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by Guido Ascari and Tiziano Ropele

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Kiel Institute for the World Economy, Düsternbrooker Weg 120, 24105 Kiel, Germany

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University of Pavia &
Bank of Italy
Via San Felice 5,
27100 Pavia, Italy
Telephone: +39 0382 986211
E-mails:
guido.ascari@unipv.it
tiziano.ropele@unimib.it

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Disinflation in a DSGE Perspective: Sacrifice Ratio or Welfare Gain Ratio? [*]

Guido Ascari[†]
University of Pavia

Tiziano Ropele
Bank of Italy

January 7, 2009

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When taken to examine disinflation monetary policies, the current workhorse DSGE model of business cycle fluctuations successfully accounts for the main stylized facts in terms of recessionary effects and sacrifice ratio. We complement the transitional analysis of the short-run costs with a rigorous welfare evaluation and show that, despite the long-lasting economic downturn, disinflation entails non-zero overall welfare gains.

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[†] *Address*: Department of Economics and Quantitative Methods, University of Pavia, Via San Felice 5, 27100 PAVIA, Italy. *Tel: +39 0382 986211; e-mail:* guido.ascari@unipv.it

1 Introduction

Disinflation is a long-standing issue in monetary economics.

On the empirical side, there is ample evidence that disinflations yield short-run output losses. Indisputably, the key indicator to gauge the real costs of disinflation has been the sacrifice ratio, calculated as the ratio between the cumulative percentage output loss, i.e., the difference between actual and potential output, and the size of disinflation. Thus, the sacrifice ratio measures the real output cost per unit of permanent decrease in inflation. A wealth of empirical studies estimated the costs of disinflation for various countries, using different econometric methodologies. In general, the findings vary across countries, episodes or time periods and estimation methods. Gordon and King (1982) is an early assessment of the sacrifice ratio for the U.S., based on the estimation of autoregressive Phillips curves (see more recently, Andersen and Wascher, 1999). For EMU countries, Cuñado and Gracia (2003) reports estimates of the sacrifice ratio between 0.55 and 1.96. Ball (1994b) analyses specific disinflationary episodes in 19 moderate-inflation OECD countries between 1960 and 1991, and comes up with estimates of sacrifice ratio between 1.8 and 3.3 (see also Mankiw, 1999, and Zhang, 2005). Using the Vector Autoregression (VAR) methodology, Cecchetti and Rich (2001) find estimates of the sacrifice ratio between 1 and 10 for the U.S., while Durand et al. (2007) studies twelve EMU countries and reports substantially lower sacrifice ratios, namely, between 0.23 to 0.75. In summary, among different empirical studies there seems to be little disagreement on the following facts: (i) a disinflation yields a loss in output; (ii) the value of the sacrifice ratio varies across countries and time periods, but a plausible range is between 0.23 and 3.3.

On the theoretical side, however, there is a widespread view that the basic *linearized* New Keynesian DSGE model, as in Clarida, Gali and Gertler (1999), fails to replicate a costly disinflation. In a nutshell, being based on the Calvo (1983) price staggered mechanism the basic New Keynesian DSGE model only delivers price stickiness but not inflation inertia. To the contrary, inflation rate is described as a forward-looking variable that can immediately adjust after a disinflation, without any output costs.

Ball (1994a) was among the firsts to point out this inconsistency of standard sticky price models, in which a disinflation could also be followed by a boom rather than a slump (see also Burstein, 2006). Indeed, in a subsequent paper, Ball (1995) calls for imperfect credibility as a necessary device to explain the observed output costs of a disinflationary policy. More recently, Erceg and Levin (2003) and Goodfriend and King (2005) introduce imperfect credibility in a standard New Keynesian model to explain the famous Volcker disinflation (see also Nicolae and Nolan, 2006). Also Mankiw (2001) forcefully expresses the view that standard sticky price models cannot deliver inflation persistence and thus justify the costs of disinflation. Indeed, this drawback was one of the main reason that led Mankiw and Reis (2002) to propose a different model of price stickiness based on sticky information.

Nowadays, however, there is an operational model of business cycle fluctuations, based on the seminal work of Christiano et al. (2005) (CEE, henceforth). They show that a medium-scale New Keynesian model, enlarged to accommodate various nominal and real frictions, matched reasonably well the empirical fluctuations along the business cycle. Indeed, this model (or some slightly modified versions of it) has been widely and successfully employed both in empirical work (e.g., Smets and Wouters, 2003, Altig et al., 2004,) and in normative analysis (e.g., Schmitt-Grohé and Uribe, 2005).

However, it is surprising that no one has so far judged the ability of the CEE model to account for the costs of disinflation and more in general to address the issue of disinflation from a welfare perspective. This is what we do in this paper. We deliberately restrain ourselves from changing any of the features of our reference model and the structural parameters values as estimated or calibrated by CEE, and address two questions:

1. How successful is the current operational New Keynesian DSGE model of business cycle at replicating the empirically estimated costs of disinflation and sacrifice ratio, without resorting to any kind of imperfect credibility and/or information or of irrationality in expectations?

2. How costly is a credible disinflation in terms of welfare?

The answer to the first question is: quite a lot. Indeed, the simulation of the model indicates that a credible disinflation leads a prolonged decline of output and that the value of the sacrifice ratio is well in line with the available empirical evidence.

With regards to the second question, we work out a rigorous welfare evaluation of the costs of a disinflation, constructing a welfare based sacrifice ratio. Interestingly, despite the prolonged slump in output, we show that *a disinflation implies small welfare gains*. The size of these welfare gains is very small: equal to a permanent increase in initial steady state consumption of 0.06-0.07% each period per each point of diminished inflation. More precisely, small long-run gains prevail on even smaller short run costs. Indeed, surprisingly enough, the short run costs of a disinflation are negligible, despite the transitional economic downturn.

