Investor borrowing heterogeneity in a Kiyotaki–Moore style macro model[☆]

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HIGHLIGHTS

- We present a modification to the Kiyotaki–Moore collateral constraint model.
- We allow for heterogeneity in investors ability to borrow from collateral.
- We calibrate the model to the debt-ratio distribution of US non-financial firms.
- The heterogeneous investors model leads to stronger financial amplification.

ARTICLE INFO

ABSTRACT

We introduce heterogeneity in investors' ability to borrow from collateral in a Kiyotaki–Moore style macro model, calibrated to the quintiles of the leverage-ratio distribution of US non-financial firms. Financial amplification intensifies, because of stronger asset price reactions of highly levered investors.

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1. Introduction

We present an extension of a core macroeconomic model with financial frictions to account for heterogeneity in investors' ability to borrow from collateral. The literature leading this field, both the seminal contributions of Kiyotaki and Moore (1997) and Bernanke et al. (1999), but also the large literature thereafter,[2] retains a high degree of aggregation: there typically exists a *representative* financially-constrained agent, and existing models are typically calibrated to match an economy-wide average of leverage ratios. In the data, observed leverage ratios (assets to net worth) of US non-financial firms are, on average, at around 1.5–2 (see, e.g. CGFS, 2009), with wide cross-sectional variation. Leverage ratios of financial intermediators are substantially higher.[3]

We take the framework of Kiyotaki and Moore (1997) and, instead of a representative investor, introduce different investor types that each can manage different types of capital, which are collateralizable to different degrees. We calibrate the model to match observed leverage ratios of the quintiles of the distribution of leverage ratios for US non-financial firms, using the dataset of Rauh and Sufi (2010). We find that the model with heterogeneous investors produces a more pronounced financial

3 Leverage ratios of US commercial banks at the center of the crisis were in the range of 15–20, those of US investment banks around 25–30 (CGFS, 2009).

amplification of shocks compared to a model version where collateral constraint parameters are calibrated to the economy-wide average (homogeneous investors). This is because investors with the highest leverage are the drivers of asset prices, not the economy-wide average. Asset price drops in response to negative productivity shocks are therefore stronger in the heterogeneous investors model, tightening financial constraints of all investors, and leading to additional amplification.

2. The model

The model economy is populated by a representative saver (patient), investors (impatient), and a representative firm. Investors borrow from savers to invest in capital; they each manage a specific type of capital, which they rent to firms for use in production, and which is collateralizable to different degrees. For simplicity, we assume labor supply and investors' types of capital in fixed supply.

2.1. Investors

There is a continuum of investors, with measure 1. Investors come in I different types, each investor i's size is given by n_i, where $\sum_i n_i = 1$. Preferences of investor i, for $i = 1, .., I$, are:

$$E_t \sum_{s=t}^{\infty} \left(\beta^{I_i}\right)^s \frac{\left(C_t^{I_i}\right)^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $C_t^{I_i}$ is consumption of a homogeneous final good, σ the coefficient of relative risk aversion and β^{I_i} investor i's discount factor. The budget constraint reads:

$$C_t^{I_i} + q_{i,t} K_{i,t}^{I_i} = L^{I_i} W_t^{I_i} + \left(q_{i,t} + R_{Ki,t}\right) K_{i,t-1}^{I_i}$$
$$+ B_{i,t}^{I_i} - R_{i,t-1} B_{i,t-1}^{I_i}. \tag{2}$$

$K_{i,t}^{I_i}$ is holdings of a type-i fixed asset, $q_{i,t}$ the asset price, $R_{Ki,t}$ the return the asset earns, $W_t^{I_i}$ is wage income, L^{I_i} her (inelastically supplied) labor input, $B_{i,t}^{I_i}$ is debt issued to savers, and $R_{i,t-1} B_{i,t-1}^{I_i}$ the payment on previously incurred debt. Investors also face a constraint on total leverage due to an inability to commit to repayment. Total debt of investor i is restricted to be no greater than κ_i times the market value of her assets, where $\kappa_i \leq 1$:

$$B_{i,t}^{I_i} \leq \kappa_i q_{i,t} K_{it}^{I_i}. \tag{3}$$

Borrowing constraint parameters κ_i differ across investors, because investors hold different types of capital, which are collateralizable to different degrees. In addition, lenders' liquidation technologies w.r.t. different investors may differ, because informational asymmetries may be differently strongly pronounced. The first-order conditions of investor i's optimal choice of asset holdings, $K_{i,t}^{I_i}$, and borrowing, $B_{i,t}^{I_i}$, are

$$\left(C_t^{I_i}\right)^{-\sigma} = \beta^{I_i} E_t \left[\left(C_{t+1}^{I_i}\right)^{-\sigma}\right] R_{i,t} + \mu_t^{I_i}, \tag{4}$$

$$q_{i,t}\left(C_t^{I_i}\right)^{-\sigma} = \beta^{I_i} E_t \left[\left(C_{t+1}^{I_i}\right)^{-\sigma}\left(q_{i,t+1} + R_{Ki,t+1}\right)\right] + \mu_t^{I_i} q_{i,t}. \tag{5}$$

