent with the timing assumptions of the standard Choleski identification. They assume that output and prices in the theoretical model are determined before the realisation of the monetary policy shock. Consequently, inflation and the output gap do not respond contemporaneously to the monetary shock. They still find that there are large differences between the true IRFs and the estimated IRFs.

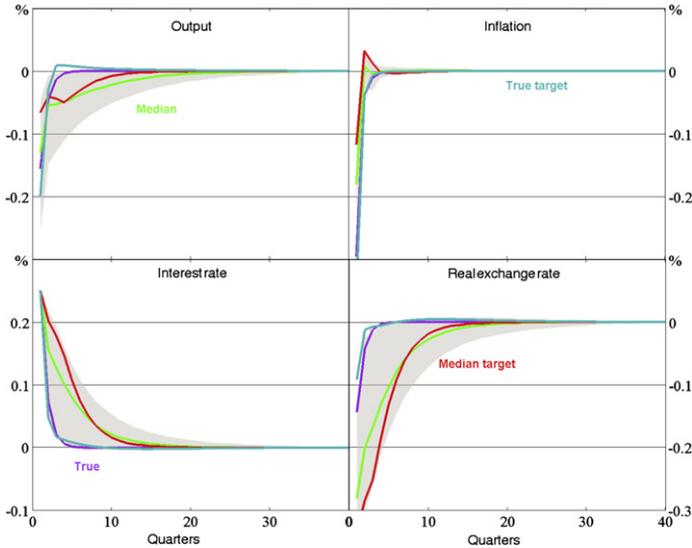


Fig. 4. Baseline sign-restricted VAR – Impulse Responses to a Monetary Policy Shock.

response to different types of shocks. Full details of the sign-restricted VAR methodology are provided in [Appendix A](#).

The set of sign restrictions adopted in the paper is presented in [Table 2](#). Given that the foreign variables enter the DSGE model as exogenous processes, we assume that domestic shocks do not affect the foreign variables, while the response of the domestic variables to the foreign shocks are left unrestricted. We also remain agnostic about the response of the exchange rate to all of the shocks in the model. In particular, we leave the response of the real exchange rate to a domestic monetary policy shock unrestricted because we want to see whether the sign restrictions on other variables are sufficient to identifying impulse responses, which are free of exchange rate puzzles. We avoid price and output puzzles by assuming that inflation and output fall in response to a contractionary monetary policy shock. The sign restrictions are imposed for the impact quarter only.<sup>7</sup> In contrast, [Scholl and Uhlig \(2008\)](#) and [Paustian \(2007\)](#) allow the restrictions to be imposed for a longer period of time. We return to this issue in [Section 4.3](#).

[Fig. 4](#) compares the responses of the variables to a monetary policy shock under the sign-restricted VAR model with those from the true model (the lilac line). The shaded area represents the area between the 5th and 95th percentiles of the responses generated from the sign-restricted VAR algorithm, and the green line plots the median of the set of identified responses. [Fry and Pagan \(2010\)](#) have criticised the practise of using the median of the distribution of responses as a location measure, since the median at each horizon and for each variable may be obtained from different candidate models. They suggest using a single unique draw that is closest to the median impulse responses for all variables. Accordingly, the red line plots this so-called ‘median target’ (MT) measure.<sup>8</sup> The same criticism applies to any other percentile measure such as the shaded area presented here. We also show a unique draw that minimises the distance from the true impulses (shown by the blue line and labelled as the ‘true target’ (TT)).

<sup>7</sup> It is worth emphasising that we impose strict exogeneity of the foreign variables, that is, the feedback from the domestic variables on the foreign variables is always zero, not just on the impact period.

<sup>8</sup> Though we focus here on the monetary policy shock, this measure finds a unique draw that minimises the distance from the medians for *all* identified shocks.