Finally, we want to raise a methodological consideration. Unlike the standard practice in the literature of approximating the model structural equations, here we simulate numerically the original non-linear model. In our view, this is crucial because taking linear or log-linear approximations may rule out some important transmission mechanisms. Yun (2005), for instance, emphasizes the role of relative price dispersion, often neglected in linear models, in driving his results for optimal monetary policy. Also, money is non superneutral in the CEE model. In this case, Ascari and Merkl (2007) shows that the use of log-linear approximations to study a disinflation can lead to misleading results, since a disinflation implies a movement from one steady state to another one.

2 An Operational Model of the Business Cycle

To study the effects of disinflationary monetary policy we rely on the operational medium scale New Keynesian DSGE model developed in CEE and then taken on, among others, in Smets and Wouters (2003) and Schmitt-Grohé and Uribe (2005, 2007). In this section we discuss some key features of the model and leave to the Appendix a brief description of the structural equations and parameters calibration.

The model features both real and nominal frictions, which are deemed to be crucial to replicate the dynamic properties of the business cycle (see CEE for US or Smets and Wouters, 2003, for the Euro Area). Real frictions include: monopolistic competition in goods and labor markets, internal habit in consumption, variable capital utilization and adjustment costs in investment decisions. As for nominal frictions: prices and wages are sticky à la Calvo with a clause of indexation. In particular, each period only a fraction of prices and wages are set optimally; those prices and wages that cannot be reoptimized are automatically adjusted to keep up with the inflation rate occurred in previous period. Finally, money balances enter the model in two ways: households derive direct utility from holding real money balances (i.e., assumption of money-in-the-utility function) and entrepreneurs must hold nominal money balances to pay wages before production (i.e., assumption of cash-in-advance).

We depart from our reference models with regards to monetary policy. We assume the central bank sets the short-term nominal interest rate, i.e., i_t, according to the non-linear rule defined by

$$\frac{1+i_t}{1+i^*} = \left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi}, \text{ with } \phi > 1 \tag{1}$$

where π_t, π^* and i^* represent the inflation rate, the inflation target and the nominal interest rate target, respectively. Notice, from the standard consumption Euler equation, it must hold that $1 + i^* = (1 + \pi^*)/\beta$, where β is the representative household's subjective discount factor.

Two distinct features of (1) are worth stressing. Firstly, our postulated nominal interest rate targeting rule does not respond to the output gap. The reason for this choice is the following. We think that a credible cold-turkey disinflation and countercyclical monetary policy behavior cannot coexist. Indeed, after implementing a permanent reduction of inflation target, any attempt to soften the output decline at the expenses of higher inflation, may question monetary authority's credibility to curb inflation. Secondly, our postulated nominal interest rate rule lacks an inertial term. Again, we think that central bank's attitude ought to be history independent. Especially at the time the disinflation is implemented, the short-term nominal interest rate ought to be adjusted freely in the light of new lower inflation target.

Before analyzing the costs of disinflation, it is important to highlight two things. The first consideration has to do with the deterministic steady state relationship between output and inflation. Although the degree of indexation in prices and wages is calibrated equal to one, money is non-superneutral. This latter result is due to the cash-in-advance constraint on intermediate firms to pay wage bill. As illustrated in CEE, in this case the real marginal cost schedule depends on the nominal interest rate. Albeit this hypothesis is important to match the empirical impulse response functions and the overall short-run dynamics, it also affects the deterministic steady state. Even with full price and wage indexation, positive trend inflation yields real output cost. Indeed, the higher the level of trend inflation, the larger the labor costs for the firms, and, *ceteris paribus*, the lower the wage paid to workers. In response, households reduce their labor supply and employment falls. Firms in turn decrease their capital stock, because labor and capital are complements in the production function. Eventually, the level of output decreases. The long-run Phillips Curve is not vertical.[1] Given CEE calibration these effects are rather minor: a permanent 1% reduction in inflation implies roughly a 0.1% increase in steady state output.[2]

The second consideration we want to draw attention to is methodological and concerns the solution of the model. We have just seen that in the CEE model money is non superneutral. This means that changes in trend inflation have effects on the steady state level of output. In our view, then, whenever a policy experiment leads to a transition between two steady states one should restrain from using standard solution methods based on local approximation. In these instances, it would be preferable and definitely more accurate to use non-linear solutions. And this is what we do in this paper. We sim-

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[1] From an empirical point of view, it has been difficult to tackle this issue within the VAR literature as the Blanchard and Quah (1989) restriction, i.e. no long-run effects of aggregate demand shock on output, is typically used as an identifying restriction (see e.g., Cecchetti and Rich, 2001). However, when this restriction is not imposed, it is not granted that output goes back exactly to its initial level (see Collard et al. 2006, Fève et al., 2007).

[2] It is important to stress that the assumption of full indexation in prices and wages rules out potential real effects arising from nominal rigidities. It is well-known that a positive steady state inflation rate increases steady state price and wage dispersion in the absence of full indexation yielding an inefficiency loss on aggregate production (e.g., Ascari, 2004, Schmitt-Grohé and Uribe, 2005, Yun, 2005). In other words, with partial wage and/or price indexation the real effects of long-run inflation, and thus also the effects on welfare, would be much larger.

ulate the perfect foresight transition path by numerically solving the non linear model in DYNARE.[3]

3 The short-run effects of disinflation

In this section we study the short-run effects of disinflation in the non linear operational New Keynesian DSGE model. However, before doing that, we define the notion of disinflation in the context of our theoretical model. Earlier to the disinflation, the economy is at a steady state characterized by a positive trend inflation π, which is pinned down by the inflation target π^*_{old}, i.e., $\pi = \pi^*_{\text{old}}$. At a certain period, say $t = 0$, the central bank reduces unexpectedly, instantaneously and credibly the inflation target from π^*_{old} to π^*_{new} implementing what is commonly known as a cold-turkey disinflation. Agents acknowledge the reduction of inflation target is permanent and do not expect any other policy surprise. Effectively, our disinflation experiment entails a transition between two steady states in a perfect foresight non linear model.