Variable $\mu_t^{I_i}$ represents the shadow value of relaxing the leverage constraint by one unit. When $\mu_t^{I_i} > 0$, the expected return on the asset exceeds the cost of borrowing, and the collateral constraint (3) holds with equality (resulting from the Kuhn–Tucker condition).[4]

2.2. Savers

There is a representative saver, of measure 1, with preferences:

$$E_t \sum_{s=t}^{\infty} \left(\beta^S\right)^s \frac{\left(C_t^S\right)^{1-\sigma}}{1-\sigma}, \tag{6}$$

where C_t^S and β^S are the saver's consumption and discount factor, respectively. Savers are more patient than investors, so that $\beta^S > \beta^{I_i}$, $\forall i$. The saver maximizes (6) subject to:

$$C_s^S + \sum_i \left(B_{is}^S - R_{is-1} B_{is-1}^S\right) + \sum_i q_{is} K_{is}^S$$
$$= L^S W_s^S + \sum_i q_{is} K_{is-1}^S + G\left(K_{s-1}^S\right). \tag{7}$$

K_{is}^S denotes the saver's holdings of type-i fixed assets, and $-B_{is}^S$ denotes lending to the i'th investor. The saver obtains wage W_t^S on her (inelastically supplied) labor. The saver uses all i-types of fixed assets, K_{is-1}^S, as inputs into a backyard production function, given by $Y_t^S = G\left(K_{s-1}^S\right) = Z\left(K_{t-1}^S\right)^{\omega}$, with $G'\left(K_{s-1}^S\right) > 0$ and $G''\left(K_{s-1}^S\right) < 0$. The saver's aggregate fixed asset holdings are modeled as a CES-composite of the individual i-types of fixed assets, with substitution elasticity θ:

$$K_t^S = \left[\sum_i n_i^{\frac{1}{\theta}} \left(K_{i,t}^S\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{8}$$

The saver's first-order conditions for optimal choices of K_{is}^S and B_{is}^S, $\forall i$, are given by:

$$\left(C_t^S\right)^{-\sigma} = \beta^S E_t \left[\left(C_{t+1}^S\right)^{-\sigma}\right] R_{i,t}, \tag{9}$$

$$q_{i,t}\left(C_t^S\right)^{-\sigma} = \beta^S E_t \left[\left(C_{t+1}^S\right)^{-\sigma}\left(q_{i,t+1}\right.\right.$$
$$\left.\left. + \omega Z\left(K_{t-1}^S\right)^{(\omega-1)} \frac{n_i^{\frac{1}{\theta}} \left(K_{i,t-1}^S\right)^{\frac{-1}{\theta}}}{\left(K_{t-1}^S\right)^{-\frac{1}{\theta}}}\right)\right]. \tag{10}$$

2.3. Firms

The final good firm produces using labor and the fixed asset with standard production function, $Y_t^I = A_t L^{\varepsilon} \left(K_{t-1}^I\right)^{\varepsilon}$. A_t is productivity. Aggregate employment L is constant (because labor supply is inelastic) and normalized to 1. K_{t-1}^I is a CES-composite of the individual i-types of assets:

$$K_t^I = \left[\sum_i n_i^{\frac{1}{\theta}} \left(K_{it}^{I_i}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{11}$$

First-order conditions give:

$$W_t = (1-\varepsilon) A_t \left(K_t^I\right)^{\varepsilon-1}, \tag{12}$$

$$R_{Ki,t} = \varepsilon A_t \left(K_t^I\right)^{(\varepsilon-1)} \frac{n_i^{\frac{1}{\theta}} \left(K_t^{I_i}\right)^{\frac{-1}{\theta}}}{\left(K_t^I\right)^{-\frac{1}{\theta}}}. \tag{13}$$

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[4] The assumption that investors are more impatient guarantees that investors' leverage constraints hold with equality at the deterministic steady-state, but, be-

cause of precautionary motives, not generally in a stochastic world. Nevertheless, with small shock volatility and sufficiently low investors' discount factor, constraints are likely binding. Our proposed model is too stylized in some dimensions. E.g., it is unrealistic to assume that all US non-financial firms are constrained. It would be possible to introduce a fraction of firms that are unconstrained. Similarly, we assume only short-term (one-period) debt, whereas in reality firms may become constrained, because they took on long-term debt under more optimistic lending conditions. All these are possible interesting avenues for future research.

