Handout Class 3

Advanced Macroeconomics, Fall 2011 - Alessandro Di Nola

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1 The model economy

1.1 The representative household problem

We assume that the representative household owns the capital stock, takes sequences of wages and interest rates as given and in every period chooses consumption and capital to be broght into tomorrow (intertemporal trade-off). It also faces an intratemporal trade-off between leisure and consumption. We also assume that households own the firm, i.e. are claimants of the firms profits Π_t. Formally, the representative household solves the following problem:

$$\max_{\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \gamma H_t \right] \right\}$$

s.t.

$$\begin{aligned} C_t + K_{t+1} &= W_t H_t + r_t K_t + (1-\delta) K_t + \Pi_t \\ C_t,\ K_{t+1} &\geq 0 \\ K_0 &> 0 \ \text{ given} \end{aligned}$$

The solution to the household's problem is represented by the following equations:

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \left[r_{t+1} + 1 - \delta \right] \right\}, \tag{1}$$

$$\gamma = \frac{W_t}{C_t}, \tag{2}$$

$$C_t + K_{t+1} = W_t H_t + r_t K_t + (1-\delta) K_t + \Pi_t. \tag{3}$$

1.2 The representative firm's problem

The representative firm's problem is, given a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$,

$$\Pi_0 = \max_{\{Y_t, K_t, H_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} [Y_t - r_t K_t - W_t H_t] \right\}$$

s.t.

$$\begin{aligned} Y_t &= z_t K_t^{\theta} \left(\eta^t H_t\right)^{1-\theta}, \\ \log(z_{t+1}) &= (1-\rho)\log(z) + \rho \log(z_t) + \varepsilon_{t+1} \end{aligned}$$

where η^t denotes labor-augmenting technological progress. Actually this is not a true dynamic problem since the variables chosen in period t, (Y_t, K_t, H_t) do not affect the constraints nor returns at later periods. Therefore the above problem is equivalent to

$$\max_{\{K_t, H_t\}} z_t K_t^{\theta} \left(\eta^t H_t\right)^{1-\theta} - r_t K_t - W_t H_t$$

and the optimality conditions are

$$(1-\theta)\frac{Y_t}{H_t} = W_t, \tag{4}$$

$$\theta \frac{Y_t}{K_t} = r_t, \tag{5}$$

$$Y_t = z_t K_t^{\theta} \left(\eta^t H_t\right)^{1-\theta}. \tag{6}$$

Note that the constant returns to scale assumption together with perfect competition implies that the firm's profits in period t are

$$\begin{aligned} \Pi_t &= Y_t - r_t K_t - W_t H_t \\ &= Y_t - \theta \frac{Y_t}{K_t} K_t - (1-\theta)\frac{Y_t}{H_t} H_t \\ &= 0 \end{aligned}$$

It follows that the representative household, as owner of the firm, does not receive any profits in equilibrium.

1.3 Definition of Equilibrium

Now we are ready to state formally what we mean by competitive equilibrium.

Definition 1 *A competitive equilibrium (C.E.) is a sequence of prices* $\mathbf{q} = \{w_t, r_t\}_{t=0}^{\infty}$, *allocations for the representative household* $\mathbf{h} = \left\{C_t, H_t^s, K_{t+1}^s\right\}_{t=0}^{\infty}$ *and allocations for the representative firm* $\mathbf{f} = \left\{H_t^d, K_t^d\right\}_{t=0}^{\infty}$ *such that given* $\mathbf{q}$*:*

1. **h** solves the household maximization problem;

2. **f** solves the firm maximization problem;

3. all markets clear, i.e. $H_t^s = H_t^d$, $K_t^s = K_t^d$, and $C_t + K_{t+1} - (1-\delta)K_t = Y_t$.

Now we can state the two fundamental theorems of welfare economics. Remember that we are in a perfect competition environment with a convex production technology, so both theorems apply.

