Technical Appendix to: "Inflation Stabilization in a Medium-Scale Macroeconomic Model"

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Abstract

These technical notes derive in detail the equilibrium conditions in stationary form of the model in authors' paper "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model." It also derives in detal the steady state of the model.

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1 The Model

1.1 Households

The economy is assumed to be populated by a large representative family with a continuum of members. Consumption and hours worked are identical across family members. The household's preferences are defined over per capita consumption, c_t, and per capita labor effort, h_t, and are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, h_t), \tag{1}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, and strictly concave. Preferences display internal habit formation, measured by the parameter $b \in [0,1)$. The consumption good is assumed to be a composite made of a continuum of differentiated goods c_{it} indexed by $i \in [0,1]$ via the aggregator

$$c_t = \left[\int_0^1 {c_{it}}^{1-1/\eta} di \right]^{1/(1-1/\eta)}, \tag{2}$$

where the parameter $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods.

For any given level of consumption of the composite good, purchases of each individual variety of goods $i \in [0,1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where P_{it} denotes the nominal price of a good of variety i at time t. The demand for goods of variety i is then given by

$$c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} c_t, \tag{3}$$

where P_t is a nominal price index defined as

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \tag{4}$$

This price index has the property that the minimum cost of a bundle of intermediate goods yielding c_t units of the composite good is given by $P_t c_t$.

Labor decisions are made by a central authority within the household, a union, which supplies labor monopolistically to a continuum of labor markets of measure 1 indexed by $j \in [0,1]$. In each labor market j, the union faces a demand for labor given by $\left(W_t^j/W_t\right)^{-\tilde{\eta}} h_t^d$. Here W_t^j denotes the nominal wage charged by the union in labor market j at time t, W_t is an index of nominal wages prevailing in the economy, and h_t^d is a measure of aggregate labor demand by firms. We postpone a formal derivation of this labor demand function until we consider the firm's problem. In each particular labor market, the union takes W_t and h_t^d as exogenous.[1] Given the wage it charges in each labor market $j \in [0,1]$, the union is assumed to supply enough labor, h_t^j, to satisfy demand. That is,

$$h_t^j = \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d, \tag{5}$$

where $w_t^j \equiv W_t^j/P_t$ and $w_t \equiv W_t/P_t$. In addition, the total number of hours allocated to the different labor markets must satisfy the resource constraint $h_t = \int_0^1 h_t^j dj$. Combining this restriction with equation (5), we obtain

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} dj. \tag{6}$$

Our setup of imperfectly competitive labor markets departs from most existing expositions of models with nominal wage inertia (e.g., Erceg, et al., 2000). For in these models, it is assumed that each household supplies a differentiated type of labor input. This assumption introduces equilibrium heterogeneity across households in the number of hours worked. To avoid this heterogeneity from spilling over into consumption heterogeneity, it is typically assumed that preferences are separable in consumption and hours and that financial markets exist that allow agents to fully insure against employment risk. Our formulation has the advantage that it avoids the need to assume both separability of preferences in leisure and consumption and the existence of such insurance markets. As we will explain later in more detail, our specification gives rise to a wage-inflation Phillips curve with a larger coefficient on the wage-markup gap than the model with employment heterogeneity across households.

The household is assumed to own physical capital, k_t, which accumulates according to the following law of motion

$$k_{t+1} = (1-\delta)k_t + i_t\left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{7}$$

[1] The case in which the union takes aggregate labor variables as endogenous can be interpreted as an environment with highly centralized labor unions. Higher-level labor organizations play an important role in some European and Latin American countries, but are less prominent in the United States.

where i_t denotes gross investment and δ is a parameter denoting the rate of depreciation of physical capital. The function $\mathcal{S}$ introduces investment adjustment costs. It is assumed that in the steady state, the function $\mathcal{S}$ satisfies $\mathcal{S} = \mathcal{S}' = 0$ and $\mathcal{S}'' > 0$. These assumptions imply the absence of adjustment costs up to first-order in the vicinity of the deterministic steady state.

As in Fisher (2005) and Altig et al. (2004), it is assumed that investment is subject to permanent investment-specific technology shocks. Fisher argues that this type of shock is needed to explain the observed secular decline in the relative price of investment goods in terms of consumption goods. More importantly, Fisher argues that investment-specific technology shocks account for about 50 percent of aggregate fluctuations at business-cycle frequencies in the postwar U.S. economy. (As we will discuss below, Altig et al., 2005, find significantly smaller numbers in the context of the model studied in our paper.)

We assume that investment goods are produced from consumption goods by means of a linear technology whereby Υ_t units of consumption goods yield one unit of investment goods, where Υ_t denotes an exogenous, permanent technology shock in period t. The growth rate of Υ_t is assumed to follow an AR(1) process of the form:

$$\hat{\mu}_{\Upsilon,t} = \rho_{\mu_\Upsilon} \hat{\mu}_{\Upsilon,t-1} + \epsilon_{\mu_\Upsilon,t},$$

where $\hat{\mu}_{\Upsilon,t} \equiv \ln(\mu_{\Upsilon,t}/\mu_\Upsilon)$ denotes the percentage deviation of the gross growth rate of investment specific technological change and μ_Υ denotes the steady-state growth rate of Υ_t.

Owners of physical capital can control the intensity at which this factor is utilized. Formally, we let u_t measure capacity utilization in period t. We assume that using the stock of capital with intensity u_t entails a cost of $\Upsilon_t^{-1} a(u_t) k_t$ units of the composite final good. The function a is assumed to satisfy $a(1) = 0$, and $a'(1), a''(1) > 0$. Both the specification of capital adjustment costs and capacity utilization costs are somewhat peculiar. More standard formulations assume that adjustment costs depend on the level of investment rather than on its growth rate, as is assumed here. Also, costs of capacity utilization typically take the form of a higher rate of depreciation of physical capital. The modeling choice here is guided by the need to fit the response of investment and capacity utilization to a monetary shock in the U.S. economy. For further discussion of this issue, see Christiano, Eichenbaum, and Evans (2005) and Altig et al. (2004).

Households rent the capital stock to firms at the real rental rate r_t^k per unit of capital. Total income stemming from the rental of capital is given by $r_t^k u_t k_t$. The investment good is assumed to be a composite good made with the aggregator function shown in equation (2). Thus, the demand for each intermediate good $i \in [0, 1]$ for investment purposes, i_{it}, is given by $i_{it} = \Upsilon_t^{-1} i_t \left(P_{it}/P_t\right)^{-\eta}$.

As in our earlier related work (Schmitt-Grohé and Uribe, 2004), we motivate a demand for money by households by assuming that purchases of consumption goods are subject to a proportional transaction cost that is increasing in consumption-based money velocity. Formally, the purchase of each unit of consumption entails a cost given by $\ell(v_t)$. Here,

$$v_t \equiv \frac{c_t}{m_t^h} \tag{8}$$

is the ratio of consumption to real money balances held by the household, which we denote by m_t^h. The transaction cost function ℓ satisfies the following assumptions: (a) $\ell(v)$ is nonnegative and twice continuously differentiable; (b) There exists a level of velocity $\underline{v} > 0$, to which we refer as the satiation level of money, such that $\ell(\underline{v}) = \ell'(\underline{v}) = 0$; (c) $(v - \underline{v})\ell'(v) > 0$ for $v \neq \underline{v}$; and (d) $2\ell'(v) + v\ell''(v) > 0$ for all $v \geq \underline{v}$. Assumption (a) implies that the transaction process does not generate resources. Assumption (b) ensures that the Friedman rule, i.e., a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the associated distortions in the intra and intertemporal allocation of consumption and leisure vanish when the nominal interest rate is zero. Assumption (c) guarantees that in equilibrium money velocity is always greater than or equal to the satiation level $\underline{\text{v}}$. As will become clear shortly, assumption (d) ensures that the demand for money is decreasing in the nominal interest rate. Assumption (d) is weaker than the more common assumption of strict convexity of the transaction cost function.

Households are assumed to have access to a complete set of nominal state-contingent assets. Specifically, each period $t \geq 0$, consumers can purchase any desired state-contingent nominal payment X_{t+1}^h in period $t+1$ at the dollar cost $E_t r_{t,t+1} X_{t+1}^h$. The variable $r_{t,t+1}$ denotes a stochastic nominal discount factor between periods t and $t+1$. Households pay real lump-sum taxess in the amount τ_t per period. The household's period-by-period budget constraint is given by:

$$\begin{aligned} E_t r_{t,t+1} x_{t+1}^h + c_t[1 + \ell(v_t)] + \Upsilon_t^{-1}[i_t + a(u_t)k_t] + m_t^h + \tau_t \quad = \quad & \frac{x_t^h + m_{t-1}^h}{\pi_t} + r_t^k u_t k_t \\ & + \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d dj + \phi_t. \end{aligned} \tag{9}$$

The variable $x_t^h/\pi_t \equiv X_t^h/P_t$ denotes the real payoff in period t of nominal state-contingent assets purchased in period $t-1$. The variable ϕ_t denotes dividends received from the ownership of firms and $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of consumer-price inflation.

We introduce wage stickiness in the model by assuming that each period the household (or unions) cannot set the nominal wage optimally in a fraction $\tilde{\alpha} \in [0,1)$ of randomly chosen labor markets. In these markets, the wage rate is indexed to average real wage growth and to the previous period's consumer-price inflation according to the rule $W_t^j = W_{t-1}^j(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}$, where $\tilde{\chi} \in [0,1]$ is a parameter measuring the degree of wage indexation. When $\tilde{\chi}$ equals 0, there is no wage indexation. When $\tilde{\chi}$ equals 1, there is full wage indexation to long-run real wage growth and to past consumer price inflation.

