1 Closed Economy Model

1.1 Dynamic System

We can now combine all of our equations together.

Our model has 24 of the following variables: $\psi_{t+1,t} = \frac{\partial\Psi(K_{t+1},K_t)}{\partial K_{t+1}}, \psi_{t+2,t+1} = \frac{\partial\Psi(K_{t+2},K_{t+1})}{\partial K_{t+1}}, w_t = \frac{W_t}{P_t}, m_t = \frac{M_t}{P_t}, q_t = \frac{Q_t}{P_t}, d_t = \frac{D_t}{P_t}, \Pi_t = \frac{P_{t+1}}{P_t}, \Psi_{t+1,t}, X_t, \mu_t, Y_t, I_t, K_t, C_t, H_t, R_t, \Phi_t, \Lambda_t, \Xi_t, a_t, v_t, \chi_t, e_t, A_t,$ and 24 equations.

Consumer's FOCs

$$\Phi_t = C_t^{(\gamma-1)/\gamma} + e_t^{1/\gamma} m_t^{(\gamma-1)/\gamma} \tag{1}$$

$$w_t = \frac{\Phi_t C_t^{\frac{1}{\gamma}} \eta}{a_t(1-H_t)} \tag{2}$$

$$R_t^{-1} = \beta E_t \left\{ \left(\frac{a_{t+1}}{a_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\gamma}} \left(\frac{\Phi_{t+1}}{\Phi_t}\right)^{-1} (\Pi_{t+1})^{-1} \right\} \tag{3}$$

$$C_t e_t = m_t (1 - R_t^{-1})^{\gamma} \tag{4}$$

$$\psi_{t+1,t} = \frac{\phi_K}{\mu} \left(\frac{K_{t+1}}{\mu K_t} - 1 \right) \tag{5}$$

$$\psi_{t+2,t+1} = \frac{\phi_K}{2} \left(\frac{K_{t+2}}{\mu K_{t+1}} \right)^2 - \frac{\phi_K}{\mu} \left(\frac{K_{t+2}}{\mu K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} \tag{6}$$

$$\left(\frac{1}{\chi_t} + \psi_{t+1,t} \right) = \beta E_t \left\{ \left(\frac{a_{t+1}}{a_t}\right) \left(\frac{\Phi_{t+1}}{\Phi_t}\right)^{-1} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\gamma}} \left[q_{t+1} + \frac{1-\delta}{\chi_{t+1}} - \psi_{t+2,t+1} \right] \right\} \tag{7}$$

Firm's FOCs

$$\Lambda_t = \frac{a_t}{\Phi_t C_t^{\frac{1}{\gamma}}} \tag{8}$$

$$\Lambda_t H_t w_t = (1-\alpha) \Xi_t Y_t \tag{9}$$

$$\Lambda_t K_t q_t = \alpha \Xi_t Y_t \tag{10}$$

$$\phi_p \Lambda_t \left(\frac{\Pi_t}{\mu} - 1 \right) \left(\frac{\Pi_t}{\mu} \right) = (1-\theta)\Lambda_t + \theta \Xi_t + (\beta \phi_p) E_t \left\{ \Lambda_{t+1} \left(\frac{\Pi_{t+1}}{\mu} - 1 \right) \left(\frac{\Pi_{t+1}}{\mu} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \right\} \tag{11}$$

Technological Constraints and Laws of Motion

$$Y_t = A_t K_t^{\alpha} [X_t H_t]^{1-\alpha} \tag{12}$$

$$K_{t+1} = (1-\delta) K_t + \chi_t I_t \tag{13}$$

$$\Psi(K_{t+1}, K_t) = \frac{\phi_K}{2} \left(\frac{K_{t+1}}{\mu K_t} - 1 \right)^2 K_t \tag{14}$$

$$d_t = Y_t - w_t H_t - q_t K_t - \frac{\phi_p}{2} \left(\frac{\Pi_t}{\mu} - 1 \right)^2 Y_t \tag{15}$$

$$w_t H_t + q_t K_t + d_t = C_t + I_t + \Psi(K_{t+1}, K_t) \tag{16}$$

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\Pi} + v_t \tag{17}$$

Stochastic Processes

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a \tag{18}$$

$$\ln(\chi_t) = \rho_\chi \ln(\chi_{t-1}) + \varepsilon_t^\chi \tag{19}$$

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \varepsilon_t^v \tag{20}$$

$$\ln(e_t) = \rho_e \ln(e_{t-1}) + (1-\rho_e) \ln(e) + \varepsilon_t^e \tag{21}$$

$$\mu_t = X_t / X_{t-1} = exp(\gamma + \varepsilon_t^X) \tag{22}$$

$$ln(X_t) = ln(X_{t-1}) + \gamma + \varepsilon_t^X \tag{23}$$

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \varepsilon_t^A \tag{24}$$

Steady-State

$$\mu = exp(\gamma) \tag{25}$$