Definition 2 *A feasible allocation* $\mathbf{x} = \{K_{t+1}, C_t, H_t\}_{t=0}^{\infty}$ *is Pareto optimal if there is no other feasible allocation that yields higher utility to the agent.*

Theorem 3 *(**First Welfare Theorem**). A C.E. allocation is Pareto optimal.*

Proof. (By contradiction). Assume that $\mathbf{x}$ is a CE (with prices $\mathbf{q}$) and assume that it is not Pareto-optimal, i.e. there exists another feasible allocation $\mathbf{x}' \neq \mathbf{x}$ that yields higher utility to the agent. Since, given $\mathbf{q}$, $\mathbf{x}$ was maximizing the household's utility, it must be that $\mathbf{x}'$ violates household's budget constraint, i.e.

$$C_t' + K_{t+1}' > W_t H_t' + r_t K_t' + (1-\delta)K_t'$$

Moreover, since $\mathbf{x}'$ is feasible it must satisfy $C_t' + K_{t+1}' - (1-\delta)K_t' = Y_t'$ But then,rearranging terms we have that

$$\Pi' = Y_t' - W_t H_t' - r_t K_t' > 0$$

This contradicts that $\mathbf{x}$ is a CE since $\mathbf{x}$ does not maximize firm's profit (recall that $\Pi = 0$). ■

Theorem 4 *(**Second Welfare Theorem**).A Pareto optimal allocation* $\mathbf{x}$ *can always be obtained as a C.E. for some prices.*

Proof. Omitted. ■

Collecting all equations together the CE allocation of the model can be characterized by the following non-linear equations:

$$\begin{aligned}
\frac{1}{C_t} &= \beta E_t \left\{ \frac{1}{C_{t+1}} \left[r_{t+1} + 1 - \delta \right] \right\}, & (7)\\
\gamma &= \frac{W_t}{C_t}, & (8)\\
C_t + K_{t+1} &= W_t H_t + r_t K_t + (1-\delta) K_t, & (9)\\
(1-\theta)\frac{Y_t}{H_t} &= W_t, & (10)\\
\theta \frac{Y_t}{K_t} &= r_t, & (11)\\
Y_t &= z_t K_t^{\theta} \left(\eta^t H_t \right)^{1-\theta}, & (12)\\
C_t + K_{t+1} - (1-\delta) K_t &= Y_t, & (13)\\
\log(z_{t+1}) &= (1-\rho)\log(z) + \rho \log(z_t) + \varepsilon_{t+1}. & (14)
\end{aligned}$$

We have 8 equations and 7 variables: $C_t, H_t, K_t, Y_t, W_t, r_t, z_t$. Is there something wrong? Actually not, since Walras' Law tells us that once the labor and capital market are in equilibrium also the goods market clears, and therefore the goods market equilibrium equation (13) can be safely omitted. In fact equation (13) can be obtained by combining (9), (10) and (11), hence it is redundant.

1.4 Stationary system

The equilibrium characterized above is non-stationary due to labour-augmenting technological progress. We need to transform the system into a stationary one by using per efficiency unit variables instead of per capita ones:

$$\begin{aligned}
\frac{1}{c_t} &= \frac{\beta}{\eta} E_t \left\{ \frac{1}{c_{t+1}} \left[r_{t+1} + 1 - \delta \right] \right\}, & (15)\\
\gamma &= \frac{w_t}{c_t}, & (16)\\
c_t + \eta k_{t+1} &= y_t + (1-\delta) k_t, & (17)\\
(1-\theta)\frac{y_t}{h_t} &= w_t, & (18)\\
\theta \frac{y_t}{k_t} &= r_t, & (19)\\
y_t &= z_t k_t^{\theta} h_t^{1-\theta}, & (20)\\
\log(z_{t+1}) &= (1-\rho)\log(z) + \rho \log(z_t) + \varepsilon_{t+1}. & (21)
\end{aligned}$$

where $c_t = C_t/\eta^t$, $y_t = Y_t/\eta^t$, $k_t = K_t/\eta^t$, $w_t = W_t/\eta^t$ are the transformed per efficiency unit variables and $h_t = H_t$.