The household chooses processes for c_t, h_t, x_{t+1}^h, w_t^j, k_{t+1}, i_t, u_t, and m_t^h so as to maximize the utility function (1) subject to (6)-(9), the wage stickiness friction, and a no-Ponzi-game constraint, taking as given the processes w_t, r_t^k, h_t^d, $r_{t,t+1}$, π_t, ϕ_t, and τ_t and the initial conditions x_0^h, k_0, and m_{-1}^h. The household's optimal plan must satisfy constraints (6)-(9). In addition, letting $\beta^t\lambda_t w_t\tilde{\mu}_t$, $\beta^t\lambda_t q_t$, and $\beta^t\lambda_t$ denote Lagrange multipliers associated with constraints (6), (7), and (9), respectively, the Lagrangian associated with the household's optimization problem is

$$\begin{aligned}
\mathcal{L} \;=\; & E_0\sum_{t=0}^{\infty}\beta^t\left\{U(c_t - bc_{t-1}, h_t)\right. \\
&+\lambda_t\left[h_t^d\int_0^1 w_t^i\left(\frac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di + r_t^k u_t k_t + \phi_t - \tau_t\right. \\
&\left.-c_t\left[1+\ell\left(\frac{c_t}{m_t^h}\right)\right] - \Upsilon_t^{-1}[i_t + a(u_t)k_t] - r_{t,t+1}x_{t+1}^h - m_t^h + \frac{m_{t-1}^h + x_t^h}{\pi_t}\right] \\
&+\frac{\lambda_t w_t}{\tilde{\mu}_t}\left[h_t - h_t^d\int_0^1\left(\frac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di\right] \\
&\left.+\lambda_t q_t\left[(1-\delta)k_t + i_t\left[1-\mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right] - k_{t+1}\right]\right\}.
\end{aligned}$$

The first-order conditions with respect to c_t, x_{t+1}^h, h_t, k_{t+1}, i_t, m_t^h, u_t, and w_t^i, in that order, are given by

$$U_c(c_t - bc_{t-1}, h_t) - b\beta E_t U_c(c_{t+1} - bc_t, h_{t+1}) = \lambda_t[1 + \ell(v_t) + v_t\ell'(v_t)], \tag{10}$$

$$\lambda_t r_{t,t+1} = \beta\lambda_{t+1}\frac{P_t}{P_{t+1}} \tag{11}$$

$$-U_h(c_t - bc_{t-1}, h_t) = \frac{\lambda_t w_t}{\tilde{\mu}_t}, \tag{12}$$

$$\lambda_t q_t = \beta E_t\lambda_{t+1}\left[r_{t+1}^k u_{t+1} - \Upsilon_{t+1}^{-1}a(u_{t+1}) + q_{t+1}(1-\delta)\right], \tag{13}$$

$$\Upsilon_t^{-1}\lambda_t = \lambda_t q_t \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right)\mathcal{S}'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E_t \lambda_{t+1} q_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \mathcal{S}'\left(\frac{i_{t+1}}{i_t}\right) \quad (14)$$

$$v_t^2 \ell'(v_t) = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}. \quad (15)$$

$$r_t^k = \Upsilon_t^{-1} a'(u_t) \quad (16)$$

$$w_t^i = \begin{cases} \tilde{w}_t & \text{if } w_t^i \text{ is set optimally in } t \\ w_{t-1}^i(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}/\pi_t & \text{otherwise} \end{cases},$$

where $\tilde{w}_t$ denotes the real wage prevailing in the $1-\tilde{\alpha}$ labor markets in which the union can set wages optimally in period t. Let $\tilde{h}_t$ denote the level of labor effort supplied to those markets. Because the labor demand curve faced by the union is identical across all labor markets, and because the cost of supplying labor is the same for all markets, one can assume that wage rates, $\tilde{w}_t$, and employment, $\tilde{h}_t$, are identical across all labor markets updating wages in a given period. By equation (5), we have that $\tilde{w}_t^{\tilde{\eta}}\tilde{h}_t = w^{\tilde{\eta}} h_t^d$. It is of use to track the evolution of real wages in a particular labor market. In any labor market j where the wage is set optimally in period t, the real wage in that period is $\tilde{w}_t$. If in period $t+1$ wages are not reoptimized in that market, the real wage is $\tilde{w}_t(\mu_{z^*}\pi_t)^{\tilde{\chi}}/\pi_{t+1}$. This is because the nominal wage is indexed by $\tilde{\chi}$ percent of the sum of past price inflation and long-run real wage growth. In general, s periods after the last reoptimization, the real wage is $\tilde{w}_t \prod_{k=1}^{s}\left(\frac{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}{\pi_{t+k}}\right)$. To derive the household's first-order condition with respect to the wage rate in those markets where the wage rate is set optimally in the current period, it is convenient to reproduce the parts of the Lagrangian given above that are relevant for this purpose,

$$\mathcal{L}^w = E_t \sum_{s=0}^{\infty} (\tilde{\alpha}\beta)^s \lambda_{t+s} h_{t+s}^d w_{t+s}^{\tilde{\eta}} \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\tilde{w}_t^{1-\tilde{\eta}} \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{-1} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\tilde{w}_t^{-\tilde{\eta}}\right].$$

The first-order condition with respect to $\tilde{w}_t$ is

$$0 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} w_{t+s}^{\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right].$$

Using equation (12) to eliminate $\tilde{\mu}_{t+s}$, we obtain that the real wage $\tilde{w}_t$ must satisfy

$$0 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)} - \frac{-U_{ht+s}}{\lambda_{t+s}}\right].$$

This expression states that in labor markets in which the wage rate is reoptimized in period

t, the real wage is set so as to equate the union's future expected average marginal revenue to the average marginal cost of supplying labor. The union's marginal revenue s periods after its last wage reoptimization is given by $\frac{\tilde{\eta}-1}{\tilde{\eta}}\tilde{w}_t \prod_{k=1}^{s}\left(\frac{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}{\pi_{t+k}}\right)$. Here, $\tilde{\eta}/(\tilde{\eta}-1)$ represents the markup of wages over marginal cost of labor that would prevail in the absence of wage stickiness. The factor $\prod_{k=1}^{s}\left(\frac{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}{\pi_{t+k}}\right)$ in the expression for marginal revenue reflects the fact that as time goes by without a chance to reoptimize, the real wage declines as the price level increases when wages are imperfectly indexed. In turn, the marginal cost of supplying labor is given by the marginal rate of substitution between consumption and leisure, or $\frac{-U_{ht+s}}{\lambda_{t+s}} = \frac{w_{t+s}}{\tilde{\mu}_{t+s}}$. The variable $\tilde{\mu}_t$ is a wedge between the disutility of labor and the average real wage prevailing in the economy. Thus, $\tilde{\mu}_t$ can be interpreted as the average markup that unions impose on the labor market. The weights used to compute the average difference between marginal revenue and marginal cost are decreasing in time and increasing in the amount of labor supplied to the market.

We wish to write the wage-setting equation in recursive form. To this end, define

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t E_t \sum_{s=0}^{\infty}(\beta\tilde{\alpha})^s \lambda_{t+s}\left(\frac{w_{t+s}}{\tilde{w}_t}\right)^{\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}$$

and

$$f_t^2 = -\tilde{w}_t^{-\tilde{\eta}} E_t \sum_{s=0}^{\infty}(\beta\tilde{\alpha})^s w_{t+s}^{\tilde{\eta}} h_{t+s}^d U_{ht+s} \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}}.$$

One can express f_t^1 and f_t^2 recursively as

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t\lambda_t\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1, \tag{17}$$

$$f_t^2 = -U_{ht}\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} f_{t+1}^2. \tag{18}$$

With these definitions at hand, the wage-setting equation becomes

$$f_t^1 = f_t^2. \tag{19}$$

The household's optimality conditions imply a liquidity preference function featuring a negative relation between real balances and the short-term nominal interest rate. To see this, we first note that the absence of arbitrage opportunities in financial markets requires that the gross risk-free nominal interest rate, which we denote by R_t, be equal to the reciprocal of the price in period t of a nominal security that pays one unit of currency in every state of period $t+1$. Formally, $R_t = 1/E_t r_{t,t+1}$. This relation together with the household's optimality condition (11) implies that

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{20}$$

which is a standard Euler equation for pricing nominally risk-free assets. Combining this expression with equations (10) and (15), we obtain

$$v_t^2 \ell'(v_t) = 1 - \frac{1}{R_t}.$$

The right-hand side of this expression represents the opportunity cost of holding money, which is an increasing function of the nominal interest rate. Given the assumptions regarding the form of the transactons cost function ℓ, the left-hand side is increasing in money velocity. Thus, this expression defines a liquidity preference function that is decreasing in the nominal interest rate and unit elastic in consumption.

1.2 Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0, 1]$ produces output using as factor inputs capital services, k_{it}, and labor services, h_{it}. The production technology is given by

$$F(k_{it}, z_t h_{it}) - \psi z_t^*,$$

where the function F is assumed to be homogenous of degree one, concave, and strictly increasing in both arguments. The variable z_t denotes an aggregate, exogenous, and stochastic neutral productivity shock. The parameter $\psi > 0$ introduces fixed costs of operating a firm in each period. In turn, the presence of fixed costs implies that the production function exhibits increasing returns to scale. We model fixed costs to ensure a realistic profit-to-output ratio in steady state. Finally, we follow Altig et al. (2005) and assume that fixed costs are subject to permanent shocks, z_t^*, with

$$\frac{z_t^*}{z_t} = \Upsilon_t^{\frac{\theta}{1-\theta}}.$$

This formulation of fixed costs ensures that along the balanced-growth path fixed costs do not vanish. Let $\mu_{z,t} \equiv z_t / z_{t-1}$ denote the gross growth rate of the neutral technology shock. By assumption, in the non-stochastic steady state $\mu_{z,t}$ is constant and equal to μ_z. Also, let $\hat{\mu}_{z,t} = \ln(\mu_{z,t}/\mu_z)$ denote the precentage deviation of the growth rate of neutral technology shocks. Then, the evolution of $\mu_{z,t}$ is assumed to be given by:

$$\hat{\mu}_{z,t} = \rho_{\mu_z}\hat{\mu}_{z,t-1} + \epsilon_{\mu_z,t},$$

with $\epsilon_{\mu_z,t} \sim (0, \sigma^2_{\mu_z})$.

Aggregate demand for good i, which we denote by y_{it}, is given by

$$y_{it} = (P_{it}/P_t)^{-\eta} y_t,$$

where

$$y_t \equiv c_t[1 + \ell(v_t)] + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t], \tag{21}$$

denotes aggregate absorption. The variable g_t denotes government consumption of the composite good in period t.

We rationalize a demand for money by firms by imposing that wage payments be subject to a working-capital requirement that takes the form of a cash-in-advance constraint. Formally, we impose

$$m^f_{it} = \nu w_t h_{it}, \tag{22}$$

where m^f_{it} denotes the demand for real money balances by firm i in period t and $\nu \geq 0$ is a parameter indicating the fraction of the wage bill that must be backed with monetary assets.