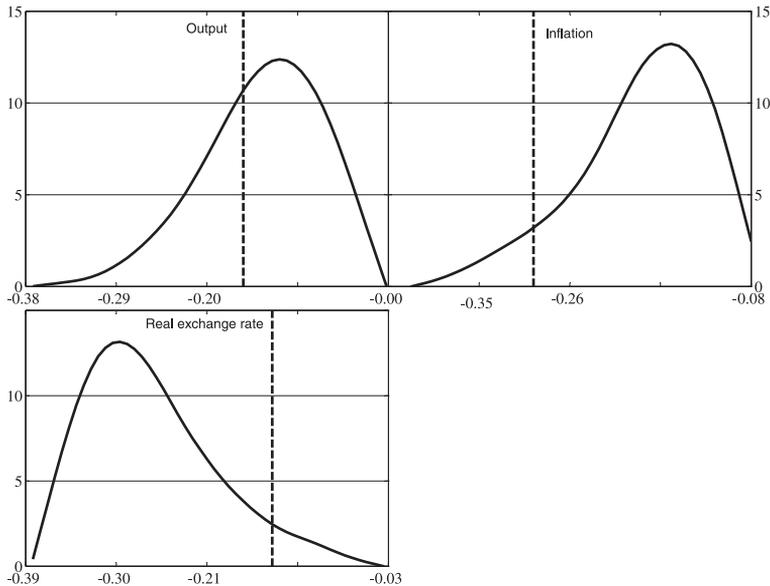


Fig. 5. Distribution of Impulse Responses to a Monetary Policy Shock on Impact.

As shown in Fig. 4, the sign-restricted VAR does a significantly better job than the recursive VAR models at replicating the true impulse responses to a contractionary monetary policy shock. The model captures correctly the sign of the real exchange rate response on impact. However, the range of responses (shown by the shaded area) is quite wide, even for those variables whose response is constrained *a priori*.<sup>9,10</sup> Responses characterising the central tendency of the sign-restricted VAR (the median and the MT measure) are more persistent than those of the DSGE model. This may be because the VAR model is only partially identified by the set of restrictions shown in Table 2. There may be unidentified shocks which happen to satisfy the sign restrictions placed on the monetary policy shock or indeed any of the other shocks we are attempting to identify. In other words, these unidentified shocks contaminate the central tendency measures that utilise all accepted draws. More specifically, there are seven variables in the model but we only identify six shocks. Hence, there is one unidentified shock on which we impose no sign restrictions. Fry and Pagan (2010) note that this can lead to the *multiple shocks problem*, in which unidentified shocks can be similar to shocks which have been identified using sign restrictions. It is also worth noting, however, that it is not possible to distinguish a negative productivity shock from a positive cost-push shock.

These results suggest that the median does not necessarily capture the true model, as it is often thought to do. This finding is highlighted in Fig. 5 which shows the distributions of the sign-restricted VAR impulse responses of output, inflation and the real exchange rate to the monetary policy shock on impact; the vertical dashed lines show the true responses. For instance, on impact the true responses of output, inflation and the real exchange rate are located on the 36th, 9th and 94th percentiles, respectively; nowhere near the 50th percentile – the median. The unique draw closest to the true impulse responses (the TT measure) is better than the central tendency measures by construction, but there are still some small discrepancies. Moreover, the biases are even more prominent for the

<sup>9</sup> As a cross-check, we also restrict the real exchange rate to appreciate in the impact quarter following a contractionary monetary policy shock. This has little effect on the range of responses of domestic output, inflation and the interest rate.

<sup>10</sup> It has been argued that in order to reduce the dispersion in the set of models, that is, the width of the band, additional quantitative information about the likely magnitude (or the shape) of the impulse responses might be required (Uhlig, 2005; Kilian and Murray, 2010).

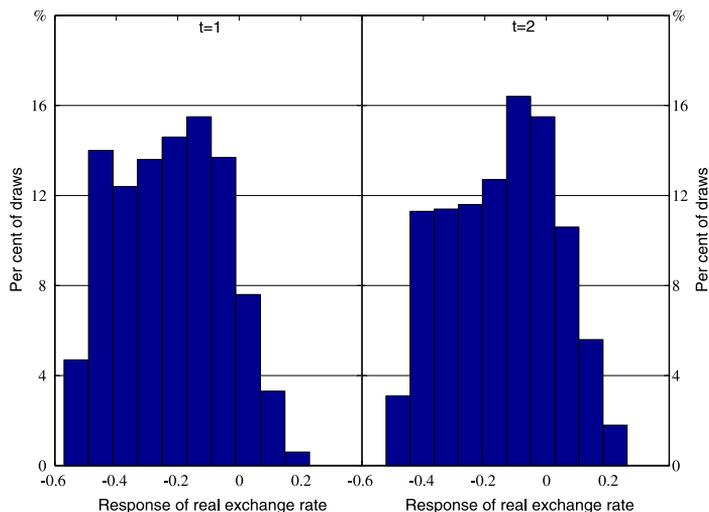


Fig. 6. Identifying the Domestic and Foreign Monetary Policy Shock Only – Response of the Real Exchange Rate.

identified demand and productivity shocks (see Figs. 9 and 10 in Appendix C), probably due to the presence of the unidentified shock in the sign-restricted VAR model.