As regards the new inflation target we consider three cases: $\pi^*_{\text{new}} = \{0\%, 1\%, 2\%\}$. Disinflations aimed at achieving an inflation target of 1-2% are interesting for at least two reasons. Such targets come near to the actual inflation objectives at work in many central banks, e.g., the Reserve Bank of New Zealand, the Bank of Canada, the Bank of England and the European Central Bank.[4] Furthermore, an inflation target of 2% is not far-off from the recent estimates of US Federal Reserve's implicit inflation target.[5] Instead, the reason for studying cold-turkey disinflations aimed at achieving full price stability, i.e., $\pi^*_{\text{new}} = 0$, is more theory-based as the recent literature on optimal monetary policy has thoroughly stressed and emphasized the reasons why full price stability is socially desirable (see, e.g., Woodford, 2003). Finally, we present results both for $\phi = 1.5$ and

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[3] For further details on DYNARE see the webpage: http://www.cepremap.cnrs.fr/dynare/.

[4] Both in New Zealand and Canada the numerical inflation target extends from 1 to 3%. In the United Kingdom the explicit inflation objective is currently 2.5%, while in the Eurozone the European Central Bank has an inflation objective below, but close to, 2%.

[5] Leigh (2008) finds that in the period 1990-2004 the US Federal Reserve's implicit inflation target varied in the range 1-3%.

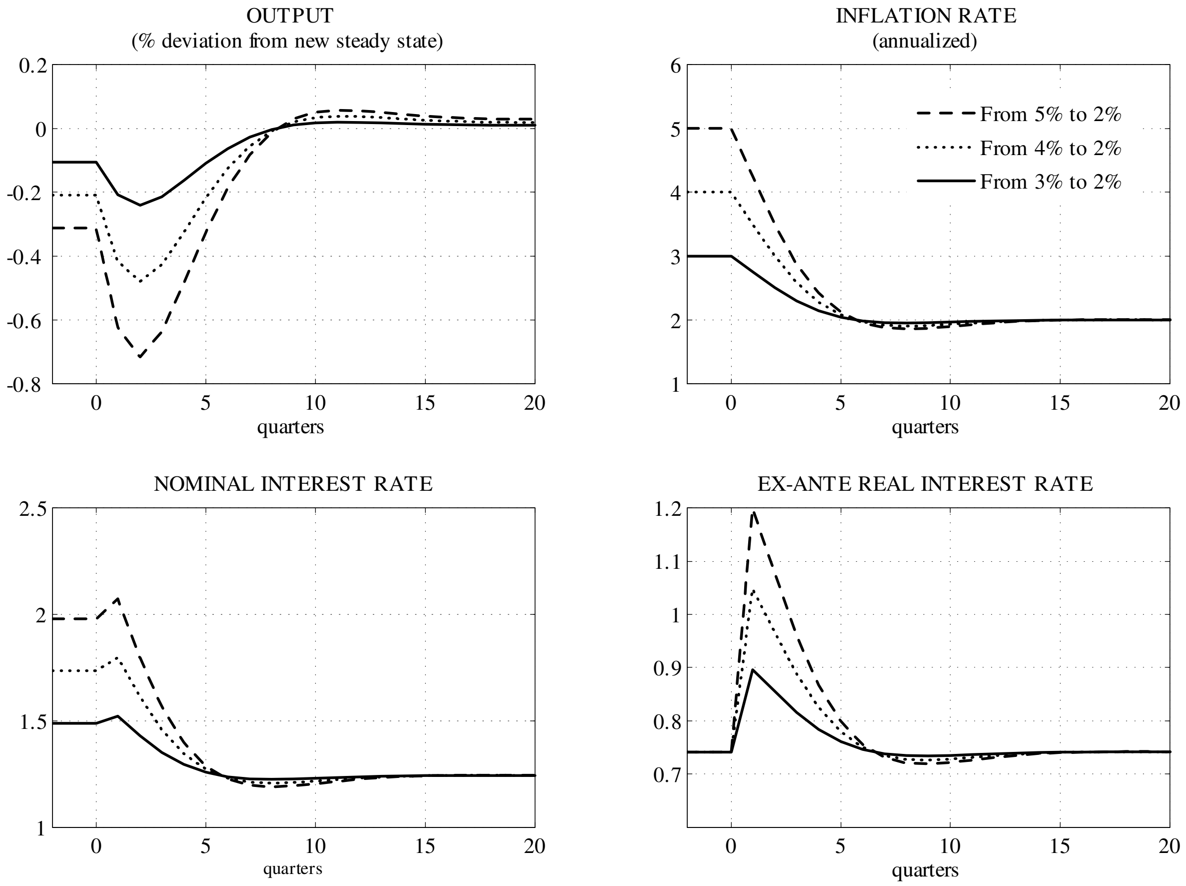

Figure 1: Cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$ with $\phi = 1.5$.

$\phi = 3$.

Figures 1 illustrates the dynamic adjustment of output, inflation, nominal and real interest rate after cold-turkey disinflation aimed at achieving $\pi^*_{\text{new}} = 2\%$, when $\phi = 1.5$. Each panel reports transition path starting off from different initial values of trend inflation, namely, $\pi^*_{\text{old}} = \{3\%, 4\%, 5\%\}$. In the non-linear CEE operational model, cold-turkey disinflations come with a sizable recession; the rate of inflation is highly persistent and gradually decreases towards the new target. Nominal and real interest rates increase on impact and then slowly revert to steady state.