1.5 Steady-state

In a stationary steady-state without any shocks the steady-state value of z_t is $\overline{z}$. The Euler equation gives

$$r = \frac{1}{\widetilde{\beta}} - 1 + \delta,$$

where $\widetilde{\beta} = \frac{\beta}{\eta}$. From (9) and (11) we can find k and c as functions of y:

$$k = \frac{\theta}{r} y$$

$$c = \left[1 + \frac{\theta}{r}(1 - \delta - \eta)\right] y$$

Then the steady-state level of output is

$$y = \left[z\left(\frac{\theta}{r}\right)^{\theta}\right]^{\frac{1}{1-\theta}} h$$

and

$$w = \gamma c$$

$$h = \left(\frac{1-\theta}{\gamma}\right)\left[1 - \frac{\theta}{r}(\eta - 1 + \delta)\right]$$

To simplify the notation, let $\lambda \equiv \eta - 1 + \delta$.

1.6 Linearized system

A first-order Taylor expansion about the steady-state yields the following system of linear equations:

$$E_t c_{t+1} - \beta r E_t r_{t+1} = c_t, \quad (22)$$

$$0 = w_t - c_t, \quad (23)$$

$$\eta \frac{k}{y} k_{t+1} = y_t + (1-\delta)\frac{k}{y} k_t - \frac{c}{y} c_t, \quad (24)$$

$$0 = y_t - k_t - r_t, \quad (25)$$

$$0 = y_t - h_t - w_t, \quad (26)$$

$$0 = -y_t + z_t + \theta k_t + (1-\theta) h_t, \quad (27)$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}. \quad (28)$$

1.7 System in matrix form

Now we try to solve the model following the same procedure of handout 2. First we recast equations (22) to (28) in following matrix form:

$$\mathbf{A}E_t\mathbf{s}_{t+1}^0 = \mathbf{B}\mathbf{s}_t^0 + \mathbf{C}z_t \tag{29}$$

where $\mathbf{s}_t^0 = [k_t,\ y_t,\ r_t,\ w_t,\ h_t,\ c_t]^T$ and the matrices $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ have dimension $6*6$, $6*6$ and $6*1$ respectively. Then the subsequent step would be to premultiply both sides of (29) by $\mathbf{A}^{-1}$, *provided that* $\mathbf{A}$ *is non singular*, and to apply the Jordan decomposition to $\mathbf{D} \equiv \mathbf{A}^{-1}\mathbf{B}$ in order to diagonalize the system. In this case we have:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -\beta r & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \eta\frac{k}{y} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ (1-\delta)\frac{k}{y} & 1 & 0 & 0 & 0 & -\frac{c}{y} \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ \theta & -1 & 0 & 0 & 1-\theta & 0 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

1.8 Blanchard-Kahn under singularity problem

At this point a serious problem arises: in the baseline RBC model A is not invertible (it has $\mathbf{det} = 0$). This is a general feature when you include "static variables" like y_t or h_t in your model. For the static equations (such as production function or FOC for labor), in fact, the $t+1$ coefficients are all zero by construction. This implies that the matrix A is singular.

In the literature on solving linear RA models there are basically three ways to deal with such non-invertibility problem:

- Eliminate all static variables from the model so that you are left with two dynamic equations for c_t and k_t (plus the stochastic process for z_t)

- Rewrite the model the other way:

$$\mathbf{A}\mathbf{E}_t\left(\mathbf{x}_{t+1}\right) = \mathbf{B}\mathbf{x}_t$$

Then, even if A is not invertible, B is generally invertible, so you can write

$$\mathbf{x}_t = \mathbf{B}^{-1}\mathbf{A}\mathbf{E}_t\left(\mathbf{x}_{t+1}\right)$$

and apply the eigenvalue-eigenvector decomposition to $\mathbf{B}^{-1}\mathbf{A}$.