It is likely that the number of identified shocks and identification restrictions employed matters for the performance of the sign-restricted VAR model. The results above are based on identifying six shocks with restrictions on six of the variables. If instead, we only identify the monetary policy shocks (both foreign and domestic), the exchange rate puzzle re-emerges. The results are summarised in Fig. 6, which shows histograms of the response of the real exchange to an unexpected tightening of monetary policy on impact (the left panel in the figure) and one period after (the right panel). It can be seen that around 10 and 24 per cent of the 1000 draws imply a depreciation of the real exchange rate on impact and one quarter after the shock, respectively. In addition, uncertainty surrounding the responses of all other variables increases slightly. This suggests that the identified monetary policy shock is contaminated with features of other structural shocks that are left unidentified (the ‘multiple shocks problem’) and as a result the ‘agnostic’ sign restriction approach of (Scholl and Uhlig(2008)) may not be able to recover ‘true’ impulse responses. In short, it appears that the likelihood of recovering the correct sign of the exchange rate increases with the number of identified shocks.

#### 4.3. Extensions

In addition to the presence of unidentified shocks, there are other reasons why there may be biases inherent in the sign-restricted VAR results which are worth examining. These include: the number of lags in the VAR model; the number of sign restriction periods; and the relative strength of the ‘true’ shock signal.

Given that we use only a subset of model variables in the VAR, we may be introducing truncation bias by estimating a finite order VAR model (see Ravenna, 2007). Kapetanios, Pagan and Scott (2007) investigate this question in a simulation exercise. They find that 50 lags were required to produce

**Table 3**  
Acceptance rate.

Lag length	$L = 1$	$L = 2$	$L = 4$	$L = 8$	$L = 16$	$L = 32$	$L = 50$
Total/Accepted	40.023	39.483	39.106	38.94	36.2810	40.367	42.021

Note: Entries indicate the average number of draws required to find a decomposition that satisfies the sign restrictions given in Table 2.

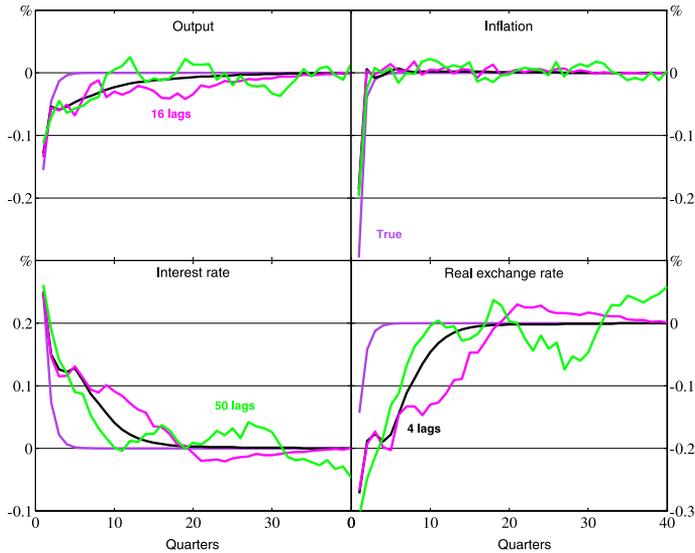


Fig. 7. Sign-restricted VAR with Different Lags – Impulse Responses to a Monetary Policy Shock.

estimated impulse responses that are essentially indistinguishable from the true values.<sup>11</sup> If increasing the lag length were to improve the model fit in our experiment, it is plausible that the number of draws required to yield a model which satisfies the identifying restrictions should decline with the lag length. However, this turns out not to be the case. Table 3 shows the average number of draws required to find a decomposition that satisfies the sign restrictions given in Table 2 for different lag length specifications. As the lag length increases, the number of these draws initially decreases, but the sign restriction algorithm requires a greater number of draws to find a satisfactory draw when the lag length is increased beyond 16. Fig. 7 shows the impulses responses with different lag lengths. There is very little, if any, evidence that increasing the lag length improves the accuracy of the estimated impulse responses. (We also ran this experiment with 1000 simulated observations, doubling the sample size did not alter this conclusion.)

It has been argued that a longer horizon over which the sign restrictions are enforced may be required in order to better match the theoretical responses. According to Paustian (2007), however, as the horizon for the sign restrictions is extended, the distribution of the responses actually becomes centred further away from the true impact responses. This is likely with our simulated data as well, given the instantaneous response of the model variables to the shocks; although imposing sign restrictions over two quarters yields broadly unchanged impulse responses.

