Employment Dynamics

The model consists of two sector, formal and informal. Workers in the informal sector are self-employed, whereas the formal labor market is characterized by search frictions à la Mortensen-Pissarides.

Labor force is normalized to one:

$$1 = N_{F,t} + N_{I,t} + U_t \tag{1}$$

In each period, define the mass of workers searching for a job (the *searchers*) in the following way:

$$S_t = 1 - (1-\rho)N_{F,t-1} - N_{I,t} \tag{2}$$

The matching technology is given by

$$\mathcal{M}(S_t, v_t) = \mathcal{M} S_t^{\xi} v_t^{1-\xi} \tag{3}$$

The vacancy filling rate is

$$q(\theta_t) = \frac{\mathcal{M}_t}{v_t} = \mathcal{M}\theta_t^{-\xi} \tag{4}$$

and the probability for a searcher of finding a job is

$$f(\theta_t) = \frac{\mathcal{M}_t}{S_t} = \mathcal{M}\theta_t^{1-\xi} \tag{5}$$

where

$$\theta_t = \frac{v_t}{S_t} \tag{6}$$

is the labor market tightness. Given these equations, employment in the formal sector evolves according to

$$N_{F,t} = (1-\rho)N_{F,t-1} + v_t q(\theta_t) \tag{7}$$

The Representative Household

The representative household work informally as a self-employed, work in the formal sector as an employee, consumes a consumption bundle made by formal and informal goods, and demand money since transactions in the informal sector have to be regulated in cash. He maximizes the following utility

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log \tilde{C}_t - \chi \frac{N_{I,t}^{1+\varphi}}{1+\varphi} \right\} \tag{8}$$

where

$$\tilde{C}_t = \left(\omega C_{F,t}^{\eta} + (1-\omega) C_{I,t}^{\eta}\right)^{\frac{1}{\eta}} \tag{9}$$

subject to a budget constraint

$$P_t C_{F,t}+P_{I,t}C_{I,t}+E_t Q_{t,t+1}B_{t+1}+M_t-M_{t-1} = W_{F,t}N_{F,t}(1-\tau_t^w)+P_{I,t}Y_{I,t}+B_t+bU_t+D_{F,t} \quad (10)$$

a CIA constraint which motivates money demand

$$M_t \geq P_{I,t}C_{I,t} \quad (11)$$

and the informal technology constraint

$$Y_{I,t} = z_{I,t}N_{I,t} \quad (12)$$

Making use of equations (1), (2) and (7), the variables $N_{F,t}$ and U_t can be conveniently written in a way to make the trade-off between working as an informal self-employed and participating in the formal labor market explicit:

$$N_{F,t} = (1-\rho)(1-f(\theta_t))N_{F,t-1} + f(\theta_t)(1-N_{I,t}) \quad (13)$$

$$U_t = 1-(1-\rho)(1-f(\theta_t))N_{F,t-1} - f(\theta_t) - (1-f(\theta_t))N_{I,t} \quad (14)$$

The first order conditions for this problem are as follows:

$$\frac{\omega}{\tilde{C}_t}\left(\frac{\tilde{C}_t}{C_{F,t}}\right)^{1-\eta} = (1+\tau_t^c)\lambda_t \quad (15)$$

$$\frac{(1-\omega)}{\tilde{C}_t}\left(\frac{\tilde{C}_t}{C_{I,t}}\right)^{1-\eta} = p_{I,t}\lambda_t(1+\nu_t) \quad (16)$$

$$p_{I,t}z_{I,t} = \frac{\chi N_{I,t}^{\varphi}}{\lambda_t} + f(\theta_t)(1-\tau_t^w)w_{F,t} + (1-f(\theta_t))b \quad (17)$$

$$E_t Q_{t,t+1} = \frac{1}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\Pi_{t+1}} \quad (18)$$

$$\nu_t = \left(1-R_t^{-1}\right) \quad (19)$$

The Final Good Producers

The final good producer in the formal sector operates under perfect competition. The final good in both sectors is a CES aggregate of a continuum of aggregates:

$$Y_{F,t} = \left(\int_0^1 Y_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

