0.1 A Simple Two-country Model

The world has two countries, the home country and the foreign country. The foreign country variables are distinguished from the home country by a superscript star. All the variables are in per capita terms unless otherwise stated.

The representative agent in the home country maximize expected lifetime utility, given by $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$, where γ is the paramter for the intertemporal substitution, $1/\gamma$ is the elasticity of substitution (EOS). Firms in each country produce a single consumption/investment good that can be traded internationally without frictions.

The budeget counstraint in home country is given by

$$c_t + I_t + B_{t+1} = f(k_t) + (1 + r_t^b)B_t$$

where $I_t = k_{t+1} - (1-\delta)k_t$ is the investment and B_{t+1} is internaional bond.

In a cometitive equilibrium, the first-order conditions for home country is given:

$$u'(c_t) = \beta(1 + r_{t+1}^b)u'(c_{t+1}) \tag{1}$$

$$u'(c_t) = \beta[f'(k_{t+1}) + 1 - \delta]u'(c_{t+1}) \tag{2}$$

where $u'(c_t) = c_t^{-\gamma}$.

The foreing country has simiar optimization conditons. The international bond clearing condition gives us

$$B_t + B_t^* = 0$$

This condtion is equivalent to the following world goods marekt clearing

$$c_t + I_t + c_t^* + I_t^* = f(k_t) + f(k_t^*)$$

The saving rate and current account in home country are given below

$$S = \frac{f(k) - c}{f(k)}. \tag{3}$$

$$CA = B_{t+1} - B_t \tag{4}$$

We can show numerically how asymmetric EOS parameter γ affect the saving rate and current account dynamics in face of TFP shock.