It is possible that if the variance of the monetary policy shock is small it may be difficult for the VAR model to properly identify monetary policy innovations. Faust and Rogers (2003) are unable to find policy shocks that generate interest rate and exchange rate responses consistent with UIP, and conclude that US monetary policy shocks may explain less of the observed exchange rate variability than previously believed. Paustian (2007) also reviews this possibility and concludes that the variance of the shock under study must be sufficiently large in order to deliver the correct sign of the unconstrained impulse response. However, we can show that our modelling does not suffer this particular problem. Even if the the variance of the monetary policy shock ( $\sigma_r$ ) in the underlying DSGE model is reduced (we tested

<sup>11</sup> Their empirical model suffers from the missing variables problem. There are 26 variables in their theoretical model but only six in the empirical one.

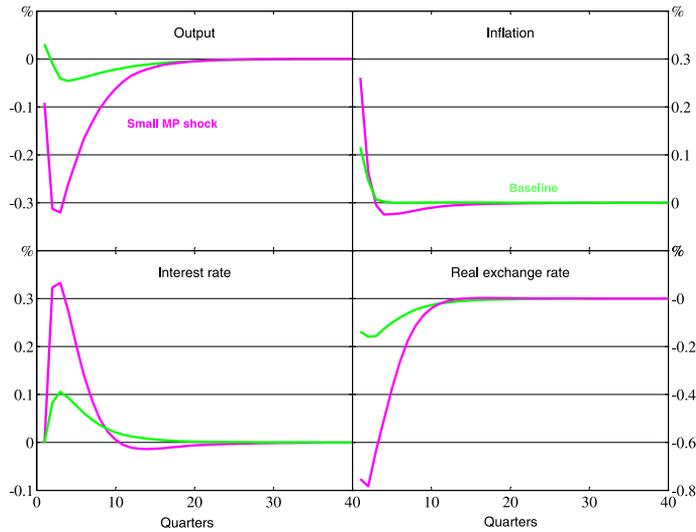


Fig. 8. Bias with Small and Large Monetary Policy Shocks.

lowering  $\sigma_r$  1000 fold), the sign of the real exchange rate response is correctly identified using our sign-restricted VAR. Although, as shown in Fig. 8, decreasing the variance of the monetary policy shock increases estimation biases (measured as the deviation from the ‘true’ impulse response).

## 5. Conclusion

This paper investigates the ability of vector autoregressive (VAR) models to properly identify monetary policy shocks with data simulated from a small open economy DSGE model estimated using Australian data. Overall, it finds that sign restriction models do reasonably well at estimating the responses of macroeconomic variables to monetary policy shocks, particularly compared to VAR models based on a recursive identification structure.

Using an identification procedure that is agnostic regarding the direction of the exchange rate response, the paper examines the ability of sign-restricted VAR models to overcome puzzles related to the real exchange rate. It finds that the sign restriction approach recovers the impulse responses (free of the exchange rate puzzles) reasonably well, provided that a sufficient number of shocks are uniquely identified; if only the monetary policy shocks are identified, the exchange rate puzzle remains. This suggests that identification schemes that are too parsimonious may fail to recover the ‘true’ impulse responses. The paper also finds that measures of central tendency can be misleading and that the true impulses hardly ever coincide with the median. This casts doubt on the common notion that the median impulses are ‘most probable’.

There are several directions in which the analysis presented in this paper could be extended. One such avenue would allow for time-varying parameters in the data-generating process. It would be interesting to see whether regime-switching (or time-varying parameter) sign-restricted VAR models would be able to capture the break in the data-generating process at all.

## Acknowledgements

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### Appendix A. Sign restriction algorithm

Consider a general VAR(p) model with  $n$  variables  $Y_t$ :

$$BY_t = A(L)Y_{t-1} + \varepsilon_t \tag{17}$$

where:  $A(L) = A_1L + \dots + A_pL^p$  is a  $p$ th order matrix polynomial;  $B$  is a  $(n \times n)$  matrix of coefficients that reflect the contemporaneous relationships among  $Y_t$ ; and  $\varepsilon_t$  is a set of  $(n \times 1)$  normally distributed structural disturbances with mean zero and variance covariance matrix  $\Sigma$ ,  $\Sigma_{i,j} = 0 \forall i \neq j$ . The structural representation has the following reduced-form:

$$Y_t = \Pi(L)Y_{t-1} + e_t \tag{18}$$

where  $\Pi(L) = B^{-1}A(L)$  and  $e_t$  is a set of  $(n \times 1)$  normally distributed reduced-form errors with mean zero and variance covariance matrix  $V$ ,  $V_{i,j} \neq 0 \forall i,j$ . The aim is to map the statistical relationships summarised by the reduced-form errors  $e_t$  back into economic relationships described by  $\varepsilon_t$ . Let  $P = B^{-1}$ . The reduced-form errors are related to the structural disturbances in the following manner:

$$e_t = P\varepsilon_t \text{ and } V = E(e_t e_t') = HH' \tag{19}$$

for some matrix  $H$  such that  $HH' = P\Sigma P'$ . An identification problem arises if there are not enough restrictions to uniquely pin down  $H$  from the matrix  $V$ .

The central idea behind SVAR analysis is to decompose the set of reduced-form shocks, characterised by  $V$ , into a set of orthogonal structural disturbances characterised by  $\Sigma$ . However, there are an infinite number of ways in which this orthogonality condition can be achieved. Let  $H$  be an orthogonal decomposition of  $V = HH'$ . The multiplicity arises from the fact that for any orthonormal matrix  $Q$  (where  $QQ' = I$ ), such that  $V = HQQ'H'$ ,  $\tilde{H}\tilde{H}'$  is also an admissible decomposition of  $V$ , where  $\tilde{H} = HQ$ . This decomposition produces a new set of uncorrelated shocks  $\varepsilon_t = \tilde{H}e_t$ , without imposing zero-type restrictions on the model.

Define an  $(n \times n)$  orthonormal rotation matrix  $Q$  such that:

$$Q = \prod_{i=1}^{n-1} \prod_{j=i+1}^n Q_{i,j}(\theta_{i,j}) \tag{20}$$

where

$$Q_{i,j}(\theta_{i,j}) = \begin{bmatrix} & & \text{col } i & & \text{col } j & & & & \\ & & \downarrow & & \downarrow & & & & \\ & & & \dots & & & & & \\ \text{row } i \rightarrow & 1 & \cos(\theta_{i,j}) & \dots & -\sin(\theta_{i,j}) & & & & \\ & & \dots & 1 & \dots & & & & \\ \text{row } j \rightarrow & & \sin(\theta_{i,j}) & \dots & \cos(\theta_{i,j}) & & & & \\ & & & & & \dots & & & \\ & & & & & & & & 1 \end{bmatrix}$$

where  $\theta_{i,j} \in [0, \pi]$ . This provides a way of systematically exploring the space of all VMA representations by searching over the range of values  $\theta_{i,j}$ . We generate the  $Q$ s randomly from a uniform distribution using the following algorithm:

1. Estimate the VAR to obtain the reduced-form variance covariance matrix  $V$ .
2. For both the foreign and domestic block, draw a vector  $\theta_{i,j}$  from a uniform  $[0, \pi]$  distribution.
3. Calculate  $Q = \prod_{i=1}^{n-1} \prod_{j=i+1}^n Q_{i,j}(\theta_{i,j})$ .
4. Use the candidate rotation matrix  $Q$  to compute  $\varepsilon_t = HQe_t$  and its corresponding structural IRFs for domestic and foreign shocks.

5. Check whether the IRFs satisfy all the sign restrictions described in Table 2. If so, keep the draw, if not, drop the draw.
6. Repeat (2)–(5) until 1000 draws that satisfy the restrictions are found.

**Appendix B. Data description and sources**

*US GDP:* Real GDP (constant prices, sa). Source: Datastream, Code – USGDP...D.

*US underlying consumer price index:* US CPI excluding food and energy (sa). Source: Datastream, Code – USCPXFDEF.

*Federal Funds rate:* Nominal US Federal Funds rate. Source: Datastream, Code – USFDTRG.

*Australian GDP:* Real non-farm GDP (chain-linked, sa). Source: National income, Expenditure and Product, ABS Cat No 5206.0, Table 20.

*Australian underlying consumer price index:* Trimmed-mean consumer price index excluding interest and taxes. Source: Reserve Bank of Australia.

*RBA Cash rate:* Nominal official cash rate. Source: Reserve Bank of Australia.

*Real exchange rate:* Real US\$/AU\$ exchange rate (March 1995 = 100). Source: Reserve Bank of Australia.