When the central bank permanently reduces the inflation target only a fraction of intermediate firms set optimal prices, because of the Calvo staggered adjustment mechanism.[6] Discounting the forthcoming decline of output, necessary to bring down inflation, optimizing firms lower their prices. Remaining firms that instead are not allowed to optimize simply index their unchanged prices to previous period's inflation rate. As a matter of fact, they increase their prices by $1 + \pi^*_{\text{old}}$. As shown in Figure 1, of these two conflicting pricing decisions the latter prevails. Aggregate price index continues increasing but a slower rate. Thus, inflation rate decelerates.

As inflation does not immediately jump onto the new target, the central bank responds to the positive inflation gap $(\pi_1 - \pi^*_{\text{new}})$ with a monetary policy contraction. The central bank temporarily increases the policy rate, despite disinflation implies a lower steady state nominal interest rate. The follow-on rise of real interest rate reduce the aggregate demand: households postpone consumption and decrease investment spending. Furthermore, higher nominal interest rate increases intermediate firms' costs via the cash-in-advance constraint. Real wage drops, households supply less labor and intermediate firms reduce the rate of capital utilization. Taken as a whole, the level of output falls. In successive periods, inflation rate continues to adjust towards the new lower target while the central bank starts cutting the nominal interest rate. Nonetheless, the real interest rate remains above steady state for several quarters. The economy enters a recession and the level of output achieves the bottom in the second quarter. At last, the economy is successfully disinflated in about 15 quarters.

Figure 1 further shows that neither the qualitative dynamic adjustment nor the lapse of the recession and the time duration for inflation to reach the new steady state are affected by the initial level of trend inflation.[7] What the level of π^*_{old} does affect, however, is the amplitude of output fluctuation during the transition. As shown in the first column of Table 1, the percentage output drop (in deviation from the new steady state level) at the trough substantially worsen as π^*_{old} increases. At the trough, output drops by 0.25% for a disinflation from 3 to 2%, whereas it drops by 0.71% for

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[6] Clearly, also for wage setters' behavior the same reasoning follows through. Here, however, we primarily comment on intermediate firms' behavior and inflation dynamics.

[7] We chose not to plot the dynamic adjustments for cold-turkey disinflations aimed at $\pi^*_{\text{new}} = 1\%$ and $\pi^*_{\text{new}} = 0$ as the transitions are qualitatively very similar to those in Figure 1.

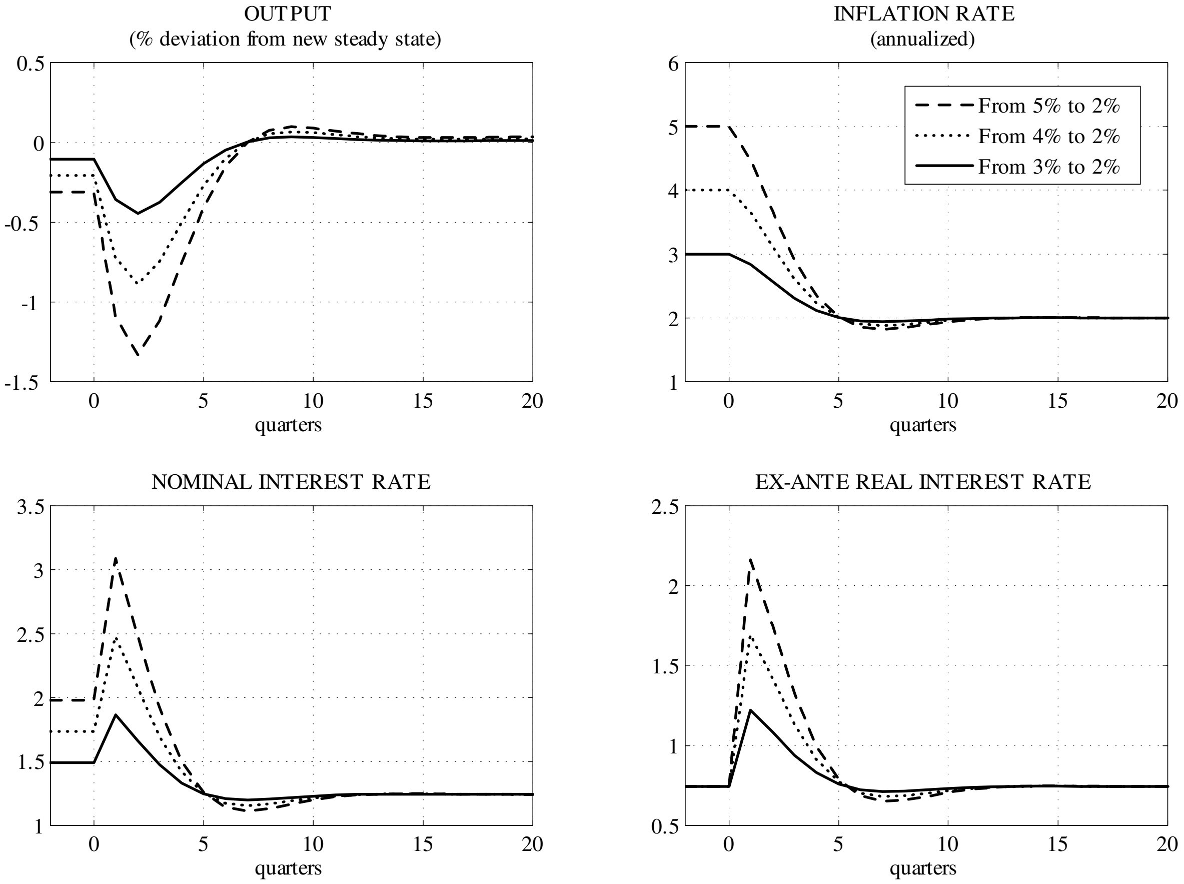

Figure 2: Cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$ with $\phi = 3$.

a disinflation from 5 to 2%. Intuitively, higher values of π^*_{old} make optimizing firms to cut prices more strongly, yielding to a larger drop of inflation and a greater rise of real interest rate. It is interesting to note that regardless of the new inflation target, either π^*_{new} =1% or π^*_{new} =0, the percentage output drops at the trough are of the same magnitude for a given disinflation size, i.e., $\pi^*_{\text{old}} - \pi^*_{\text{new}}$.