- Use the generalized Schur decomposition.

In the subsequent analysis I will follow the first approach, namely eliminating all static variables from the model. For the reader's convenience I report down the equilibrium equations of the model:

$$\begin{aligned}
E_t c_{t+1} - \beta r E_t r_{t+1} &= c_t, && (30)\\
w_t &= c_t, && (31)\\
\eta \frac{k}{y} k_{t+1} &= y_t + (1-\delta)\frac{k}{y} k_t - \frac{c}{y} c_t, && (32)\\
y_t - r_t &= k_t, && (33)\\
y_t - w_t - h_t &= 0, && (34)\\
y_t + (\theta - 1) h_t &= z_t + \theta k_t, && (35)\\
z_{t+1} &= \rho z_t + \varepsilon_{t+1}. && (36)
\end{aligned}$$

Let the matrix form be:

$$\mathbf{A}\begin{bmatrix} k_{t+1} \\ \mathbf{f}_{t+1} \end{bmatrix} = \mathbf{B}\begin{bmatrix} k_t \\ \mathbf{f}_t \end{bmatrix} + \mathbf{C} z_t$$

where $\mathbf{f}_t = [y_t,\ r_t,\ w_t,\ h_t,\ c_t]^T$ is the vector of endogenous non-predetermined (or forward-looking, or control) variables, k_t is the only endogenous predetermined (or state) variable and z_t is the exogenous state variable. Clearly A is not invertible. I do the following steps:

1. Divide the variables into a vector of dynamic variables (i.e. indexed by t and $t+1$) and static variables (i.e. indexed only by t)

2. Substitute out the static variables

Let the vector of dynamic variables be $\mathbf{s}_t^0 = [k_t,\ c_t]^T$ and the vector of static variables be $\mathbf{f}_t^0 = [y_t,\ r_t,\ w_t,\ h_t]^T$. The dynamic equations are

$$E_t c_{t+1} - \beta r E_t r_{t+1} = c_t$$

$$\eta \frac{k}{y} k_{t+1} = y_t + (1-\delta)\frac{k}{y} k_t - \frac{c}{y} c_t$$

We can collect them in

$$\mathbf{D} E_t \mathbf{s}_{t+1}^0 + \mathbf{F} E_t \mathbf{f}_{t+1}^0 = \mathbf{G}\mathbf{s}_t^0 + \mathbf{H}\mathbf{f}_t^0$$

hence

$$\mathbf{D}=\begin{bmatrix}0 & 1\\ \eta\frac{k}{y} & 0\end{bmatrix}\quad \mathbf{F}=\begin{bmatrix}0 & -\beta r & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\quad \mathbf{G}=\begin{bmatrix}0 & 1\\ (1-\delta)\frac{k}{y} & -\frac{c}{y}\end{bmatrix}$$

and

$$\mathbf{H}=\begin{bmatrix}0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}.$$

The static variables are instead:

$$\begin{aligned} w_t &= c_t, && (37)\\ y_t - r_t &= k_t, && (38)\\ y_t - w_t - h_t &= 0, && (39)\\ y_t + (\theta-1)h_t &= z_t + \theta k_t, && (40)\end{aligned}$$

which can be written as:

$$\mathbf{A}\mathbf{f}_t^0 = \mathbf{B}\mathbf{s}_t^0 + \mathbf{C}z_t$$

where

$$\mathbf{A}=\begin{bmatrix}0 & 0 & 1 & 0\\ 1 & -1 & 0 & 0\\ 1 & 0 & -1 & -1\\ 1 & 0 & 0 & \theta-1\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}0 & 1\\ 1 & 0\\ 0 & 0\\ \theta & 0\end{bmatrix}\quad \mathbf{C}=\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}$$