**Appendix C. Supplementary figures**

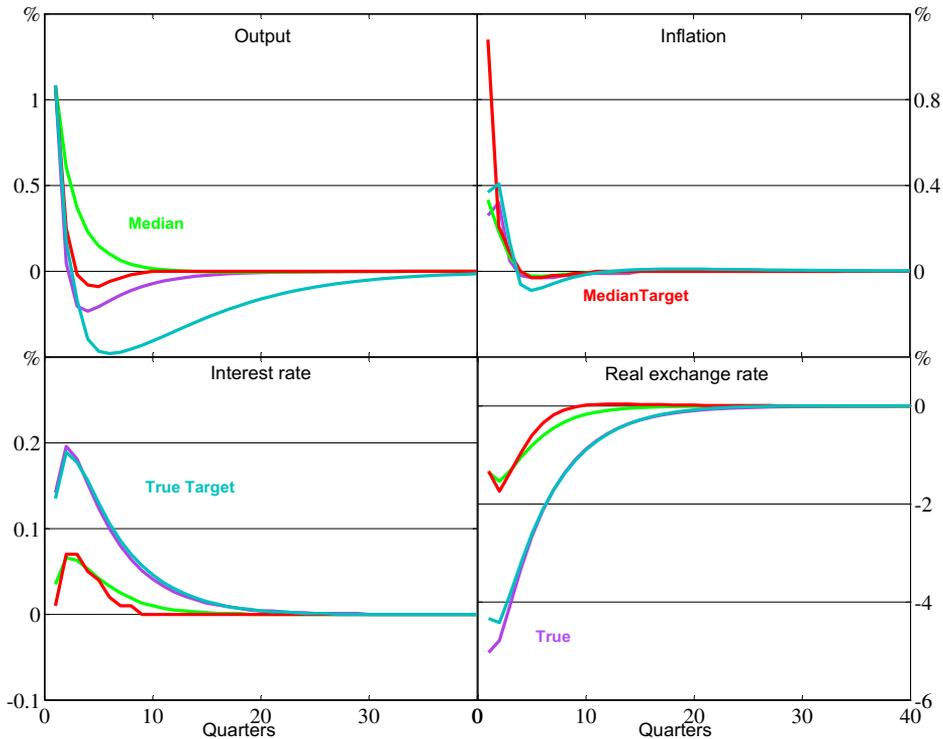


Fig. 9. Impulse Responses to a Demand Shock.

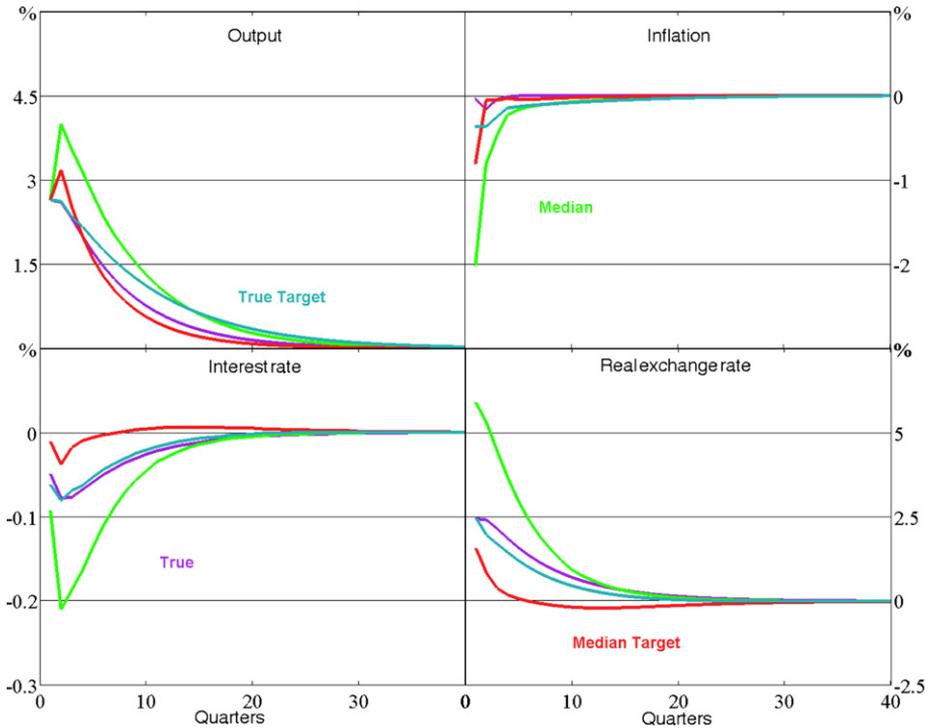


Fig. 10. Impulse Responses to a Productivity Shock.

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