Figure 2 illustrates the dynamic adjustment of output, inflation, nominal and real interest rate after cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$, when $\phi = 3$. The effects of having a more hawkish central bank are intuitive. In general, the monetary policy is more restrictive (see the notable hike of nominal interest rate) and the output downturn more severe (see the Table 1) . Nonetheless, adjusting firms seems to behave much like as in previous case (i.e., when $\phi = 1.5$). As a matter of fact the adjustment path of inflation is surprisingly similar to the top-right panel in Figure 1. There is just a small difference in terms of adjustment speed. Indeed, with $\phi = 3$ the cold-turkey disinflation is accomplished in about 12 quarters.

So, we have seen that cold-turkey disinflations yield a notable recession but how large are these short-run output costs? To answer this question we directly borrow from the empirical literature on disinflation (see e.g., Gordon and King, 1982) and define a model-consistent sacrifice ratio. In particular,

$$\mathrm{SR} = -\frac{1}{\pi^*_{\mathrm{old}} - \pi^*_{\mathrm{new}}} \sum_{t=0}^{T} \left(\frac{Y_t - Y_{\mathrm{new}}}{Y_{\mathrm{new}}} \right), \tag{2}$$

where Y_{new} represents the steady state level of output at π^*_{new}. Thus, our measure indicates the cumulative percentage output loss the economy has to sacrifice to achieve a 1% permanent reduction of steady state inflation. Two features of (2) are worthy to notice. Firstly, we define the sacrifice ratio by calculating the output loss in deviation from the new steady state. Secondly, we sum up the percentage output losses over the first T periods. In particular, the value of T is chosen to reflect the number of periods inflation takes to settle down to the new inflation target[8].

Table 1 reports values of the model-consistent sacrifice ratios calculated both for $\phi = 1.5$ (and $T = 15$) and $\phi = 3$ (and $T = 12$). The first thing we want to stress here is that the theoretical sacrifice ratios are positive and in line with the existing empirical estimates (see the Introduction section). In particular, the sacrifice ratio turns out to be approximately equal to 1.05 when $\phi = 1.5$; whereas it takes up a slightly larger value, i.e., 1.62, when the central bank is relatively more concerned with inflation stabilization around the target, i.e., when $\phi = 3$. In fact, we have seen that in this latter case the ensuing recession after the cold-turkey disinflation is more severe. Notwithstanding, the size of disinflation does not seem to affect the sacrifice ratio. Varying the size of disinflation leads a roughly proportional rescaling of output transition paths and this leaves practically unchanged the value of the sacrifice ratio.

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[8] In particular we truncate the horizon at a point where the distance between actual inflation and the new inflation target is (in absolute value) less than 10^{-3}.

		$\phi = 1.5$		$\phi = 3$	
π^*_{old}	π^*_{new}	Output at trough	SR (T=15)	Output at trough	SR (T=12)
3%	2%	-0.24	1.04	-0.44	1.59
4%	2%	-0.47	1.03	-0.88	1.60
5%	2%	-0.71	1.02	-1.32	1.61
2%	1%	-0.24	1.05	-0.45	1.61
3%	1%	-0.48	1.04	-0.90	1.62
4%	1%	-0.72	1.03	-1.34	1.63
1%	0	-0.24	1.06	-0.45	1.63
2%	0	-0.49	1.05	-0.91	1.64
3%	0	-0.73	1.05	-1.36	1.65

Table 1: Short-run costs of disinflation. Output at the trough is expressed in percentage deviation from the new steady state level.

In summary, in the medium-scale operational New Keynesian DSGE model a cold turkey permanent reduction of trend inflation entails sizable short-run output costs. To bring down trend inflation, say, from 4 to 2%, by means of a credible cold-turkey disinflation the economy would have to sacrifice a cumulative output loss of either 2.1 or 3.2% in relation to the type of interest rate rule. The inflation adjustment would then be completed in about 4 or 3 years.

4 A welfare based measure of the cost of disinflation

As already noted in Gordon and King (1982), the output loss from disinflation does not contain *per se* policy implications. A careful assessment must be made of the welfare cost of lost output *and* the welfare benefits of lower inflation. On this latter point, the recent monetary policy literature has largely emphasized the reasons why achieving full price stability is desirable (see Woodford, 2003 and the references therein). One notable advantage of working with microfounded structural model is that they provide a natural welfare metric, namely the representative household's value function. Hence, we can calculate a welfare based indicator of the costs of disinflations, rather than just focussing on an empirical based one as the sacrifice ratio.

Mimicking the construction of the sacrifice ratio, a measure of the welfare loss caused by disinflation may be calculated as the difference between the value function at time zero, i.e., V_0 (when the disinflation is actually implemented) and the value function at the initial steady state inflation, i.e., $\mathrm{V}_{\mathrm{old}}$ (as if the disinflation was not implemented). More formally, our welfare based sacrifice ratio can be defined as

$$\mathrm{WSR} = -\left(\frac{\mathrm{V}_0 - \mathrm{V}_{\mathrm{old}}}{\pi^*_{\mathrm{old}} - \pi^*_{\mathrm{new}}}\right). \tag{3}$$

Notice that V_0 represents the discounted sum of future stream of instantaneous utility, as such it measures both the transition dynamics and the long-run effects of the disinflation. Paralleling the standard sacrifice ratio definition, $\mathrm{WSR} > 0$, if $\mathrm{V}_0 - \mathrm{V}_{\mathrm{old}} < 0$. That is, the welfare-based sacrifice ratio is positive if the disinflation reduces welfare .