Firms incur financial costs in the amount $(1 - R_t^{-1})m^f_{it}$ stemming from the need to hold money to satisfy the working-capital constraint. Letting the variable ϕ_{it} denote real distributed profits, the period-by-period budget constraint of firm i can then be written as

$$E_t r_{t,t+1} x^f_{it+1} + m^f_{it} - \frac{x^f_{it} + m^f_{it-1}}{\pi_t} = \left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r^k_t k_{it} - w_t h_{it} - \phi_{it},$$

where $E_t r_{t,t+1} x^f_{it+1}$ denotes the total real cost of one-period state-contingent assets that the firm purchases in period t in terms of the composite good.[2] We assume that the firm must satisfy demand at the posted price. Formally, we impose

$$F(k_{it}, z_t h_{it}) - \psi z^*_t \geq \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t. \tag{23}$$

[2]Implicit in this specification of the firm's budget constraint is the assumption that firms rent capital services from a centralized market. This is a common assumption in the related literature (e.g., Christiano et al., 2003; Kollmann, 2003; Carlstrom and Fuerst, 2003; and Rotemberg and Woodford, 1992). A polar assumption is that capital is firm specific, as in Woodford (2003, chapter 5.3) and Sveen and Weinke (2003). Both assumptions are clearly extreme. A more realistic treatment of investment dynamics would incorporate a mix of firm-specific and homogeneous capital.

The objective of the firm is to choose contingent plans for P_{it}, h_{it}, k_{it}, x^f_{it+1}, and m^f_{it} so as to maximize the present discounted value of dividend payments, given by

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \phi_{it+s},$$

where $r_{t,t+s} \equiv \prod_{k=1}^{s} r_{t+k-1,t+k}$, for $s \geq 1$, denotes the stochastic nominal discount factor between t and $t+s$, and $r_{t,t} \equiv 1$. Firms are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games.

Clearly, because $r_{t,t+s}$ represents both the firm's stochastic discount factor and the market pricing kernel for financial assets, and because the firm's objective function is linear in asset holdings, it follows that any asset accumulation plan of the firm satisfying the no-Ponzi constraint is optimal. Suppose, without loss of generality, that the firm manages its portfolio so that its financial position at the beginning of each period is nil. Formally, assume that $x^f_{it+1} + m^f_{it} = 0$ at all dates and states. Note that this financial strategy makes x^f_{it+1} state noncontingent. In this case, distributed dividends take the form

$$\phi_{it} = \left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r^k_t k_{it} - w_t h_{it} - (1 - R_t^{-1}) m^f_{it}. \tag{24}$$

For this expression to hold in period zero, we impose the initial condition $x^f_{i0} + m^f_{i-1} = 0$. The last term on the right-hand side of the above expression for dividends represents the firm's financial costs associated with the cash-in-advance constraint on wages. This financial cost is increasing in the opportunity cost of holding money, $1 - R_t^{-1}$, which in turn is an increasing function of the short-term nominal interest rate R_t.

Letting $r_{t,t+s} P_{t+s} \mathrm{mc}_{it+s}$ denote the Lagrange multiplier associated with constraint (23), the first-order conditions of the firm's maximization problem with respect to capital and labor services are, respectively,

$$\mathrm{mc}_{it} z_t F_2(k_{it}, z_t h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right] \tag{25}$$

and

$$\mathrm{mc}_{it} F_1(k_{it}, z_t h_{it}) = r^k_t. \tag{26}$$

It is clear from these optimality conditions that the presence of a working-capital requirement introduces a financial cost of labor that is increasing in the nominal interest rate. We note also that because all firms face the same factor prices and because they all have access to the same production technology with the function F being linearly homogeneous, marginal costs, mc_{it}, are identical across firms. Indeed, because the above first-order conditions hold for all firms independently of whether they are allowed to reset prices optimally, marginal costs are identical across all firms in the economy.

Prices are assumed to be sticky à la Calvo (1983) and Yun (1996). Specifically, each period $t \geq 0$ a fraction $\alpha \in [0,1)$ of randomly picked firms is not allowed to optimally set the nominal price of the good they produce. Instead, these firms index their prices to past inflation according to the rule $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$. The interpretation of the parameter χ is the similar to that of its wage counterpart $\tilde{\chi}$. The remaining $1-\alpha$ firms choose prices optimally. Consider the price-setting problem faced by a firm that has the opportunity to reoptimize the price in period t. This price, which we denote by $\tilde{P}_t$, is set so as to maximize the expected present discounted value of profits. That is, $\tilde{P}_t$ maximizes the following Lagrangian:

$$\begin{aligned}\mathcal{L} \;=\; & E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left\{ \left(\frac{\tilde{P}_t}{P_t}\right)^{1-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{1-\eta} y_{t+s} - r_{t+s}^k k_{it+s} - w_{t+s} h_{it+s}[1 + \nu(1 - R_{t+s}^{-1})] \right. \\ & \left. + \text{mc}_{it+s} \left[F(k_{it+s}, z_{t+s} h_{it+s}) - \psi z_{t+s}^* - \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{-\eta} y_{t+s} \right] \right\}.\end{aligned}$$

The first-order condition with respect to $\tilde{P}_t$ is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{-\eta} y_{t+s} \left[\frac{\eta - 1}{\eta} \left(\frac{\tilde{P}_t}{P_t}\right) \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{it+s} \right] = 0. \tag{27}$$

According to this expression, optimizing firms set nominal prices so as to equate average future expected marginal revenues to average future expected marginal costs. The weights used in calculating these averages are decreasing with time and increasing in the size of the demand for the good produced by the firm. Under flexible prices ($\alpha = 0$), the above optimality condition reduces to a static relation equating marginal costs to marginal revenues period by period.

It will prove useful to express this first-order condition recursively. To that end, let

$$x_t^1 \;\equiv\; E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} mc_{it+s} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta-1} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{(1+\eta)/\eta}}\right)^{-\eta}$$

and

$$x_t^2 \;\equiv\; E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{\eta/(\eta-1)}}\right)^{1-\eta}.$$

Express x_t^1 and x_t^2 recursively as

$$x_t^1 = y_t \text{mc}_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} (\tilde{p}_t/\tilde{p}_{t+1})^{-\eta-1} \left(\frac{\pi_t^\chi}{\pi_{t+1}}\right)^{-\eta} x_{t+1}^1, \tag{28}$$

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^\chi}{\pi_{t+1}}\right)^{1-\eta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} x_{t+1}^2. \tag{29}$$

Then we can write the first-order condition with respect to $\tilde{P}_t$ as

$$\eta x_t^1 = (\eta - 1) x_t^2. \tag{30}$$

The labor input used by firm $i \in [0, 1]$, denoted h_{it}, is assumed to be a composite made of a continuum of differentiated labor services, h_{it}^j indexed by $j \in [0, 1]$. Formally,

$$h_{it} = \left[\int_0^1 {h_{it}^j}^{1-1/\tilde{\eta}} dj\right]^{1/(1-1/\tilde{\eta})}, \tag{31}$$

where the parameter $\tilde{\eta} > 1$ denotes the intratemporal elasticity of substitution across different types of activities. For any given level of h_{it}, the demand for each variety of labor $j \in [0, 1]$ in period t must solve the dual problem of minimizing total labor cost, $\int_0^1 W_t^j h_{it}^j dj$, subject to the aggregation constraint (31), where W_t^j denotes the nominal wage rate paid to labor of variety j at time t. The optimal demand for labor of type j is then given by

$$h_{it}^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_{it}, \tag{32}$$

where W_t is a nominal wage index given by

$$W_t \equiv \left[\int_0^1 {W_t^j}^{1-\tilde{\eta}} dj\right]^{\frac{1}{1-\tilde{\eta}}}. \tag{33}$$

This wage index has the property that the minimum cost of a bundle of intermediate labor inputs yielding h_{it} units of the composite labor is given by $W_t h_{it}$.

1.3 The Government

Each period, the government consumes g_t units of the composite good. We assume that the government minimizes the cost of producing g_t. As a result, public demand for each variety $i \in [0, 1]$ of differentiated goods g_{it} is given by $g_{it} = (P_{it}/P_t)^{-\eta} g_t$.

We assume that along the balanced-growth path the share of government spending in value added is constant, that is, we impose $\lim_{j\to\infty} E_t g_{t+j}/y_{t+j} = s_g$, where s_g is a constant indicating the share of government consumption in value added. To this end we impose:

$$g_t = z_t^* \bar{g}_t,$$

where $\bar{g}_t$ is an exogenous stationary stochastic process. This assumption ensures that government purchases and output are cointegrated. We impose the following law of motion for $\bar{g}_t$:

$$\ln\left(\frac{\bar{g}_t}{\bar{g}}\right) = \rho_{\bar{g}} \ln\left(\frac{\bar{g}_{t-1}}{\bar{g}}\right) + \epsilon_{\bar{g},t}.$$

The government issues money given in real terms by $m_t \equiv m_t^h + \int_0^1 m_{it}^f di$. For simplicity, we assume that government debt is zero at time zero and that the fiscal authority levies lump-sum taxes, τ_t to bridge any gap between seignorage income and government expenditures, that is, $\tau_t = g_t - (m_t - m_{t-1}/\pi_t)$. As a consequence, goverment debt is nil at all times.

We postpone the presentation of the monetary policy regime until after we characterize a competitive equilibrium.