Table 1
Parameters.

| Parameter		Value	
Discount factor, savers	β^S	0.99	
Discount factor, investors	β^{I_i}	0.97	
Risk aversion	σ	2.00	
Productivity, autocor.	ρ_A	0.9	Perri and Quadrini (1991)
Productivity, shock vol.	σ_A	0.005	Perri and Quadrini (1991)
Formal production	ϵ	0.39	Young et al. (2010)
Informal production	ω	0.08	Benhabib et al. (1991)
Labor share, savers	$l^S = \frac{L^S}{\sum_i L^{I_i} + L^S}$	0.8	
Substitution elasticity, CES	θ	5	
heterogen. investors			
Debt ratio parameters	$\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$	0.84, 0.61, 0.48, 0.34, 0.10	
Size of i-type capital, CES	n_1, n_2, n_3, n_4, n_5	0.08, 0.13, 0.14, 0.25, 0.41	
homogen. investors			
Debt ratio parameters	$\kappa_i, i = 1, \dots, 5$	0.48	
Size of i-type capital, CES	n_i	$1/I = 1/5$	
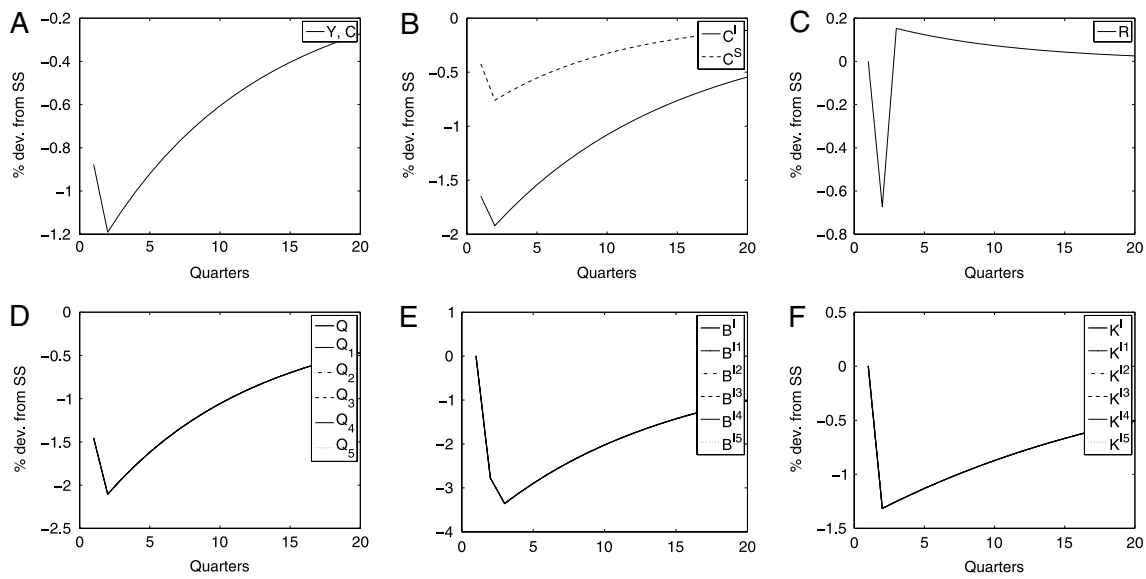
Fig. 1. Impulse responses to a 1% productivity decrease, model with homogeneous investors.