Hence the matrix form of the RBC model (step 1) is:

$$\begin{aligned} \mathbf{D}E_t\mathbf{s}_{t+1}^0 + \mathbf{F}E_t\mathbf{f}_{t+1}^0 &= \mathbf{G}\mathbf{s}_t^0 + \mathbf{H}\mathbf{f}_t^0, && (41)\\ \mathbf{A}\mathbf{f}_t^0 &= \mathbf{B}\mathbf{s}_t^0 + \mathbf{C}z_t, && (42)\\ z_{t+1} &= \rho z_t + \varepsilon_{t+1}. && (43)\end{aligned}$$

To implement step 2 delined above, we have now to get rid of the static equations. Premultiplying both sides of (42) by A^{-1} yields:

$$\mathbf{f}_t^0 = \mathbf{A}^{-1}\mathbf{B}\mathbf{s}_t^0 + \mathbf{A}^{-1}\mathbf{C}z_t \tag{44}$$

Plugging (44) into (41) delivers:

$$\mathbf{D}E_t\mathbf{s}_{t+1}^0 + \mathbf{F}\left[\mathbf{A}^{-1}\mathbf{B}E_t\mathbf{s}_{t+1}^0 + \mathbf{A}^{-1}\mathbf{C}E_t z_{t+1}\right] = \mathbf{G}\mathbf{s}_t^0 + \mathbf{H}\left[\mathbf{A}^{-1}\mathbf{B}\mathbf{s}_t^0 + \mathbf{A}^{-1}\mathbf{C}z_t\right]$$

Using the property $E_t z_{t+1} = \rho z_t$ and rearranging gives:

$$E_t\mathbf{s}_{t+1}^0 = \mathbf{K}\mathbf{s}_t^0 + \mathbf{L}z_t \tag{45}$$

where $\mathbf{K} = \left(\mathbf{D} + \mathbf{F}\mathbf{A}^{-1}\mathbf{B}\right)^{-1}\left(\mathbf{G} + \mathbf{H}\mathbf{A}^{-1}\mathbf{B}\right)$ and $\mathbf{L} = \left(\mathbf{D} + \mathbf{F}\mathbf{A}^{-1}\mathbf{B}\right)^{-1}\left(\mathbf{H}\mathbf{A}^{-1}\mathbf{C} - \mathbf{F}\mathbf{A}^{-1}\mathbf{C}\rho\right)$. Note that (45) is made only of dynamic variables and dynamic equations.

1.9 Parameter values

The following parameter values are used:

$$\begin{aligned}
\beta &= 0.99 \\
\gamma &= 0.0045 \\
\eta &= 1.0039 \\
\theta &= 0.2342 \\
z &= 6.0952 \\
\delta &= 0.025 \\
\rho &= 0.9983 \\
\sigma_e^2 &= 0.00025
\end{aligned}$$

It implies that system (45) is:

$$E_t \begin{bmatrix} k_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} 1.1373 & -0.6802 \\ 0 & 0.8882 \end{bmatrix} \begin{bmatrix} k_t \\ c_t \end{bmatrix} + \begin{bmatrix} 0.7090 \\ 0.1458 \end{bmatrix} z_t$$

You can check this by running the code `handout_class3.m` (see my webpage). Note that (45) and (43) can be written together as:

$$\mathbf{E}_t \mathbf{x}_{t+1} = \mathbf{V} \mathbf{x}_t \tag{46}$$

where

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{s}_t^0 \\ z_t \end{bmatrix} = \begin{bmatrix} k_t & c_t & z_t \end{bmatrix}^T, \quad \mathbf{V} = \begin{bmatrix} \mathbf{K} & \mathbf{L} \\ 0 & \rho \end{bmatrix}$$

or, using the parameter values given above,

$$E_t \begin{bmatrix} k_{t+1} \\ c_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} 1.1373 & -0.6802 & 0.7090 \\ 0 & 0.8882 & 0.1458 \\ 0 & 0 & 0.9983 \end{bmatrix} \begin{bmatrix} k_t \\ c_t \\ z_t \end{bmatrix}$$

