The Financial Accelerator in a New Keynesian Model: a Calibration and Estimation Exercise

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Appendix
Work based on Christensen and Dib (2008)

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1 The Model

1.1 Households

One period utility function of the household:

$$u(\cdot) = \frac{\gamma e_t}{\gamma - 1} \log \left[c_t^{\frac{\gamma-1}{\gamma}} \ + \ b_t^{1/\gamma} \left(\frac{M_t}{p_t} \right)^{\frac{\gamma-1}{\gamma}} \right] \ + \ \eta \log(1 - h_t) \tag{1}$$

where γ and η denote the constant elasticity of substitution between consumption and real balances, and the weight of leisure in the utility function, respectively. e_t and b_t are to be interpreted as a preference and money-demand shock, respectively. They follow the processes

$$\log(e_t) = \rho_e \log(e_{t-1}) + \varepsilon_{et}, \tag{2}$$

and

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \tag{3}$$

where $\rho_e, \rho_b \in (-1, 1)$, b is a constant, while ε_{et} and ε_{bt} are serially uncorrelated, normally distributed shocks with zero means and standard deviations σ_e and σ_b, respectively.

The household faces the following budget constraint:

$$P_t c_t \ + \ M_t \ + \ D_t \leq W_t h_t \ + \ R_{t-1} D_{t-1} \ + \ M_{t-1} \ + \ T_t \ + \ \Omega_t \tag{4}$$

where D_{t-1} and M_{t-1} are the nominal deposits in a financial intermediary and the nominal money balances, respectively, of the household at the beginning of period t. Deposits D_t pay the gross nominal interest rate R_t between t and $t + 1$ and W_t is the nominal wage. In addition, households receive a lump-sum transfer T_t from the monetary authority, as well as dividend payments Ω_t from retail firms. Finally, the household chooses c_t, M_t, h_t and D_t to maximize their lifetime utility, subject to 4.

First order conditions:

$$\frac{e_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t \tag{5}$$

$$\frac{e_t b_t^{1/\gamma} m^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{6}$$

$$\frac{\eta}{1 - h_t} = \lambda_t w_t \tag{7}$$

$$\frac{\lambda_t}{R_t} = \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{8}$$

1.2 Entrepreneurs

The optimal demand for capital (FOC) is:

$$E_t f_{t+1} = E_t \left[\frac{z_{t+1} + (1-\delta) q_{t+1}}{q_t} \right], \tag{9}$$

where $E_t f_{t+1}$ is the expected marginal external financing cost at $t+1$, δ is the capital depreciation rate and z_{t+1} is the marginal productivity of capital at $t+1$. In turn, the demand for capital implies:

$$E_t f_{t+1} = E_t \left[S \left(\frac{n_{t+1}}{q_t k_{t+1}} \right) R_t / \pi_{t+1} \right], \tag{10}$$

with $S'(\cdot) < 0$ and $S(1) = 1$. I set $S(\cdot)$ such that in addition with these characteristics it yields a consistent value for S in the steady state, given the steady state values of n, k, R, π and f. In particular, I define

$$S(n_{t+1}, q_t, k_{t+1}) = \left(\frac{n_{t+1}}{q_t k_{t+1}} \right)^{-1/\kappa}. \tag{11}$$

with $\kappa > 0$ is set consistently with the steady state values for n, k, q, and S. Aggregate entrepreneurial net worth evolves according to

$$n_{t+1} = \nu [f_t q_{t-1} k_t - E_{t-1} f_t (q_{t-1} k_t - n_t)], \tag{12}$$

which is nothing but the real return on capital minus the cost of borrowing of surviving entrepreneurs. Dying firms simply consume any capital remains and depart from the scene. Entrepreneurs use a constant-returns-to-scale technology to produce output y_t, given by

$$y_t \leq k_t^{\alpha} (A_t h_t)^{1-\alpha}, \quad \alpha \in (0,1), \tag{13}$$

where A_t is a technology shock common to all entrepreneurs ruled by the stationary first-order autoregressive process