1.4 Aggregation

We limit attention to a symmetric equilibrium in which all firms that have the opportunigy to change their price optimally at a given time choose the same price. It then follows from (4) that the aggregate price index can be written as $P_t^{1-\eta} = \alpha(P_{t-1}\pi_{t-1}^{\chi})^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta}$. Dividing this expression through by $P_t^{1-\eta}$ one obtains

$$1 = \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}. \tag{34}$$

1.4.1 Market Clearing in the Final Goods Market

Naturally, the set of equilibrium conditions includes a resource constraint. Such a restriction is typically of the type $F(k_t, z_t h_t) - \psi z_t^* = c_t[1+\ell(v_t)] + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]$. In the present model, however, this restriction is not valid. This is because the model implies relative price dispersion across varieties. This price dispersion, which is induced by the assumed nature of price stickiness, is inefficient and entails output loss. To see this, consider the following expression stating that supply must equal demand at the firm level:

$$F(k_{it}, z_t h_{it}) - \psi z_t^* = \left\{[1+\ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]\right\}\left(\frac{P_{it}}{P_t}\right)^{-\eta}.$$

Integrating over all firms and taking into account that (a) the capital-labor ratio is common across firms, (b) that the aggregate demand for the composite labor input, h_t^d, satisfies

$$h_t^d = \int_0^1 h_{it} di,$$

and that (c) the aggregate effective level of capital, $u_t k_t$ satisfies

$$u_t k_t = \int_0^1 k_{it} di,$$

we obtain

$$z_t h_t^d F\left(\frac{u_t k_t}{z_t h_t^d}, 1\right) - \psi z_t^* = \left\{[1+\ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]\right\} \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di.$$

Let $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di$. Then we have

$$\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di \\
&= (1-\alpha)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} + (1-\alpha)\alpha\left(\frac{\tilde{P}_{t-1}\pi_{t-1}^{\chi}}{P_t}\right)^{-\eta} + (1-\alpha)\alpha^2\left(\frac{\tilde{P}_{t-2}\pi_{t-1}^{\chi}\pi_{t-2}^{\chi}}{P_t}\right)^{-\eta} + \ldots \\
&= (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(\frac{\tilde{P}_{t-j}\prod_{s=1}^{j}\pi_{t-j-1+s}^{\chi}}{P_t}\right)^{-\eta} \\
&= (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta} s_{t-1}.
\end{aligned}$$

Summarizing, the resource constraint in the present model is given by the following two expressions

$$F(u_t k_t, z_t h_t^d) - \psi z_t^* = \left\{[1+\ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]\right\} s_t \tag{35}$$

and

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta} s_{t-1}, \tag{36}$$

with s_{-1} given. The state variable s_t summarizes the resource costs induced by the inefficient price dispersion featured in the Calvo model in equilibrium. Three observations are in order about the price dispersion measure s_t. First, s_t is bounded below by 1. That is, price dispersion is always a costly distortion in this model. To see that s_t is bounded below by 1, let $v_{it} \equiv (P_{it}/P_t)^{1-\eta}$. It follows from the definition of the price index given in equation (4) that $\left[\int_0^1 v_{it}\right]^{\eta/(\eta-1)} = 1$. Also, by definition we have $s_t = \int_0^1 v_{it}^{\eta/(\eta-1)}$. Then, taking into account that $\eta/(\eta-1) > 1$, Jensen's inequality implies that $1 = \left[\int_0^1 v_{it}\right]^{\eta/(\eta-1)} \leq \int_0^1 v_{it}^{\eta/(\eta-1)} = s_t$. Second, in an economy where the non-stochastic level of inflation is nil (i.e., when $\pi = 1$) or where prices are fully indexed to any variable ω_t with the property that its deterministic steady-state level equals the deterministic steady-state value of inflation (i.e., $\omega = \pi$), then the variable s_t follows, up to first order, the univariate autoregressive process $\hat{s}_t = \alpha \hat{s}_{t-1}$. In these cases, the price dispersion measure s_t has no first-order real consequences for the stationary distribution of any endogenous variable of the model. This means that studies that restrict attention to linear approximations to the equilibrium conditions are justified to ignore the variable s_t if the model features no price dispersion in the deterministic steady state. But s_t matters up to first order when the deterministic steady state features movements in relative prices across goods varieties. More importantly, the price dispersion variable s_t must be taken into account if one is interested in higher-order approximations to the equilibrium conditions even if relative prices are stable in the deterministic steady state. Omitting s_t in higher-order expansions would amount to leaving out certain higher-order terms while including others. Finally, when prices are fully flexible, $\alpha = 0$, we have that $\tilde{p}_t = 1$ and thus $s_t = 1$. (Obviously, in a flexible-price equilibrium there is no price dispersion across varieties.)

As discussed above, equilibrium marginal costs and capital-labor ratios are identical across firms. Therefore, one can aggregate the firm's optimality conditions with respect to labor and capital, equations (25) and (26), as

$$\mathrm{mc}_t z_t F_2(u_t k_t, z_t h_t^d) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right] \tag{37}$$

and

$$\mathrm{mc}_t F_1(u_t k_t, z_t h_t^d) = r_t^k. \tag{38}$$

1.4.2 Market Clearing in the Labor Market

It follows from equation (32) that the aggregate demand for labor of type $j \in [0,1]$, which we denote by $h_t^j \equiv \int_0^1 h_{it}^j di$, is given by

$$h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d, \tag{39}$$

where $h_t^d \equiv \int_0^1 h_{it} di$ denotes the aggregate demand for the composite labor input. Taking into account that at any point in time the nominal wage rate is identical across all labor markets at which wages are allowed to change optimally, we have that labor demand in each of those markets is

$$\tilde{h}_t = \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} h_t^d.$$

Combining this expression with equation (39), describing the demand for labor of type $j \in [0, 1]$, and with the time constraint (6), which must hold with equality, we can write

$$h_t = (1-\tilde{\alpha}) h_t^d \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s} \prod_{k=1}^{s} (\mu_{z^*} \pi_{t+k-s-1})^{\tilde{\chi}}}{W_t}\right)^{-\tilde{\eta}}.$$

Let $\tilde{s}_t \equiv (1-\tilde{\alpha}) \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s} \prod_{k=1}^{s} (\mu_{z^*} \pi_{t+k-s-1})^{\tilde{\chi}}}{W_t}\right)^{-\tilde{\eta}}$. The variable $\tilde{s}_t$ measures the degree of wage dispersion across different types of labor. The above expression can be written as

$$h_t = \tilde{s}_t h_t^d. \tag{40}$$

The state variable $\tilde{s}_t$ evolves over time according to

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}} \left(\frac{\pi_t}{(\mu_{z^*} \pi_{t-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \tilde{s}_{t-1}. \tag{41}$$

We note that because all job varieties are ex-ante identical, any wage dispersion is inefficient. This is reflected in the fact that $\tilde{s}_t$ is bounded below by 1. The proof of this statement is identical to that offered earlier for the fact that s_t is bounded below by unity. To see this, note that $\tilde{s}_t$ can be written as $\tilde{s}_t = \int_0^1 \left(\frac{W_{it}}{W_t}\right)^{-\tilde{\eta}} di$. This inefficiency introduces a wedge that makes the number of hours supplied to the market, h_t, larger than the number of productive units of labor input, h_t^d. In an environment without long-run wage dispersion, the dead-weight loss created by wage dispersion is nil up to first order. Formally, a first-order approximation of the law of motion of $\tilde{s}_t$ yields a univariate autoregressive process of the form $\hat{\tilde{s}}_t = \tilde{\alpha} \hat{\tilde{s}}_{t-1}$, as long as there is no wage dispersion in the deterministic steady state. When wages are fully flexible, $\tilde{\alpha} = 0$, wage dispersion disappears, and thus $\tilde{s}_t$ equals 1.

It follows from our definition of the wage index given in equation (33) that in equilibrium the real wage rate must satisfy

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\alpha}) \tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{(\mu_{z^*} \pi_{t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}. \tag{42}$$

Aggregating the expression for firm's profits given in equation (24) yields

$$\phi_t = y_t - r_t^k u_t k_t - w_t h_t^d - \nu(1 - R_t^{-1}) w_t h_t^d. \tag{43}$$

In equilibrium, real money holdings can be expressed as

$$m_t = m_t^h + \nu w_t h_t^d, \tag{44}$$

and the goverment budget constraint is given by

$$\tau_t = g_t - (m_t - m_{t-1}/\pi_t). \tag{45}$$

1.5 Competitive Equilibrium

A stationary competitive equilibrium is a set of stationary processes u_t, c_t, h_t, i_t, k_{t+1}, v_t, m_t^h, m_t, λ_t, π_t, w_t, $\tilde{\mu}_t$, q_t, r_t^k, ϕ_t, f_t^1, f_t^2, $\tilde{w}_t$, h_t^d, y_t, mc_t, x_t^1, x_t^2, $\tilde{p}_t$, s_t, $\tilde{s}_t$, and τ_t satisfying (7), (8), (10), (12)-(21), (28)-(30), (34)-(38), and (40)-(45), given exogenous stochastic processes $\{g_t, z_t, \Upsilon_t\}_{t=0}^{\infty}$, the policy process, R_t, and initial conditions c_{-1}, w_{-1}, s_{-1}, $\tilde{s}_{-1}$, π_{-1}, i_{-1}, and k_0.

2 Complete Set of Equilibrium Conditions

$$k_{t+1} = (1-\delta)k_t + i_t\left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right]$$

$$v_t \equiv \frac{c_t}{m_t^h}$$

$$U_c(c_t - bc_{t-1}, h_t) - b\beta E_t U_c(c_{t+1} - bc_t, h_{t+1}) = \lambda_t[1 + \ell(v_t) + v_t \ell'(v_t)]$$

$$-U_h(c_t - bc_{t-1}, h_t) = \frac{\lambda_t w_t}{\tilde{\mu}_t},$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1}\left[r_{t+1}^k u_{t+1} - \Upsilon_{t+1}^{-1} a(u_{t+1}) + q_{t+1}(1-\delta)\right],$$

$$\Upsilon_t^{-1}\lambda_t = \lambda_t q_t\left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right)\mathcal{S}'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E_t \lambda_{t+1} q_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \mathcal{S}'\left(\frac{i_{t+1}}{i_t}\right)$$

$$v_t^2 \ell'(v_t) = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}.$$

$$r_t^k = \Upsilon_t^{-1} a'(u_t)$$

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t \lambda_t \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1,$$

$$f_t^2 = -U_{ht}\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} f_{t+1}^2$$

$$f_t^1 = f_t^2.$$

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}$$

$$y_t = c_t[1+\ell(v_t)] + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]$$

$$x_t^1 = y_t \mathrm{mc}_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}(\tilde{p}_t/\tilde{p}_{t+1})^{-\eta-1}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\eta} x_{t+1}^1$$

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\eta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} x_{t+1}^2$$

$$\eta x_t^1 = (\eta-1)x_t^2$$

$$1 = \alpha \pi_t^{\eta-1}\pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}.$$

$$F(u_t k_t, z_t h_t^d) - \psi z_t^* = \left\{[1+\ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]\right\} s_t$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta} s_{t-1}$$

$$\mathrm{mc}_t z_t F_2(u_t k_t, z_t h_t^d) = w_t\left[1 + \nu\frac{R_t - 1}{R_t}\right]$$

$$\mathrm{mc}_t F_1(u_t k_t, z_t h_t^d) = r_t^k$$

$$h_t = \tilde{s}_t h_t^d$$

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}}\left(\frac{\pi_t}{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \tilde{s}_{t-1}$$