The consumption equivalent measure

A policy maker is interested in the welfare cost of implementing a disinflationary policy, but given that the utility function is not cardinal, a measure based on the value function is not very revealing. The difference (V_0-$\mathrm{V}_{\mathrm{old}}$) needs to be converted in consumption equivalent units. The consumption equivalent measure defines the constant fraction of consumption that households should give away in each period in the starting steady state, that equates the value function households would obtain if the disinflation is implemented. Thus, it measures how much households have to suffer in terms of consumption loss, in order to reduce the inflation rate permanently of a certain amount.

The derivation of the welfare based measure in terms of consumption equivalent units is straightforward. The initial value function, in case the central bank does not disinflate the economy and keeps inflation target permanently at π^*_{old}, is given by

$$\mathrm{V}_{\text{old}} = \frac{1}{1-\beta}\left[\ln(1-b)c_{\text{old}} - \frac{\phi_0}{2}h^2_{\text{old}} + \frac{\left(m^h_{\text{old}}\right)^{1-\sigma_m}}{1-\sigma_m}\right], \tag{4}$$

where c_{old}, h_{old} and m^h_{old} denote respectively consumption, hours worked and real money balances held by the households in the initial steady state; ϕ_0 and σ_m are structural parameters.[9] Given the value of V_0, available from the numerical solution of the model, we then have to find the constant fraction of steady state consumption, i.e., λ, that solves the following equation

$$\mathrm{V}_0 = \frac{1}{1-\beta}\left[\ln(1-b)(1-\lambda)c_{\text{old}} - \frac{\phi_0}{2}h^2_{\text{old}} + \frac{\left(m^h_{\text{old}}\right)^{1-\sigma_m}}{1-\sigma_m}\right]. \tag{5}$$

Thus, the consumption equivalent measure is given by

$$\lambda = 1 - \exp\left[(1-\beta)(\mathrm{V}_0 - \mathrm{V}_{\text{old}})\right]. \tag{6}$$

Finally, our proposed welfare based sacrifice ratio is obtained as[10]

$$\mathrm{SR}^{\mathbb{W}} = \frac{\lambda}{\pi^*_{\text{old}} - \pi^*_{\text{new}}}. \tag{7}$$

The first column of Table 2 reports the values of $\mathrm{SR}^{\mathbb{W}}$. The main result can be stated as:

Result 1. Our proposed welfare based sacrifice ratio calculated in a medium scale New Keynesian DSGE model for different disinflation experiments assumes negative values. This means that disinflation is welfare improving.[11]

Therefore, when discussing about the effects of disinflation policies it would be more appropriate to use the notion of *welfare gain ratio*, rather than *sacrifice ratio* as in

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[9] See the Appendix for further details.

[10] Note that there is no minus in front of this ratio, to maintain a positive sign for a loss. Indeed, if V_0-$\mathrm{V}_{\text{old}} < 0$, that is, disinflation brings about a welfare loss, then $\lambda > 0$, and vice versa.

[11] This result does not depend on the inclusion of real money balances in the utility function. We also calculated a similar measure without taking into account the gain in utility coming from the increase in real money balances in the new steady state. The measure would then be about 2/3 of the values reported in Table 2.

π^*_{old}	π^*_{new}	$\text{SR}^{\mathbb{W}}$ Total Welfare costs ($\times 10^{-2}$)		$\text{SR}^{\mathbb{W}}_{\infty}$ Long-run Welfare costs ($\times 10^{-2}$)	$\text{SR}^{\mathbb{W}}-\text{SR}^{\mathbb{W}}_{\infty}$ Short-run Welfare costs ($\times 10^{-2}$)	
		$\phi = 1.5$	$\phi = 3$		$\phi = 1.5$	$\phi = 3$
3%	2%	-6.46	-6.38	-7.23	0.77	0.85
4%	2%	-6.39	-6.32	-7.18	0.79	0.86
5%	2%	-6.35	-6.27	-7.13	0.79	0.86
2%	1%	-6.55	-6.48	-7.34	0.78	0.86
3%	1%	-6.49	-6.41	-7.29	0.80	0.87
4%	1%	-6.44	-6.36	-7.24	0.80	0.87
1%	0	-6.67	-6.59	-7.46	0.80	0.87
2%	0	-6.59	-6.52	-7.40	0.81	0.89
3%	0	-6.54	-6.46	-7.35	0.81	0.89

Table 2: Welfare-based sacrifice ratios.

the empirical literature. We think this is a novel and interesting result: the empirical literature on disinflation focuses only on the short-run costs in terms of output (or unemployment), but neglects any long-run gain. We show, to the contrary, that in a medium scale DSGE monetary model of the business cycle a disinflationary policy is welfare improving.

Moreover, note that the welfare gain from disinflating: (i) decreases with the size of the disinflation; (ii) decreases with the starting level of inflation, for a given size of disinflation.

A second notable result from Table 2 is:

Result 2. The size of $\text{SR}^{\mathbb{W}}$, however, is small: the welfare gain is equivalent to an extra

0.06% of consumption each period.

Actually, the results are possibly even more striking, if we disentangle the short-run welfare costs of a disinflation during the transition dynamics and the long-run welfare gains stemming from higher price stability. Indeed, in the standard medium scale DSGE macro model, despite a disinflation entails a large and prolonged recession, such that the implied sacrifice ratio is in line with the empirical evidence, the short-run welfare costs of such a painful adjustment path are plainly insignificant.