2.4. Equilibrium

Equilibrium in the markets for labor, fixed assets, and debt implies:

$$\sum_i L^{I_i} + L^S = L \equiv 1, \tag{14}$$

$$K_{i,t}^{I_i} + K_{i,t}^S = n_i, \quad i = 1, \dots, I, \tag{15}$$

$$B_{i,t}^{I_i} + B_{i,t}^S = 0, \quad i = 1, \dots, I. \tag{16}$$

The resource constraint is:

$$\sum_i C_t^{I_i} + C_t^S = A_t L^\varepsilon \left(K_{t-1}^I \right)^\varepsilon + G \left(K_{t-1}^S \right). \tag{17}$$

3. Parameterization

Table 1 summarizes parameter values. To save space, the table provides references used for the specification of conventional parameters. The substitution elasticity of the *i*-types of capital, θ, is less conventional and difficult to calibrate. We thus set $\theta = 1$ as a baseline, but provide sensitivity analysis also for the cases of substitutes ($\theta > 1$) and complements ($\theta < 1$). Z, the productivity in the backyard production sector, is set such that investors hold 80% of fixed assets at steady-state. The main novelty of our paper

is to take seriously the large heterogeneity in investors' leverage ratios. We consider $I = 5$ and calibrate parameters κ_i, for $i = 1, ..5$, to match the 90, 70, 50, 30, and 10-th percentiles of the distribution of debt ratios (measured as total debt to total assets at book value) of US non-financial firms, using the dataset of Rauh and Sufi (2010). This dataset contains 2453 public US non-financial firms, for the period of 1996–2006. Each investor *i*'s size is calibrated to the share of the total sales (as a proxy for value-added) of firms in quintile *i* in total sales of all firms. The weights reflect the fact that firms in the quintiles with high debt ratios are typically smaller, because small firms typically rely more on external finance. The leverage ratios (assets-to-net worth) corresponding to κ_i, for $i = 1, ..5$, are given by $\frac{1}{1-\kappa_i}$, which are 6.45, 2.54, 1.92, 1.51, and 1.11.

In a comparison case of homogeneous investors, we parameterize debt ratios to the median, $\kappa_i = 0.48$ and set $n_i = 1/n$ for all *i*.

4. Results

We present impulse responses for two model versions: the first is the case of 'homogeneous investors' ($\kappa_i = 0.48$, $n_i = 1/n$, $\forall i$). The second is the 'heterogeneous investors' case, allowing investors $i = 1, \dots, I$ to differ in their ability to borrow from collateral.

Fig. 1 presents impulse responses to a 1% productivity decrease for the case of 'homogeneous investors'. The reduced productivity
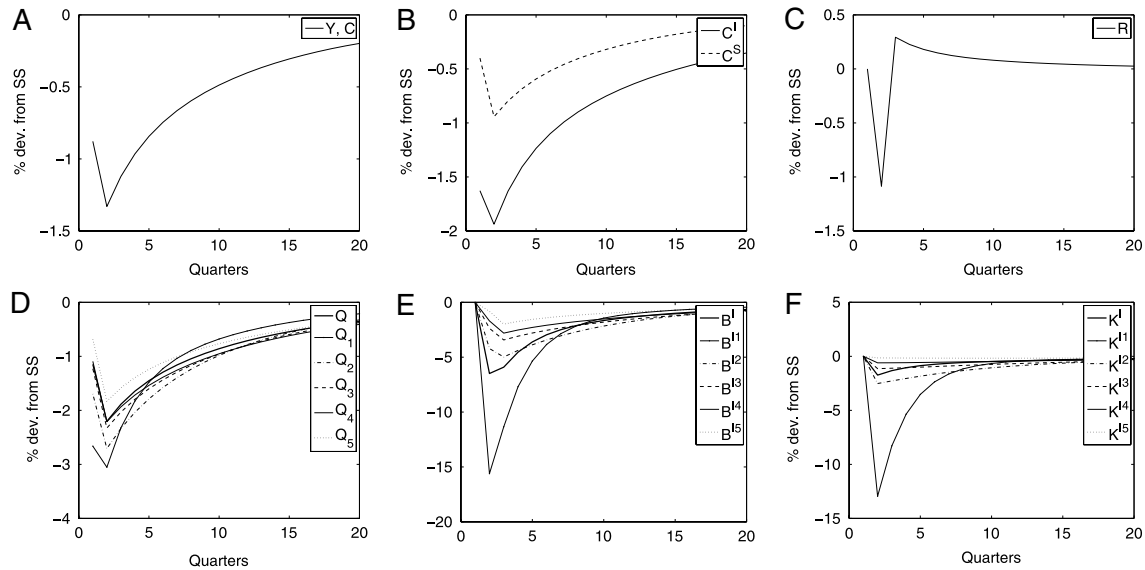
Fig. 2. Impulse responses to a 1% productivity decrease, model with heterogeneous investors.
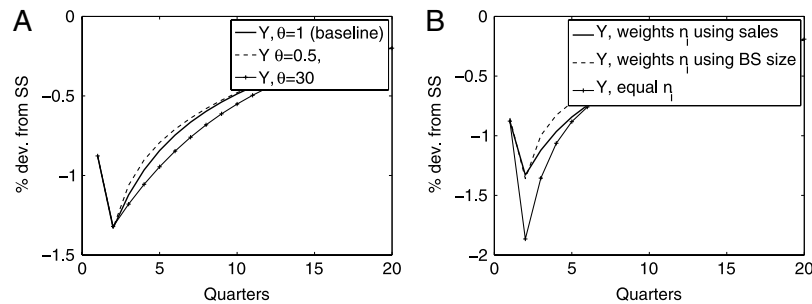