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}}\left(\frac{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}$$

$$\phi_t = y_t - r_t^k u_t k_t - w_t h_t^d - \nu(1-R_t^{-1}) w_t h_t^d$$

$$m_t = m_t^h + \nu w_t h_t^d$$

$$m_t(1-R_t^{-1}) + \tau_t = g_t$$

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2.1 Functional Forms

We use the following standard functional forms for utility and technology:

$$U = \frac{\left[(c_t - bc_{t-1})^{1-\phi_4} (1-h_t)^{\phi_4}\right]^{1-\phi_3} - 1}{1-\phi_3}$$

$$F(k,h) = k^\theta h^{1-\theta},$$

The functional form for the investment adjustment cost function is taken from Christiano, Eichenbaum, and Evans (2005).

$$\mathcal{S}\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - \mu_I\right)^2,$$

where μ_I is the steady-state growth rate of investment. Following Schmitt-Grohé and Uribe (2004) we assume that the transaction cost technology takes the form

$$\ell(v) = \phi_1 v + \phi_2/v - 2\sqrt{\phi_1\phi_2}. \tag{46}$$

The money demand function implied by the above transaction technology is of the form

$$v_t^2 = \frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{R_t - 1}{R_t},$$

Note the existence of a satiation point for consumption-based money velocity, $\underline{v}$, equal to $\sqrt{\phi_2/\phi_1}$. Also, the money demand has a unit elasticity with respect to consumption expenditures. This feature is a consequence of the assumption that transaction costs, $c\ell(c/m)$, are homogenous of degree one in consumption and real balances and is independent of the particular functional form assumed for $\ell(\cdot)$. Further, as the parameter ϕ_2 approaches zero, the transaction cost function $\ell(\cdot)$ becomes linear in velocity and the demand for money adopts the Baumol-Tobin square root form with respect to the opportunity cost of holding money, $(R-1)/R$. That is, the log-log elasticity of money demand with respect to the opportunity cost of holding money converges to 1/2, as ϕ_2 vanishes.

The costs of higher capacity utilization are parameterized as follows:

$$a(u) = \gamma_1(u-1) + \frac{\gamma_2}{2}(u-1)^2.$$

CCE estimate the ratio of γ_2/γ_1.

2.2 Equilibrium Conditions for written with the specific functional forms

$$k_{t+1} = (1-\delta)k_t + i_t\left[1 - \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - \mu_I\right)^2\right]$$

$$v_t = \frac{c_t}{m_t^h}$$

$$\begin{aligned}&(1-\phi_4)(c_t - bc_{t-1})^{(1-\phi_3)(1-\phi_4)-1}(1-h_t)^{\phi_4(1-\phi_3)} \\ -b\beta E_t(1-\phi_4)(c_{t+1}-bc_t)^{(1-\phi_3)(1-\phi_4)-1}&(1-h_{t+1})^{\phi_4(1-\phi_3)} = \lambda_t[1+\ell(v_t)+v_t\ell'(v_t)]\end{aligned}$$

$$\phi_4(c_t - bc_{t-1})^{(1-\phi_3)(1-\phi_4)}(1-h_t)^{\phi_4(1-\phi_3)-1} = \frac{\lambda_t w_t}{\tilde{\mu}_t}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1}\left[r_{t+1}^k u_{t+1} - \Upsilon_{t+1}^{-1} a(u_{t+1}) + q_{t+1}(1-\delta)\right],$$

$$\begin{aligned}\Upsilon_t^{-1}\lambda_t = \lambda_t q_t &\left[1 - \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - \mu_I\right)^2 - \left(\frac{i_t}{i_{t-1}}\right)\kappa\left(\frac{i_t}{i_{t-1}} - \mu_I\right)\right] \\ &+\beta E_t \lambda_{t+1} q_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \kappa\left(\frac{i_{t+1}}{i_t} - \mu_I\right)\end{aligned}$$

$$v_t^2\ell'(v_t) = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}.$$

$$r_t^k = \Upsilon_t^{-1} a'(u_t)$$

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t\lambda_t\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1,$$

$$f_t^2 = \left[\phi_4(c_t - bc_{t-1})^{(1-\phi_3)(1-\phi_4)}(1-h_t)^{\phi_4(1-\phi_3)-1}\right]\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} f_{t+1}^2$$

$$f_t^1 = f_t^2.$$

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}$$

$$y_t = c_t[1+\ell(v_t)] + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]$$

$$x_t^1 = y_t \mathrm{mc}_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}(\tilde{p}_t/\tilde{p}_{t+1})^{-\eta-1}\left(\frac{\pi_t^\chi}{\pi_{t+1}}\right)^{-\eta} x_{t+1}^1$$

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_t^\chi}{\pi_{t+1}}\right)^{1-\eta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} x_{t+1}^2$$

$$\eta x_t^1 = (\eta - 1)x_t^2$$

$$1 = \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}.$$

$$(u_t k_t)^\theta (z_t h_t^d)^{1-\theta} - \psi z_t^* = \left\{[1 + \ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]\right\} s_t$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^\chi}\right)^\eta s_{t-1}$$

$$\text{mc}_t z_t (1-\theta)(u_t k_t)^\theta (z_t h_t^d)^{-\theta} = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]$$

$$\text{mc}_t \theta (u_t k_t)^{\theta-1} (z_t h_t^d)^{1-\theta} = r_t^k$$

$$h_t = \tilde{s}_t h_t^d$$

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}} \left(\frac{\pi_t}{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \tilde{s}_{t-1}$$

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}$$

$$\phi_t = y_t - r_t^k u_t k_t - w_t h_t^d - \nu(1 - R_t^{-1}) w_t h_t^d$$

$$m_t = m_t^h + \nu w_t h_t^d$$

$$m_t (1 - R_t^{-1}) + \tau_t = g_t$$

2.3 Stationary Variables

This economy features two types of permanent shocks. Therefore, several variables such as output and the real wage will not be stationary along the balanced growth path. We next perform a change of variables so as to obtain a set of equilibrium conditions that involve only stationary variables. To this end let

$$C_t = \frac{c_t}{z_t^*}$$

$$I_t = \frac{i_t}{\Upsilon_t z_t^*} = \frac{i_t}{\Upsilon_t^{\frac{1}{1-\theta}} z_t}$$

$$\mu_{I,t} = \mu_{\Upsilon,t}^{\frac{1}{(1-\theta)}} \mu_{z,t}$$

$$K_{t+1} = \frac{k_{t+1}}{z_t \Upsilon^{\frac{1}{1-\theta}}}$$

$$M_t^h = \frac{m_t^h}{z_t^*}$$

$$M_t = \frac{m_t}{z_t^*}$$

$$\Lambda_t = \frac{\lambda_t}{z_t^{*(1-\phi_3)(1-\phi_4)-1}}$$

$$\mu_{\lambda,t} \equiv \frac{z_t^{*(1-\phi_3)(1-\phi_4)-1}}{z_{t-1}^{*\ (1-\phi_3)(1-\phi_4)-1}} = \mu_{z^*,t}^{(1-\phi_3)(1-\phi_4)-1}$$

$$W_t = \frac{w_t}{z_t^*}$$

$$Q_t = \Upsilon_t q_t$$

$$R_t^k = \Upsilon_t r_t^k$$

$$\tilde{W}_t = \frac{\tilde{w}_t}{z_t^*}$$

$$F_t^1 = \frac{f_t^1}{z_t^{*(1-\phi_4)(1-\phi_3)}}$$

$$F_t^2 = \frac{f_t^2}{z_t^{*(1-\phi_4)(1-\phi_3)}}$$

$$Y_t = \frac{y_t}{z_t^*}$$

$$G_t = \frac{g_t}{z_t^*}$$

$$\Phi_t = \frac{\phi_t}{z_t^*}$$

$$X_t^1 = \frac{x_t^1}{z_t^*}$$

$$X_t^2 = \frac{x_t^1}{z_t^*}$$

$$\tilde{\tau}_t = \frac{\tau_t}{z_t^*}$$

Variables that need not be transformed are: $\tilde{p}_t$, u_t, mc_t, h_t, h_t^d, s_t, $\tilde{s}_t$, π_t

2.4 Equilibrium Conditions in Stationary Variables

$$K_{t+1} = (1-\delta)\frac{K_t}{\mu_{I,t}} + I_t\left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)^2\right]$$

$$v_t = \frac{C_t}{M_t^h}$$

$$\begin{aligned}(1-\phi_4)(C_t - \mu_{z^*,t}^{-1}bC_{t-1})^{(1-\phi_3)(1-\phi_4)-1}(1-h_t)^{\phi_4(1-\phi_3)} \\ -b\beta E_t(1-\phi_4)(\mu_{z^*,t+1}C_{t+1} - bC_t)^{(1-\phi_3)(1-\phi_4)-1}(1-h_{t+1})^{\phi_4(1-\phi_3)} = \Lambda_t[1+\ell(v_t)+v_t\ell'(v_t)]\end{aligned}$$

$$\phi_4(C_t - b\mu_{z^*,t}^{-1}C_{t-1})^{(1-\phi_3)(1-\phi_4)}(1-h_t)^{\phi_4(1-\phi_3)-1} = \frac{\Lambda_t W_t}{\tilde{\mu}_t}$$

$$\Lambda_t Q_t = E_t\frac{\beta\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}\left[R_{t+1}^k u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)\right],$$

$$\begin{aligned}\Lambda_t = \Lambda_t Q_t\left[1 - \frac{\kappa}{2}\left(\frac{\mu_{I,t}I_t}{I_{t-1}} - \mu_I\right)^2 - \left(\frac{\mu_{I,t}I_t}{I_{t-1}}\right)\kappa\left(\frac{\mu_{I,t}I_t}{I_{t-1}} - \mu_I\right)\right] \\ +\beta E_t\frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}Q_{t+1}\left(\mu_{I,t+1}\frac{I_{t+1}}{I_t}\right)^2\kappa\left(\mu_{I,t+1}\frac{I_{t+1}}{I_t} - \mu_I\right)\end{aligned}$$

$$v_t^2\ell'(v_t) = 1 - \beta E_t\mu_{\Lambda,t+1}\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1}{\pi_{t+1}}.$$