To show that, we follow the same line of reasoning above and define:

(i) the long-run costs in terms of consumption equivalent units:

$$\lambda_{\infty} = 1 - \exp\left[(1-\beta)\left(\mathrm{V}_{\text{new}} - \mathrm{V}_{\text{old}}\right)\right] \tag{8}$$

where V_{new} and V_{old} denote the values function in the new and old inflation steady states. The above indicator can be expressed per unit of diminished inflation to yield a *long-run welfare based sacrifice ratio:*[12]

$$\mathrm{SR}_{\infty}^{\mathbb{W}} = \frac{\lambda_{\infty}}{\pi_{\text{old}}^{*} - \pi_{\text{new}}^{*}}; \tag{9}$$

(ii) the *short-run welfare based sacrifice ratio* is then given by

$$\mathrm{SR}^{\mathbb{W}} - \mathrm{SR}_{\infty}^{\mathbb{W}} = \frac{\exp\left[(1-\beta)\left(\mathrm{V}_{\text{new}} - \mathrm{V}_{\text{old}}\right)\right] - \exp\left[(1-\beta)\left(\mathrm{V}_{0} - \mathrm{V}_{\text{old}}\right)\right]}{\pi_{\text{old}}^{*} - \pi_{\text{new}}^{*}}. \tag{10}$$

Table 2 reports the long-run welfare gains and the short-run welfare costs in consumption equivalent units for various disinflation experiments. The order of magnitude of the short-run welfare costs is, roughly about 0.008-0.009% of initial consumption. Therefore, the long-run gains quantitatively dominate, though being themselves very small (roughly 0.07%). The main message from Table 2 is that a disinflation is going to be welfare improving of the order of an increase of initial consumption of 0.06-0.07% each period per point of diminished inflation. That is, the welfare effects of a disinflation are barely relevant, despite high short-run costs in terms of output losses.

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[12] Note that we use a coherent definition as above also for $\mathrm{SR}_{\infty}^{\mathbb{W}}$. Indeed, if V_{new}-$\mathrm{V}_{\text{old}} < 0$ (that is if disinflation brings about a welfare loss) then $\lambda > 0$, and vice versa.

This stands in sharp contrast with the consensus view about the effects of a credible disinflation. What is the intuition for these results? To illustrate this point, let us consider the case with $\phi = 3$. Figure 3 displays the path of consumption and employment, expressed in deviation from the new steady state, together with value of the utility function. The disinflation induces a prolonged recession that cause both consumption and employment to be below their new (and higher) steady state value for some periods. Consumption and employment, however, has opposite effects on the utility function of the representative agent. It follows, therefore, that the net effects of the recession on the utility of the representative agent is ambiguous. Indeed, the decrease in consumption dominates in the impact period, dragging the utility function down. Already from the second period, however, the effects of the dynamics of employment takes over, and the utility function is above its new higher long-run value. Moreover, it will stay there for all the other periods of the recession. This is because the drop in employment is bigger in percentage terms, and slightly more sluggish. It follows that the positive effect of employment is quite effective in counterbalancing the negative effect of lower consumption. Overall the transition, thus, entails a short-run cost, as shown above, but of a negligible order of magnitude. Finally, also the value of the utility function without counting the real money balances term is visualized in Figure 3, so to make clear that the role of the real money balances term in the utility function in the above results is nil.

This result obviously hinges on the representative agent assumption, that is, on complete markets and risk-sharing. That is, the welfare analysis based on a representative agent framework can not take into account, for example, the fact that some people may suffer a very big drop in utility during recessions because they lose their jobs and do not have access to financial markets. Such heterogeneity and composition effect is missing by construction. However, we believe our results have two notable interpretations. First, taken at face value, our findings simply show that disinflations, in particular, and recessions, in general, could be less of a problem than we normally think, if the economy could provide an efficient risk-sharing mechanism amongst agents (either through capital markets, or some public welfare system). In this sense, this is once again the Lucas' negligible costs of business cycle result. Second, if one, instead, is skeptical about the