$$\log(A_t) = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At} \tag{14}$$

where $\rho_A \in (0,1)$, $A > 0$ is constant and ε_{At} is normally distributed with zero mean and standard deviation σ_A. Entrepreneurs solve the typical maximization problem under perfect competition. The FOCs of this problem are:

$$z_t = \alpha \xi_t \frac{y_t}{k_t} \tag{15}$$

$$w_t = (1-\alpha)\xi_t \frac{y_t}{h_t} \tag{16}$$

$$y_t = k_t^{\alpha}(A_t h_t)^{1-\alpha} \tag{17}$$

1.3 Capital producers

Capital producers use a linear technology to solve the following problem:

$$\max_{i_t} E_t \left[q_t x_t i_t - i_t - \frac{\chi}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 k_t \right], \tag{18}$$

where the last term of the squared bracket shows quadratic capital adjustment costs, x_t is an investment-specific shock and i_t represents investment goods (expressed in units of consumption units of the final good), which are combined with the existing capital stock to produce new capital goods, k_{t+1}. The FOC is given by

$$q_t x_t - 1 - \chi \left(\frac{i_t}{k_t} - \delta \right) = 0, \tag{19}$$

while the investment specific shock follows

$$\log(x_t) = \rho_x \log(x_{t-1}) + \varepsilon_{xt}, \tag{20}$$

with $\rho_x \in (0,1)$ and ε_{xt} is normally distributed with mean zero and standard deviation σ_x. In turn, the aggregate capital stock evolves according to

$$k_{t+1} = x_t i_t + (1-\delta)k_t. \tag{21}$$

1.4 Retailers

Retailers solve the problem:

$$\max_{\widetilde{p}_t(j)} E_0 \left[\sum_{n=1}^{\infty} (\beta\phi)^l \lambda_{t+l} \Omega_{t+l}(j)/p_{t+l} \right] \tag{22}$$

subject to the demand function

$$y_{t+l}(j) = \left(\frac{\widetilde{p}_t(j)}{p_{t+l}} \right)^{-\theta} y_{t+l}, \tag{23}$$

with the nominal profit function $\Omega_{t+l}(j) = (\pi^l \tilde{p}_t(j) - p_{t+l}\xi_{t+l})y_{t+l}(j)$. The FOC for $\tilde{p}_t(j)$ is

$$\tilde{p}_t(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l}(j) \xi_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l}(j) \pi^l / p_{t+l}} \tag{24}$$

and the aggregate price index can be expressed as

$$1 = \phi \left(\frac{\pi}{\pi_t} \right)^{1-\theta} + (1 - \phi) \left(\frac{\tilde{p}_t}{p_t} \right)^{1-\theta} \tag{25}$$

In turn, we have to find a recursive expression for $\tilde{p}_t(j)$. Since we only consider the symmetric equilibrium, we can drop the subscript j. In fact, we can write

$$\frac{\tilde{p}_t}{p_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l} \xi_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l} \pi^l \prod_{i=1}^{l} \pi_{t+i}^{-1}} \tag{26}$$

and define

$$(\theta - 1) \underbrace{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l} \pi^l \prod_{i=1}^{l} \pi_{t+i}^{-1} \frac{\tilde{p}_t}{p_t}}_{g_t^2} = \theta \underbrace{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l} \xi_{t+l}}_{g_t^1} \tag{27}$$

We now find recursive expressions for g_t^1 and g_t^2. For the first,

$$g_t^1 = \lambda_t y_t \xi_t + (\beta\phi) E_t \lambda_{t+l} y_{t+l} \xi_{t+l} + (\beta\phi)^2 E_t \lambda_{t+2} y_{t+2} \xi_{t+2} \dots \tag{28}$$

$$g_t^1 = \lambda_t y_t \xi_t + (\beta\phi) E_t g_{t+1}^1. \tag{29}$$

Similarly, for the second:

$$g_t^2 = \lambda_t y_t \frac{\tilde{p}_t}{p_t} + (\beta\phi) E_t \lambda_{t+l} y_{t+l} \frac{\pi}{\pi_{t+1}} \frac{\tilde{p}_t}{p_t} + (\beta\phi)^2 E_t \lambda_{t+2} y_{t+2} \frac{\pi^2}{\pi_{t+1} \pi_{t+2}} \frac{\tilde{p}_t}{p_t} \dots \tag{30}$$