$$R_t^k = a'(u_t)$$

$$F_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}_t\Lambda_t\left(\frac{W_t}{\tilde{W}_t}\right)^{\tilde{\eta}}h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\mu_{z^*,t+1}\tilde{W}_{t+1}}{\tilde{W}_t}\right)^{\tilde{\eta}-1}\mu_{\Lambda,t+1}\mu_{z^*,t+1}F_{t+1}^1,$$

$$\begin{aligned}F_t^2 = \left[\phi_4(C_t - b\mu_{z^*,t}^{-1}C_{t-1})^{(1-\phi_3)(1-\phi_4)}(1-h_t)^{\phi_4(1-\phi_3)-1}\right]\left(\frac{W_t}{\tilde{W}_t}\right)^{\tilde{\eta}}h_t^d + \\ \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}}\left(\frac{\mu_{z^*,t+1}\tilde{W}_{t+1}}{\tilde{W}_t}\right)^{\tilde{\eta}}\mu_{\Lambda,t+1}\mu_{z^*,t+1}F_{t+1}^2\end{aligned}$$

$$F_t^1 = F_t^2.$$

$$\Lambda_t = \beta R_t E_t\mu_{\Lambda,t+1}\frac{\Lambda_{t+1}}{\pi_{t+1}}$$

$$Y_t = C_t[1+\ell(v_t)] + G_t + [I_t + a(u_t)\mu_{I,t}^{-1}K_t]$$

$$X_t^1 = Y_t \text{mc}_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\mu_{\Lambda,t+1}\Lambda_{t+1}}{\Lambda_t} (\tilde{p}_t/\tilde{p}_{t+1})^{-\eta-1} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\eta} \mu_{z^*,t+1} X_{t+1}^1$$

$$X_t^2 = Y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\mu_{\Lambda,t+1}\Lambda_{t+1}}{\Lambda_t} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\eta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} \mu_{z^*,t+1} X_{t+1}^2$$

$$\eta X_t^1 = (\eta - 1) X_t^2$$

$$1 = \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}.$$

$$(u_t \mu_{I,t}^{-1} K_t)^{\theta} h_t^{d\,1-\theta} - \psi = \{[1 + \ell(v_t)]C_t + G_t + [I_t + a(u_t)K_t/\mu_{I,t}]\} s_t$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta} s_{t-1}$$

$$\text{mc}_t(1-\theta)(u_t K_t/\mu_{I,t})^{\theta} h_t^{d\,-\theta} = W_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]$$

$$\text{mc}_t \theta (u_t K_t/\mu_{I,t})^{\theta-1} h_t^{d\,1-\theta} = R_t^k$$

$$h_t = \tilde{s}_t h_t^d$$

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{W_{t-1}}{\mu_{z^*,t} W_t}\right)^{-\tilde{\eta}} \left(\frac{\pi_t}{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \tilde{s}_{t-1}$$

$$W_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{W}_t^{1-\tilde{\eta}} + \tilde{\alpha}(W_{t-1}/\mu_{z^*,t})^{1-\tilde{\eta}} \left(\frac{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}$$

$$\Phi_t = Y_t - R_t^k u_t K_t/\mu_{I,t} - W_t h_t^d (1 + \nu(1 - R_t^{-1})$$

$$M_t = M_t^h + \nu W_t h_t^d$$

$$M_t(1 - R_t^{-1}) + \tau_t/z_t^* = G_t$$

$$\mu_{I,t} = \mu_{\Upsilon,t}\mu_{z^*,t}$$

$$\mu_{\Lambda,t} = \mu_{z^*,t}^{(1-\phi_3)(1-\phi_4)-1}$$

$$\mu_{z^*,t} = \mu_{\Upsilon,t}^{\frac{\theta}{1-\theta}} \mu_{z,t}$$

$$\mu_{z,t} \equiv \frac{z_t}{z_{t-1}} \quad \text{and} \quad \hat{\mu}_{z,t} \equiv \ln(\mu_{z,t}/\mu_z))$$

$$\hat{\mu}_{z,t} = \rho_{\mu_z}\hat{\mu}_{z,t-1} + \epsilon_{\mu_z,t} \quad \text{with } \epsilon_{\mu_z,t} \sim (0, \sigma_{\mu_z}^2)$$

$$\mu_{\Upsilon,t} \equiv \Upsilon_t/\Upsilon_{t-1} \quad \text{and} \quad \hat{\mu}_{\Upsilon,t} \equiv \ln(\mu_{\Upsilon,t}/\mu_{\Upsilon})$$

$$\hat{\mu}_{\Upsilon,t} = \rho_{\mu_\Upsilon}\hat{\mu}_{\Upsilon,t-1} + \epsilon_{\mu_\Upsilon,t} \quad \text{with } \epsilon_{\mu_\Upsilon,t} \sim (0, \sigma_{\mu_\Upsilon}^2)$$

$$\ln\left(\frac{G_t}{G}\right) = \rho_g \ln\left(\frac{G_{t-1}}{G}\right) + \epsilon_{g,t}$$

Calibration

The equilibrium conditions in terms of stationary variables listed above contain 30 equations (not counting the law of motion of the exogenous variables) and 30 variables (again not counting the exogenous variables or the policy variable R_t): (u_t, K_{t+1},C_t, I_t, $\tilde{s}_t$, h_t, h_t^d, v_t, M_t^h, Λ_t, W_t, $\tilde{\mu}_t$, Q_t, R_t^k, F_t^1, F_t^2, π_t, $\tilde{W}_t$, Y_t, X_t^1, X_t^2, mc_t, $\tilde{p}_t$, s_t, Φ_t, M_t, $\tilde{\tau}_t$, $\mu_{I,t}$, $\mu_{\Lambda,t}$, $\mu_{z^*,t}$. In addition, the equilibrium conditions feature 20 parameters (ϕ_1, ϕ_2, ϕ_3, ϕ_4, γ_1, γ_2, θ, κ, b, β, δ, $\tilde{\eta}$, $\tilde{\alpha}$, η, α, χ, $\tilde{\chi}$, ψ, ν, μ_I).

Finally, the three exogenous processes involve 9 further parameters: μ_z, μ_Υ, G, σ_{μ_z}, σ_{μ_Υ}, σ_g, ρ_{μ_z}, ρ_{μ_Υ}, ρ_g .

This means that in order to obtain values for the steady-state levels of all variables and for the deep structural parameters, we need to impose 30 restrictions.

We take most parameters from ACEL. But not the baseline case, rather the one in which they impose that the product markup is 20 percent.

The exogenous stochastic process is calibrated in ACEL as

$$\mu_\Upsilon = 1.0042$$

$$\mu_z = 1.00213$$

Note what they in fact calibrate is: $\mu_Y \equiv y_t/y_{t-1} = 1.0045$ and then they use $\mu_y = \mu_\Upsilon^{\theta/(1-\theta)}\mu_z$. I believe that the paper has a typo, it says $\mu_z = 1.00013$. The standard deviations and serial correlations are estimated.

$$\sigma_{\mu_z} = 0.0007$$

$$\sigma_{\mu_\Upsilon} = 0.0031$$

$$\rho_{\mu_z} = 0.89$$

$$\rho_{\mu_\Upsilon} = 0.20$$

The process for government purchases is taken from Christiano and Eichenbaum (AER, 1992)

$$\rho_g = 0.96$$

$$\sigma_g = 0.020$$

$$G/Y = 0.17 \quad \text{own estimates}$$

Following CEE and ACEL we assume that in the competitive equilibrium steady state:

$$U = 1$$

Discount factor (ACEL calibrate $1.03^{-1/4}$)

$$BETTA = 1.03^{(-1/4)}$$

Capital share (ACEL calibrate this)

$$THETA = 0.36$$

Reciprocal of intertemporal elasticity of substitution (ACEL assume from the start logarithm utility in consumption)

$$PHI3 = 1$$

Labor elasticity of subst (CEE and ACEL calibrate this)

$$ETATIL = 21$$

Goods elasticity of substitution (CEE, value estimated; note in ACEL they estimate a much smaller value, because of firm specific stuff, but we stick to a 20 percent markup, so we do their high markup $\lambda_f = 1.2$ case)

$$ETA = 6$$

Degree of price stickiness (as estimated in CEE) (Note that ACEL impose full indexation, in the case of high markup and homo capital their estimates imply an α of 0.8, which I judge to be too high, so I go with the old CEE estimate)

$$ALFA = 0.6$$

Degree of wage stickiness (as estimated in ACEL)

$$ALFATIL = 0.69$$

Degree of habit formation (as estimated in ACEL)

$$B = 0.69;$$

Capital adjustment cost (as estimated in ACEL, in CEE they find 2.48)

$$KAPA = 2.79;$$

Following CEE and ACEL we set the fixed cost parameter so that in the steady state of the competitive economy profits are zero.

$$\text{Profits } = 0 = Y - R^k uK/\mu_I - Wh^d - \nu(1 - R^{-1})Wh^d$$

Quarterly depreciation rate: CEE (and ACEL) set

$$\delta = 0.025$$

We draw from the estimates reported in ACEL to assign values to the following capacity utilization parameters:

$$\frac{\gamma_2}{\gamma_1} = 1.46$$

Degree of wage indexation (assumed in CEE and ACEL)

$$CHITIL = 1$$

We use

$$\pi = 1.042^{-1/4}$$

but ACEL and CEE calibrate the money growth rate.