Fig. 3. Impulse responses to a 1% productivity decrease, sensitivity analysis.

in formal production reduces wages for both investors and savers, and, because the fall is persistent, reduces the return on investment in the fixed assets for investors. These effects imply a fall in the price of the fixed asset (panel D), which causes a tightening of leverage constraints and leads investors to decrease their borrowing (panel E). Panel A of Fig. 1 shows that total output, Y_t, the sum of formal and backyard production, falls. Output in the formal sector, Y_t^I, declines both because of the direct effect of lower productivity, but also because in response to lower borrowing investors can now finance less of their holdings of fixed assets. Investors reduce their holdings of the fixed asset (panel F), which was used in final good sector. Since more of the fixed asset is allocated to backyard production, Y_t^S, increases, but this increase is not enough to compensate the fall in Y_t^I. The binding leverage constraint thus leads to an additional dip in output in period 2. Because both savers and investors have temporarily lower consumption (panel B), they reduce their savings. However, investors reduce their demand for investment funds even more strongly, since the tightening of leverage constraints forces them to reduce their total borrowing to finance investment. With binding constraints the drop in demand thus exceeds the drop in the supply of funds, and the real interest rate must fall (panel C).

Fig. 2 presents impulse responses for the case of heterogeneous investors. Qualitatively, the behavior of economic variables is similar as before. But quantitatively the responses differ markedly. Investor 1, the most levered investor, has to sharply decrease her borrowing and, as a result, her holdings of the fixed asset, by 15.61% and 12.95% respectively. The asset price corresponding to investor 1's fixed assets drops by almost 3.05% at peak—because of

the high leverage ratio this constitutes a much more pronounced asset price drop than in the 'homogeneous investors' version. Because asset prices of different types of capital are tightly linked to each other via savers' intertemporal optimality conditions, they also experience pronounced declines. The (CES-based) aggregate asset price index falls by 2.20% at peak, compared to the more moderate drop of 2.01% in the homogeneous investors model. The stronger asset prices declines, in turn, lead to a more substantial tightening of financial constraints also for the other, less levered, investors.[5] This translates into an additional financial amplification compared to the model version where investors are identical, even though *on average* leverage constraints are not more severe: output in the economy drops, at peak, by 1.33% in the model version with investors who are heterogeneous in their ability to borrow, compared to just 1.19% in the model with homogeneous investors. This constitutes an additional 11.74% amplification of the heterogeneous over the homogeneous investors model.

Fig. 3 presents sensitivity analysis of our findings, focusing on the impulse response of total output. Panel A demonstrates that variations in the substitution elasticity of different types of fixed assets have little effect on the results. This is because holdings of i-types of capital are determined mostly by the saver's willingness to lend to investors, which changes little with θ, since the saver

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[5] Even though in a very different framework, the idea that asset prices are driven not by the average (but the marginal) investor features prominently in the work of Geanakoplos (2009). In our setup asset prices are driven primarily by the agents with the highest leverage.

earns the same return ($R_{it} = R_t$, $\forall i$) on all B_{it}^S, $i = 1, \ldots, I$. Panel B varies the weights of firms in quintiles $i = 1, \ldots, I$, using firms' balance sheet (BS) size (total assets) for the computation of weights,[6] or using equal weights $n_i = 1/n$. Since high-levered investors are weighted more heavily in those alternatives, we find higher amplification.

5. Conclusion

We presented a simple modification to a stylized Kiyotaki–Moore style model with collateral constraints: allowing for multiple investor types, which each has a different ability to borrow from collateral, calibrated to match the means of the quintiles of the distribution of leverage ratios of US non-financial firms. We find that in such extension, the financial amplification and acceleration mechanism is 11.74% stronger.

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[6] Balance sheet size weights n_i, $i = 1, \ldots, I$, are $0.10, 0.15, 0.16, 0.26, 0.33$.