writing the expression for g_{t+1}^2 and multiplying it by $\beta\phi \frac{\pi}{\pi_{t+1}} \frac{\pi_t^*}{\pi_{t+1}^*}$, where $\pi_t^* = \frac{\tilde{p}_t}{p_t}$, we get:

$$g_t^2 = \lambda_t y_t \frac{\tilde{p}_t}{p_t} + (\beta\phi) E_t \frac{\pi}{\pi_{t+1}} \frac{\pi_t^*}{\pi_{t+1}^*} g_{t+1}^2 \tag{31}$$

so that the FOC for $\tilde{p}_t$ can be expressed recursively as the system composed by 29, 31 and:

$$\theta g_t^1 = (\theta - 1) g_t^2 \tag{32}$$

1.5 Monetary authority

The central bank adjusts the interest rate, R_t, in response to deviations of inflation π_t, output y_t and the money-growth rate $\mu_t = M_t / M_{t-1}$ (or $\mu_t = \frac{m_t \pi_t}{m_{t-1}}$) from their steady-state values. In particular, the monetary policy rule is described by:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\varrho_\pi} \left(\frac{y_t}{y}\right)^{\varrho_y} \left(\frac{\mu_t}{\mu}\right)^{\varrho_\mu} \exp(\varepsilon_{Rt}), \tag{33}$$

where the no-subscript variables represent steady-state values and ε_{Rt} is a normally distributed monetary policy shock with zero mean and standard deviation σ_R. For the Taylor principle to be satisfied and a unique equilibrium to exist, the condition $\varrho_\pi + \varrho_\mu > 1$ must hold.

2 Steady state

The market clearing condition implies

$$y_t = c_t + i_t. \tag{34}$$

To compute the steady state it will be helpful to work with the ratios $\frac{k}{y}$, $\frac{c}{y}$ and $\frac{i}{y}$. In fact, note that from equation 15 we have that

$$\frac{k}{y} = \alpha \frac{\xi}{z}. \tag{35}$$

Additionally, from equation 12 we see that

$$f = \frac{1}{\nu} \tag{36}$$

and from equations 9 and 32, respectively, we have that

$$z = f - 1 + \delta \tag{37}$$

$$\xi = \frac{\theta - 1}{\theta}. \tag{38}$$

Hence, $\frac{k}{y}$ is fully determined with the steady state values of ξ and z, that are entirely determined by the parameters of the model. Also note that equation 10 also implies a steady state value for the gross external finance premium, S, given that

$$f = S \frac{R}{\pi}, \tag{39}$$

and hence

$$S = \frac{\beta}{\nu}. \tag{40}$$

Next, from the law of motion for capital (21) we get

$$\frac{i}{y} = \delta \frac{k}{y} \tag{41}$$

and from the market clearing condition

$$\frac{c}{y} = 1 - \frac{i}{y}. \tag{42}$$

We next move to the FOCs of the household problem. First note that 8 implies a steady state interest rate given by

$$R = \frac{\pi}{\beta}. \tag{43}$$

Then, from 5 we have:

$$\frac{e\, c^{-\frac{1}{\gamma}}}{c^{\frac{\gamma-1}{\gamma}} + b^{1/\gamma}\, m^{\frac{\gamma-1}{\gamma}}} = \lambda. \tag{44}$$

Dividing the numerator and denominator by $c^{\frac{\gamma-1}{\gamma}}$ we get

$$\frac{e\, c^{-1}}{1 + b^{1/\gamma}\, (m/c)^{\frac{\gamma-1}{\gamma}}} = \lambda, \tag{45}$$

or, analogously,

$$\lambda\, c = \frac{e}{1 + b^{1/\gamma}\, (c/m)^{\frac{1-\gamma}{\gamma}}}. \tag{46}$$