The Jordan decomposition $\mathbf{V} = \mathbf{M} \boldsymbol{\Lambda} \mathbf{M}^{-1}$ gives the following representation of the system:

$$\mathbf{M}^{-1} \mathbf{E}_t \mathbf{x}_{t+1} = \boldsymbol{\Lambda} \mathbf{M}^{-1} \mathbf{x}_t$$

Let M_{ij} denote the (i, j) element of matrix $\mathbf{M}^{-1}$. Letting furthermore $\widetilde{x}_t = \mathbf{M}^{-1} \mathbf{x}_t$ we have the decoupled system:

$$\mathbf{E}_t \widetilde{x}_{t+1} = \boldsymbol{\Lambda} \widetilde{x}_t$$

which can also be written as

$$E_t \widetilde{x}_{i,t+1} = \lambda_i \widetilde{x}_{i,t}, \ \ \forall i = 1, 2, 3$$

Clearly if $|\lambda_i| > 1$ it is the case that $\lim_{t\to\infty} \widetilde{x}_{i,t} = \pm\infty$ *unless you set* $\widetilde{x}_{i,t} = 0\ \forall t$, which gives you the equation for the *saddlepath*. In this model we have

$$E_t \begin{bmatrix} \widetilde{x}_{1,t+1} \\ \widetilde{x}_{2,t+1} \\ \widetilde{x}_{3,t+1} \end{bmatrix} = \begin{bmatrix} 1.1373 & 0 & 0 \\ 0 & 0.8882 & 0 \\ 0 & 0 & 0.9983 \end{bmatrix} \begin{bmatrix} \widetilde{x}_{1,t} \\ \widetilde{x}_{2,t} \\ \widetilde{x}_{3,t} \end{bmatrix}$$

which implies the restriction

$$\widetilde{x}_{1,t} = 0 \iff M_{11}k_t + M_{12}c_t + M_{13}z_t = 0$$

from which

$$c_t = \eta_{ck}k_t + \eta_{cz}z_t \tag{47}$$

where $\eta_{ck} = -\frac{M_{11}}{M_{12}}, \eta_{cz} = -\frac{M_{13}}{M_{12}}$. To get the solution for capital, simply substitute (47) into the first row of (46):

$$\begin{aligned} k_{t+1} &= V_{11}k_t + V_{12}c_t + V_{13}z_t \\ &= \left(V_{11} + V_{12}\eta_{ck}\right)k_t + \left(V_{13} + V_{12}\eta_{cz}\right)z_t. \end{aligned}$$

Letting $\eta_{kk} = V_{11} + V_{12}\eta_{ck}$ and $\eta_{kz} = V_{13} + V_{12}\eta_{cz}$ the state-space form is:

$$\begin{bmatrix} k_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} \eta_{kk} & \eta_{kz} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} k_t \\ z_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varepsilon_{t+1},$$

or, more compactly,

$$\mathbf{s}_{t+1} = \mathbf{\Pi}\mathbf{s}_t + \mathbf{W}\boldsymbol{\varepsilon}_{t+1},$$

where $\mathbf{s}_t = [k_t,\ z_t]^T$. Finally it is easy to find the vector equation relating the static variables to the states:

$$\mathbf{f}_t = \mathbf{U}\mathbf{s}_t,$$

where $\mathbf{f}_t = [y_t\ r_t\ w_t\ h_t\ c_t]^T$ and $\mathbf{U}$ is a $5 * 2$ matrix.

Exercise 5 *Given the parameter values I provided you, compute the matrices* $\mathbf{\Pi}$, $\mathbf{W}$ *and* $\mathbf{U}$. *(Hint: run the file* `handout_class3.m`*).*

Exercise 6 *Given the state space form you get from previous exercise, compute impulse responses to a one-percent technology shock for capital, consumption and the interest rate. Compute also the impulse responses to a one-percent deviation of capital from its steady-state value (Hint: recall that if* $k_t = \overline{k}$*, then* $\widehat{k}_t = 0$*).*