Profit maximization gives the demand for the j^{th} intermediate good:

$$Y_{F,t}(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_{F,t} \tag{20}$$

where the associated price index is given by

$$P_t = \left(\int_0^1 P_t(j)dj\right)^{\frac{1}{1-\epsilon}}$$

Formal Sector Firms

Firms in the formal sector operates in monopolistic competition. They post vacancies, hire workers, and set their price in order to maximize profits subject to a quadratic adjustment cost. Firm i profits reads as follows:

$$\begin{aligned} D_{F,t+n}(j) = P_{t+n}(j)Y_{F,t+n}(j) - W_{F,t+n}(j)N_{F,t+n}(j) - \kappa v_{t+n}(j) \\ - \frac{\phi_{F,p}}{2}\left(\frac{P_{t+n}(j)}{P_{t+n-1}(j)} - 1\right)^2 P_{t+n}Y_{F,t+n}(j) \end{aligned} \tag{21}$$

where

$$Y_{F,t} = z_{F,t}N_{F,t} \tag{22}$$

The first order conditions of this problem are

$$\mu_t = mc_{F,t}z_{F,t} - w_{F,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\mu_{t+1} \tag{23}$$

$$\frac{\kappa}{q(\theta_t)} = \mu_t \tag{24}$$

$$\epsilon - 1 - \epsilon mc_{F,t} + \phi_{F,p}(\Pi_t - 1)\Pi_t - \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\phi_{F,p}(\Pi_{t+1} - 1)\Pi_{t+1}\frac{z_{F,t+1}N_{F,t+1}}{z_{F,t}N_{F,t}} \tag{25}$$

Nash Wage Setting

The formal sector wage is set according to a standard Nash solution. The value functions for firms, employed and unemployed workers are, respectively:

$$V_t^F = mc_{F,t}z_{F,t} - w_{F,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left\{(1-\rho)V_{t+1}^F\right\} \tag{26}$$

$$V_t^E = w_{F,t}(1-\tau_t^w) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left\{(1-\rho)V_{t+1}^E + \rho\theta_{t+1}q(\theta_{t+1})V_{t+1}^E + \rho(1-\theta_{t+1}q(\theta_{t+1}))V_{t+1}^U\right\} \tag{27}$$

$$V_t^U = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \theta_{t+1} q(\theta_{t+1}) V_{t+1}^E + (1 - \theta_{t+1} q(\theta_{t+1})) V_{t+1}^U \right\} \tag{28}$$

The Nash bargaining rule is

$$\frac{\psi}{1-\psi}(1-\tau_t^w)V_t^F = V_t^E - V_t^U \tag{29}$$

which results in the following equation for the wage rate

$$w_{F,t} = \psi mc_t z_{F,t} + \frac{(1-\psi)}{(1-\tau_t^w)} b - \psi\beta \frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\frac{\kappa}{q(\theta_{t+1})}\left\{(1-\theta_{t+1}q_{t+1})\frac{(1-\tau_{t+1}^w)}{(1-\tau_t^w)} - 1\right\} \tag{30}$$

Government and Aggregation

Public expenditure and the payment of unemployment benefits are financed through tax collection and the issue of money (I will consider both the availability and the non-availability of lump-sum taxes)

$$G_t + bU_t = \tau_t^w N_{F,t} w_{F,t} + \tau_t^c C_{F,t} + m_t - \frac{m_{t-1}}{\Pi_t} + T_t^{LS} \tag{31}$$

The aggregate resource constraints of the economy are:

$$Y_{I,t} = C_{I,t} \tag{32}$$

$$Y_{F,t} = C_{F,t} + G_t + \kappa v_t + \frac{\phi}{2}(\Pi_t - 1)^2 Y_{F,t} \tag{33}$$