From 6 and 8 we have:

$$\frac{e_t b_t^{1/\gamma} m_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \frac{\lambda_t}{R_t}, \tag{47}$$

and using 5 we get

$$e_t b_t^{1/\gamma} m_t^{-\frac{1}{\gamma}} = e_t c_t^{-\frac{1}{\gamma}} \left(\frac{R_t - 1}{R_t} \right), \tag{48}$$

which simplifies to the following steady state expression:

$$\frac{c}{m} = \left(\frac{R-1}{R} \right)^{\gamma} \frac{1}{b}, \tag{49}$$

implying

$$\lambda\, c = \frac{e}{1 + b(R - 1/R)^{1-\gamma}}. \tag{50}$$

On the other hand, the labor market equilibrium implies (from 7 and 16)

$$\frac{\eta}{1-h} = \lambda(1-\alpha)\xi\frac{y}{h} \tag{51}$$

and solving for h yields

$$h = \frac{\lambda(1-\alpha)\xi y}{\eta + \lambda(1-\alpha)\xi y} \tag{52}$$

which, in order to be expressed as a function of known steady state values, can be written as

$$h = \frac{\lambda c(1-\alpha)\xi\left(\frac{c}{y}\right)^{-1}}{\eta + \lambda c(1-\alpha)\xi\left(\frac{c}{y}\right)^{-1}} \tag{53}$$

2.1 Summary of the steady state system

The steady state is summarized by the following values: $e = A = x = q = 1$, $\pi = 0.9999$, $b = 0.0655$, $\beta = 0.9854$, $\theta = 6$, $\delta = 0.025$, $\alpha = 0.33$, $\gamma = 0.0598$, $\eta = 1.315$, $\nu = 0.9782$, $\phi = 0.7418$, $\kappa = 93$. The values for e, A, and x are consistent with the notion of no influence of shocks in the steady state; π is the sample mean of the gross inflation rate (percentage change from preceding period of GDP deflator) and q is normalized to 1 in the steady state. The value for b implies a steady state consumption to real balances ratio consistent with the observed data in the sample. β implies an annual real interest rate of 2.97%, the sample average. $\theta = 6$ is a standard value in the literature and implies a steady state mark-up of 20%; same for δ and α. γ and ϕ are estimated by the authors. η implies that the household spends about 1/3 of the time in market activities. Finally, ν is chosen such that the external finance premium is equal to $S = 1.0075$, which implies a 300 basis points spread between the T-bill and the business prime lending rate, the observed spread in the sample; the value for ν also implies that the expected lifetime of entrepreneurs is of about 45 quarters, or a bit over 11 years.

In turn, the equations describing the steady state system are:

$$R = \frac{\pi}{\beta}$$

$$\xi = \frac{\theta - 1}{\theta}$$

$$f = \frac{1}{\nu}$$

$$S = \frac{\beta}{\nu}$$

$$z = f - 1 + \delta$$

$$h = \frac{\frac{e}{1+b((R-1)/R)^{1-\gamma}}(1-\alpha)\xi\left(1-\delta\left(\alpha\frac{\xi}{z}\right)^{-1}\right)}{\eta + \frac{e}{1+b((R-1)/R)^{1-\gamma}}(1-\alpha)\xi\left(1-\delta\left(\alpha\frac{\xi}{z}\right)^{-1}\right)}$$

$$y = \left(\alpha\frac{\xi}{z}\right)^{\alpha/(1-\alpha)} A\ h$$

$$w = (1-\alpha)\xi A\left(\alpha\frac{\xi}{z}\right)^{\alpha/(1-\alpha)}$$

$$k = y\alpha\frac{\xi}{z}$$

$$i = \delta k$$

$$c = y - i$$

$$n = k\left(\frac{\beta}{\nu}\right)^{-\kappa}$$

$$m = b\ c\left(\frac{R}{R-1}\right)^{\gamma}$$

$$\lambda = \frac{e\ c^{-1/\gamma}}{c^{(\gamma-1)/\gamma} + b^{1/\gamma}m^{(\gamma-1)/\gamma}}$$

$$\mu = \pi$$

$$g^1 = \frac{\lambda\xi y}{1-\phi\beta}$$

$$g^2 = \frac{\lambda y}{1-\phi\beta}$$

3 IFR analysis

4 References

Christensen, I., and A. Dib (2008): "The Financial Accelerator in an Estimated New Keynesian Model," *Review of Economic Dynamics*, 11(1), 155–178.