Degree of price indexation (taken from Cogley and Sbordone and LOWW)

$$CHI = 0$$

Share of household money in total money (own estimate).

$$SMH = 0.44$$

M1/GDP. Sample: 1959:1-2004:3. Source: GDP NIPA and M1 FRB. Produced with $m1_g dp.m$

$$SM = 0.1695 * 4;$$

Annualized inerest rate semielasticity of money demand (ACEL)

$$\epsilon_{m^h,R} \equiv \frac{1}{4}\frac{\partial \ln(m^h)}{\partial R} = -0.81$$

We assume that μ_I that appears in the investment adjustment cost function is the steady state value, so that in steady state adjustment costs are nil.

$$\mu_I = \mu_\Upsilon \mu_{z^*}$$

Finally, the Frisch elasticity of labor supply in ACEL is 1. To have this in our model we need to set

$$H = 0.5$$

2.5 Nonstochastic Steady State

We used 4 equilibrium conditions to eliminate $\tilde{\mu}_t$, ϕ_t, m_t^h, and τ_t.

$$I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right)$$

$$v = \frac{C}{M^h}$$

$$(1-\phi_4)[C(1-b/\mu_{z^*})]^{(1-\phi_3)(1-\phi_4)-1}(1-h)^{\phi_4(1-\phi_3)}\,(1-b\beta\mu_\Lambda) = \Lambda[1+\ell(v)+v\ell'(v)]$$

$$Q = \frac{\beta\mu_\Lambda}{\mu_\Upsilon}\left[R^k u - a(u) + Q(1-\delta)\right]$$

$$Q = 1$$

$$v^2\ell'(v) = 1 - \beta\mu_\Lambda\frac{1}{\pi}$$

$$R^k = a'(u)$$

$$F^1\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right] = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}\Lambda\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^2\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right] = \left[\phi_4(C - b\mu_{z^*}^{-1}C)^{(1-\phi_3)(1-\phi_4)}(1-h)^{\phi_4(1-\phi_3)-1}\right]\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^1 = F^2$$

$$R^{-1} = \beta\mu_\Lambda\frac{1}{\pi}$$

$$Y = C[1+\ell(v)] + G + [I + a(u)\mu_I^{-1}K]$$

$$X^1\left(1 - \alpha\beta\mu_\Lambda\pi^{-(\chi-1)\eta}\mu_{z^*}\right) = Y\mathrm{mc}\tilde{p}^{-\eta-1}$$

$$X^2\left(1 - \alpha\beta\mu_\Lambda\pi^{(\chi-1)(1-\eta)}\mu_{z^*}\right) = Y\tilde{p}^{-\eta}$$

$$\eta X^1 = (\eta-1)X^2$$

$$1 = \alpha\pi^{(\eta-1)(1-\chi)} + (1-\alpha)\tilde{p}_t^{1-\eta}.$$

$$\left(u\mu_I^{-1}K\right)^\theta {h^d}^{1-\theta} - \psi = \left\{[1+\ell(v)]C + G + [I + a(u)K/\mu_I]\right\}s$$

$$s = (1-\alpha)\tilde{p}^{-\eta} + \alpha\pi_t^{(1-\chi)\eta}s$$

$$\mathrm{mc}(1-\theta)(u_tK_t/\mu_{I,t})^\theta {h_t^d}^{-\theta} = W_t\left[1 + \nu\frac{R_t-1}{R_t}\right]$$

$$\mathrm{mc}\theta(uK/\mu_I)^{\theta-1}{h^d}^{1-\theta} = R^k$$

$$h = \tilde{s}h^d$$

$$\tilde{s}\left[1 - \tilde{\alpha}(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\right] = (1-\tilde{\alpha})\left(\frac{\tilde{W}}{W}\right)^{-\tilde{\eta}}$$

$$1 = (1-\tilde{\alpha})\left(\frac{\tilde{W}}{W}\right)^{1-\tilde{\eta}} + \tilde{\alpha}\left(\mu_{z^*}\pi\right)^{(\tilde{\chi}-1)(1-\tilde{\eta})}$$

3 Finding the steady state from the calibration restrictions when $\phi_3 = 1$:

3.1 Steady State 1:

At this point we know: u, h, ϕ_3, γ_2/γ_1, θ, κ, b, β, δ, $\tilde{\eta}$, $\tilde{\alpha}$, η, α, χ, $\tilde{\chi}$, μ_z, μ_Υ, σ_{μ_z}, σ_{μ_Υ}, σ_g, ρ_{μ_z}, ρ_{μ_Υ}, ρ_g.

We need to find ϕ_1, ϕ_2, ϕ_4, γ_1, γ_2, ψ, ν, μ_I, G, and the remaining endogenous variables

$$\boxed{h = 0.5}$$

$$\boxed{u = 1}$$

$$s_K = \frac{R^k u K}{\mu_I Y} \qquad (*)$$

$$\frac{G}{Y} = 0.17 = s_g$$

$$\gamma_2 = \frac{\gamma_2}{\gamma_1} \times \gamma_1$$

$$Y = R^k u K/\mu_I + W h^d (1 + \nu(1 - R^{-1})) \qquad (**)$$

$$\frac{M^h}{M^h + \nu W h^d} = SMH \qquad (***)$$

$$\frac{M^h + \nu W h^d}{Y} = SM \qquad (****)$$

$$\epsilon_{m^h,R} = -\frac{1}{8}\,\frac{1}{R(\phi_2\, R + R - 1)}$$

$$\boxed{\mu_I = \mu_\Upsilon \mu_{z^*}}$$

$$\boxed{\mu_\Lambda = \frac{1}{\mu_{z^*}}}$$

$$I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right)$$

$$v = \frac{C}{M^h}$$

$$(1-\phi_4)[C(1-\mu_{z^*}^{-1}b)]^{(1-\phi_3)(1-\phi_4)-1}(1-h)^{\phi_4(1-\phi_3)}\,(1-b\beta\mu_\Lambda)=\Lambda[1+\ell(v)+v\ell'(v)]$$

$$Q=\frac{\beta\mu_\Lambda}{\mu_\Upsilon}\left[R^k u-a(u)+Q(1-\delta)\right]$$

$$\boxed{Q=1}$$

$$v=\sqrt{\frac{\phi_2}{\phi_1}+\frac{1}{\phi 1}(1-R^{-1})}$$

$$R^k=a'(u)=\gamma_1+\gamma_2(u-1)$$

$$F^1\left[1-\tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right]=\left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}\Lambda\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}}h^d$$

$$F^2\left[1-\tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right]=\left[\phi_4(C-b\mu_{z^*}^{-1}C)^{(1-\phi_3)(1-\phi_4)}(1-h)^{\phi_4(1-\phi_3)-1}\right]\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}}h^d$$

$$F^1=F^2$$

$$\boxed{R=\frac{\pi}{\beta\mu_\Lambda}}$$

$$Y=C[1+\ell(v)]+G+[I+a(u)\mu_I^{-1}K]$$

$$X^1\left(1-\alpha\beta\mu_\Lambda\pi^{-(\chi-1)\eta}\mu_{z^*}\right)=Y\mathrm{mc}\tilde{p}^{-\eta-1}$$

$$X^2\left(1-\alpha\beta\mu_\Lambda\pi^{(\chi-1)(1-\eta)}\mu_{z^*}\right)=Y\tilde{p}^{-\eta}$$

$$\eta X^1=(\eta-1)X^2$$

$$\boxed{\tilde{p}=\left(\frac{1-\alpha\pi^{(\eta-1)(1-\chi)}}{(1-\alpha)}\right)^{1/(1-\eta)}}$$

$$(u\mu_I^{-1}K)^\theta {h^d}^{1-\theta}-\psi=\left\{[1+\ell(v)]C+G+[I+a(u)K/\mu_I]\right\}s$$

$$s=(1-\alpha)\tilde{p}^{-\eta}+\alpha\pi_t^{(1-\chi)\eta}s$$

$$\mathrm{mc}(1-\theta)(u_tK_t/\mu_{I,t})^\theta h_t^{d-\theta}=W_t\left[1+\nu\frac{R_t-1}{R_t}\right]$$

$$\mathrm{mc}\theta(uK/\mu_I)^{\theta-1}{h^d}^{1-\theta}=R^k$$

$$h=\tilde{s}h^d$$

$$\tilde{s}\left[1-\tilde{\alpha}(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\right]=(1-\tilde{\alpha})\left(\frac{\tilde{W}}{W}\right)^{-\tilde{\eta}}$$

$$\boxed{\left(\frac{\tilde{W}}{W}\right)=\left(\frac{1-\tilde{\alpha}(\mu_{z^*}\pi)^{(\tilde{\chi}-1)(1-\tilde{\eta})}}{(1-\tilde{\alpha})}\right)^{1/(1-\tilde{\eta})}}$$

Using equations (*), (**), (***), and (****), and it follows that

$$\boxed{\nu=\frac{s_m(1-s_{mh})}{1-\theta-s_m(1-s_{mh})(1-1/R)}}$$

3.2 Steady State 2 :

At this point we know: u, h, ϕ_3, γ_2/γ_1, θ, κ, b, β, δ, $\tilde{\eta}$, $\tilde{\alpha}$, η, α, χ, $\tilde{\chi}$, μ_z, μ_Υ, σ_{μ_z}, σ_{μ_Υ}, σ_g, ρ_{μ_z}, ρ_{μ_Υ}, ρ_g, ν, μ_I. μ_Λ, Q, $\tilde{p}$, R.

I still need: ϕ_1, ϕ_2, ϕ_4, γ_1, γ_2, ψ, , G,

$$s_K = \theta = \frac{R^k u K}{\mu_I Y} \qquad (*)$$

$$\gamma_2 = \frac{\gamma_2}{\gamma_1} \times \gamma_1$$

$$Y = R^k uK/\mu_I + Wh^d(1 + \nu(1 - R^{-1})) \qquad (**)$$

$$\frac{M^h}{M^h + \nu W h^d} = SMH \qquad (***)$$

$$\frac{M^h + \nu W h^d}{Y} = SM \qquad (****)$$

$$\boxed{\phi_2 = \frac{(-8R\epsilon_{m^h,R})^{-1} + 1 - R}{R}}$$

$$I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right)$$

$$v = \frac{C}{M^h}$$

$$(1-\phi_4)[C(1-\mu_{z^*}^{-1}b)]^{-1}(1 - b\beta\mu_\Lambda) = \Lambda[1 + \ell(v) + v\ell'(v)]$$

$$\boxed{R^k = \frac{\mu_\Upsilon}{\beta\mu_\Lambda} - (1-\delta)}$$

$$v = \frac{1}{\sqrt{\phi_1}}\tilde{v}; \quad \boxed{\tilde{v} \equiv \sqrt{\phi_2 + (1 - R^{-1})}}$$

$$R^k = \gamma_1 + \gamma_2(u-1)$$

$$F^1\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right] = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}\Lambda\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^2\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right] = [\phi_4(1-h)^{-1}]\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^1 = F^2$$

$$Y = C[1 + \ell(v)] + G + [I + a(u)\mu_I^{-1}K]$$

$$\boxed{\mathrm{mc} = \frac{\tilde{p}(\eta-1)[1 - \alpha\beta\mu_\Lambda\mu_{z^*}\pi^{\eta(1-\chi)}]}{\eta[1 - \alpha\beta\mu_\Lambda\mu_{z^*}\pi^{(\chi-1)(1-\eta)}]}}$$

$$(u\mu_I^{-1}K)^\theta {h^d}^{1-\theta} - \psi = Ys$$

$$\boxed{s = \frac{(1-\alpha)\tilde{p}^{-\eta}}{1 - \alpha\pi^{(1-\chi)\eta}}}$$

$$\mathrm{mc}(1-\theta)(u_t K_t/\mu_{I,t})^\theta h_t^{d-\theta} = W_t\left[1 + \nu\frac{R_t - 1}{R_t}\right]$$

$$\mathrm{mc}\theta(uK/\mu_I)^{\theta-1}{h^d}^{1-\theta} = R^k$$

$$h = \tilde{s}h^d$$

$$\boxed{\tilde{s} = \frac{(1-\tilde{\alpha})\left(\frac{\tilde{W}}{W}\right)^{-\tilde{\eta}}}{\left[1 - \tilde{\alpha}(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\right]}}$$

3.3 Steady State 3 :

$$\gamma_2 = \frac{\gamma_2}{\gamma_1} \times \gamma_1$$

$$Y = R^k u K/\mu_I + W h^d (1 + \nu(1 - R^{-1})) \qquad (**)$$

$$I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right)$$

$$v = \frac{C}{M^h}$$

$$(1-\phi_4)[C(1-\mu_{z^*}^{-1}b)]^{-1}(1 - b\beta\mu_\Lambda) = \Lambda[1 + \ell(v) + v\ell'(v)]$$

$$v = \frac{1}{\sqrt{\phi_1}}\tilde{v}$$

$$\boxed{\gamma_1 = R^k - \gamma_2(u-1)}$$

$$F^1\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right] = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}\Lambda\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^2\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right] = [\phi_4(1-h)^{-1}]\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^1 = F^2$$

$$Y = C[1 + \ell(v)] + G + [I + a(u)\mu_I^{-1}K]$$

$$\left(u\mu_I^{-1}K\right)^\theta {h^d}^{1-\theta} - \psi = Ys$$

$$\text{mc}(1-\theta)(u_t K_t/\mu_{I,t})^\theta h_t^{d-\theta} = W_t\left[1 + \nu\frac{R_t - 1}{R_t}\right]$$

$$\boxed{K = \mu_I/u\left(\frac{R^k}{\text{mc}\theta}\right)^{1/(\theta-1)} h^d}$$

$$\boxed{h^d = \frac{h}{\tilde{s}}}$$

3.4 Steady State 4:

$$\boxed{\gamma_2 = \frac{\gamma_2}{\gamma_1} \times \gamma_1}$$

$$\boxed{Y = R^k uK/\mu_I + Wh^d(1 + \nu(1 - R^{-1}))}$$

$$\boxed{I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right)}$$

$$\boxed{\phi_1 = \left[\frac{\tilde{v}}{-\tilde{v}^2 - \phi_2 + 2\tilde{v}\sqrt{\phi_2} + \frac{1-G/Y-I/Y}{s_{mh}s_m}}\right]^2}$$

$$(1-\phi_4)[C(1-\mu_{z^*}^{-1}b)]^{-1}(1-b\beta\mu_\Lambda) = \Lambda[1+\ell(v)+v\ell'(v)]$$

$$\boxed{v = \frac{1}{\sqrt{\phi_1}}\tilde{v}}$$

$$F^1\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right] = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}\Lambda\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}}h^d$$

$$F^2\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right] = [\phi_4(1-h)^{-1}]\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}}h^d$$

$$F^1 = F^2$$

$$\boxed{C = \frac{G + [I + a(u)\mu_I^{-1}K] - Y}{[1+\ell(v)]}}$$

$$\boxed{\psi = \left(u\mu_I^{-1}K\right)^\theta {h^d}^{1-\theta} - Ys}$$

$$\boxed{W = \mathrm{mc}(1-\theta)(u_tK_t/\mu_{I,t})^\theta h_t^{d-\theta}\left[1 + \nu\frac{R_t - 1}{R_t}\right]}$$

3.5 Steady State 5:

$$\boxed{\frac{\Lambda}{(1-\phi_4)} = \frac{[C(1-\mu_{z^*}^{-1}b)]^{-1}(1-b\beta\mu_\Lambda)}{[1+\ell(v)+v\ell'(v)]}}$$

$$\boxed{\phi_4 = \frac{A}{1+A}; \quad \text{where } A = \frac{\left[1-\tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right]}{\left[1-\tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right]}\left(\frac{\tilde{\eta}-1}{\tilde{\eta}[(1-h)^{-1}]}\right)\tilde{W}\frac{\Lambda}{1-\phi_4}}$$

3.6 Knowing all structural parameters:

Suppose we know the steady state value of the policy instrument, that is, R_t, and also all structural parameters, that is, we have numerical values for: δ, μ_I, ϕ_3, ϕ_4, μ_{z^*}, b, β, μ_Λ, μ_Υ . Then for a given value of R we can find the steady state as follows: Note that in steady state $u = 1$ regardless of the value taken by the nominal interest rate. This is because γ_1 was chosen such $1 = \beta\mu_\Lambda/\mu_\Upsilon[\gamma_1 + 1 - \delta]$

3.7 Steady State 1 for $\phi_3 = 1$

$$\boxed{Q = 1}$$

$$I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right)$$

$$\boxed{\pi = R\beta\mu_\Lambda}$$

$$v = \frac{C}{M^h}$$

$$(1-\phi_4)[C(1-\mu_{z^*}^{-1}b)]^{-1}\,(1 - b\beta\mu_{z^*}\mu_\Lambda) = \Lambda[1 + \ell(v) + v\ell'(v)]$$

$$\boxed{u = 1}$$

$$\boxed{v = \sqrt{\frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{R-1}{R}}}$$

$$\boxed{R^k = \gamma_1}$$

$$F^1\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right] = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}\Lambda\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^2\left[1 - \tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right] = [\phi_4(1-h)^{-1}]\left(\frac{W}{\tilde{W}}\right)^{\tilde{\eta}} h^d$$

$$F^1 = F^2$$

$$Y = C[1 + \ell(v)] + G + I$$

$$\mathrm{mc} = \tilde{p}\frac{\eta-1}{\eta}\frac{\left(1-\alpha\beta\mu_\Lambda\pi^{-(\chi-1)\eta}\mu_{z^*}\right)}{\left(1-\alpha\beta\mu_\Lambda\pi^{(\chi-1)(1-\eta)}\mu_{z^*}\right)}$$

$$\boxed{\tilde{p} = \left(\frac{1-\alpha\pi^{(\eta-1)(1-\chi)}}{(1-\alpha)}\right)^{1/(1-\eta)}}$$

$$\left(u\mu_I^{-1}K\right)^\theta {h^d}^{1-\theta} - \psi = \{[1+\ell(v)]C + G + [I + a(u)K/\mu_I]\}\, s$$

$$\boxed{s = \frac{(1-\alpha)\tilde{p}^{-\eta}}{1-\alpha\pi^{(1-\chi)\eta}}}$$

$$W = \frac{\mathrm{mc}(1-\theta)(u_t K_t/h^d/\mu_{I,t})^\theta}{\left[1+\nu\frac{R_t-1}{R_t}\right]}$$

$$\boxed{\left(\frac{uK}{\mu_I h^d}\right) = \left(\frac{R^k}{\theta\mathrm{mc}}\right)^{1/(\theta-1)}}$$

$$h = \tilde{s}h^d$$

$$\boxed{\tilde{s} = \frac{(1-\tilde{\alpha})\left(\frac{\tilde{W}}{W}\right)^{-\tilde{\eta}}}{\left[1-\tilde{\alpha}(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\right]}}$$

$$\boxed{\left(\frac{\tilde{W}}{W}\right) = \left(\frac{1-\tilde{\alpha}(\mu_{z^*}\pi)^{(\tilde{\chi}-1)(1-\tilde{\eta})}}{(1-\tilde{\alpha})}\right)^{1/(1-\tilde{\eta})}}$$

3.8 Steady State 2

$$I = K\left(1 - \tfrac{(1-\delta)}{\mu_I}\right)$$

$$v = \tfrac{C}{M^h}$$

$$C = (1-h)\left(\tfrac{1-\phi_4}{\phi_4}\right)\left(\tfrac{\tilde{\eta}-1}{\tilde{\eta}}\right)\left(\tfrac{(1-b\beta\mu_\Lambda)}{(1-\mu_{z^*}^{-1}b)[1+\ell(v)+v\ell'(v)]}\right)\left(\tfrac{\left[1-\tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})\tilde{\eta}}\mu_\Lambda\mu_{z^*}\right]\tilde{W}}{\left[1-\tilde{\alpha}\beta(\mu_{z^*}\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}\mu_\Lambda\mu_{z^*}\right]}\right) = A(1-h)$$

$$\boxed{\mathrm{mc} = \tilde{p}^{\frac{\eta-1}{\eta}}\tfrac{\left(1-\alpha\beta\mu_\Lambda\pi^{-(\chi-1)\eta}\mu_{z^*}\right)}{\left(1-\alpha\beta\mu_\Lambda\pi^{(\chi-1)(1-\eta)}\mu_{z^*}\right)}}$$

$$\left(\tfrac{K}{\mu_I h^d}\right)^\theta h^d - \psi = \{[1+\ell(v)]C + G + I\}\, s$$

$$\boxed{W = \tfrac{\mathrm{mc}(1-\theta)(u_t K_t/h^d/\mu_{I,t})^\theta}{\left[1+\nu\frac{R_t-1}{R_t}\right]}}$$

$$h = \tilde{s}h^d$$

$$\boxed{\tilde{W} = W\tfrac{\tilde{W}}{W}}$$

3.9 Steady State 3

$$v = \tfrac{C}{M^h}$$

$$\left(\tfrac{K}{\mu_I h^d}\right)^\theta h^d - \psi = \left\{[1+\ell(v)]A(1-\tilde{s}h^d) + G + h^d\left(\tfrac{K}{\mu_I h^d}\right)(\mu_I - 1 + \delta)\right\} s$$

Write the last equation as:

$$B_1 h^d - \psi = B_2(1-\tilde{s}h^d) + sG + B_3 h^d$$

where

$$B_1 = \left(\frac{K}{\mu_I h^d}\right)^\theta$$

$$B_2 = s[1+\ell(v)]A$$

$$B_3 = s\left(\frac{K}{\mu_I h^d}\right)(\mu_I - 1 + \delta)$$

Solve for h^d

$$\boxed{h^d = \tfrac{B_2 + sG + \psi}{[B_1 + B_2\tilde{s} - B_3]}}$$