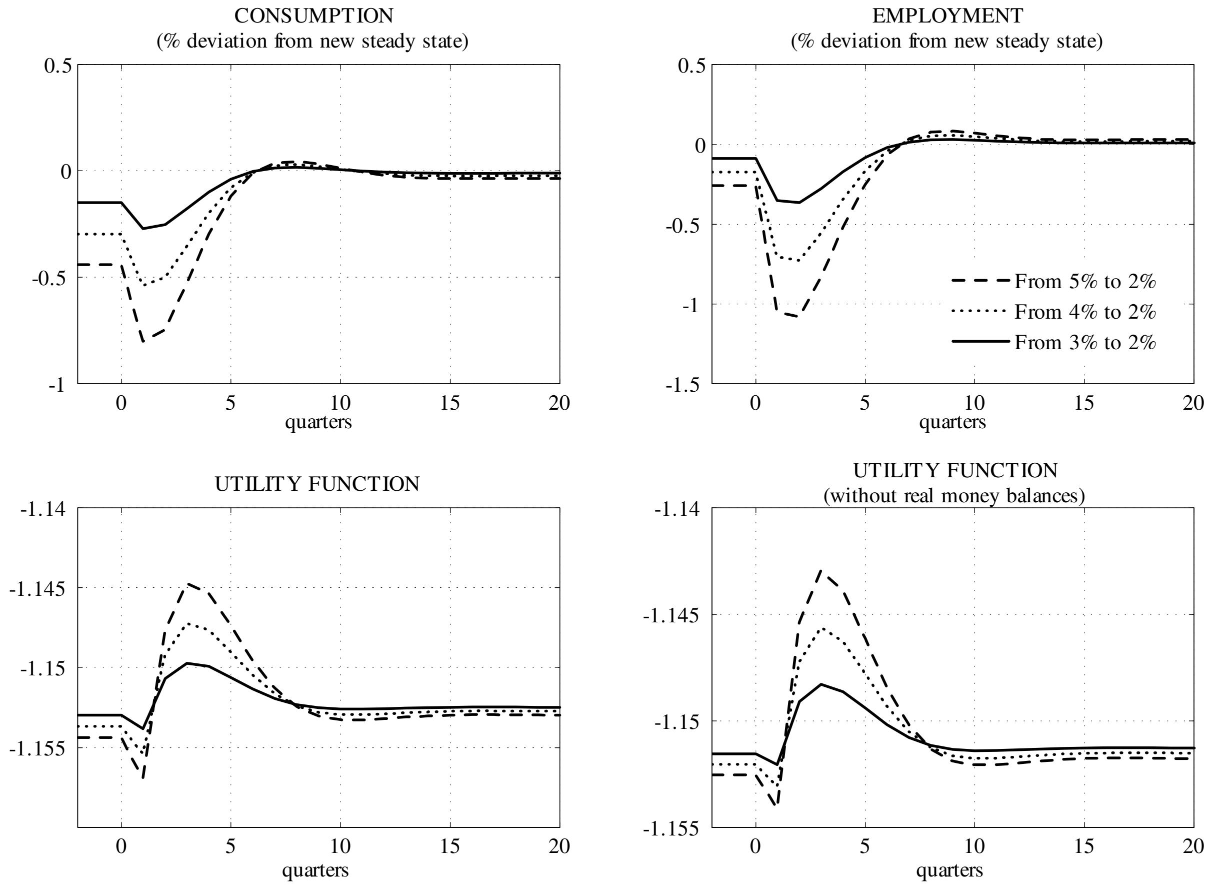

Figure 3: Cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$ with $\phi = 3$.

actual relevance of the welfare results, then, at the very least, these results cast serious shadows on using these DSGE models for welfare evaluation without "inspecting the mechanism". In particular, the whole literature on optimal policy problems or on the ranking between different monetary policy rules is bound to be based on mechanism similar to the ours.

5 Conclusions

Disinflation is an important topic in monetary economics and the subject of a large literature. However, there is a widespread consensus that the New Keynesian models can not explain the cost of disinflation observed in the data, for which they need to resort to lack of credibility or information.

The logic of the policy experiments laid out in this paper is clear and straight. We question whether the workhorse DSGE model of the US business cycle, i.e., the CEE model, can account for the sacrifice ratio and the overall adjustment dynamics after a disinflation, that is a permanent shock in the inflation rate. We think this is a sort of needed requirement for an operational monetary model.

Our results show that a perfectly credible cold-turkey disinflation entails a sizable and long-lasting recession in the CEE model. In addition, the values of the sacrifice ratio are in line with those estimated in the empirical literature.

Moreover we conduct a rigorous welfare evaluation of the costs of disinflation, proposing a welfare based sacrifice ratio. Surprisingly enough, despite a deep and prolonged recession the short-run costs of a disinflation are negligible in terms of consumption equivalent units. A disinflation would actually imply very tiny welfare gain, since in the CEE model money is not superneutral (despite full indexation), and there are very small long-run welfare gain than overcome the short-run costs.

The finding that the CEE model can replicate the main facts after a disinflation is at odds with the consensus in the literature, and may be good news for the New Keynesian models. This however does not mean that some of the model features or mechanisms should not be improved to tackle the disinflation question. Indeed, we think that testing the CEE model with respect to disinflation had proved to be useful to suggest the important aspects for current and future research.

First, the indexation is a reduced form assumption that can act as a substitute for many other more structural phenomenon. There is a macroeconomic reduced form equivalence of different microeconomic models, so that actually a similar effect can come out from irrational price setters (rule of thumbers), inattentive price setters or lack of credibility, and hence sluggish expectation adjustment.

Secondly, a Calvo time dependent price setting model would need indexation in order not have unpalatable long-run implications of a permanent change in inflation because of the large effects of price dispersion in this model. Moreover, despite the fact that we look only at moderate rate of inflation, for which the Calvo parameter defining the frequency of price adjustment can be considered constant, ideally one would like to work with a model where the changes in the average inflation level induce firms to revise their behavior[13]. In other words, a time dependent model is particularly fragile to the Lucas critique when used to analyzed changes in the average inflation rate. Last, but not least, recently Klenow and Kryvtsov (2008) shows that the many price adjustments occur on the intensive margin rather than on the extensive margin. Embedding what Klenow and Kryvtsov (2008) calls a second generation model of state dependent pricing in the CEE framework would cure all these problems at once: no need for indexation to cure the unpalatable long-run effects, shelter from the Lucas critique, and the intensive margin. Moreover, as we know from Burnstein (2006) this could generate interesting non-linearities regarding the effects of large vs. small disinflations.

Finally, our welfare results are rather surprising. The abandonment of the risk sharing assumption, together with a proper account of heterogeneity among agents regarding the impact of a recession on their welfare, may overturn our results.

Fortunately, the current research and the recent contributions to the New Keynesian literature are taking up all these challenges.

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[13] For disinflation dynamics in small models with endogenous pricing, see Almeida and Bonomo (2002), Bonomo and Carvalho (2004), Burstein (2006). .

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A The Christiano, Eichenbaum and Evans (2005) Model

In this Appendix we describe the CEE model, following closely the outline in Schmitt-Grhoe and Uribe (2005).

Households

There is a continuum of infinitely-lived households whose expected intertemporal utility function is given by

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) \right\}. \tag{11}$$

where E_0 defines the mathematical expectation operator conditional on the information set available at time 0, β is the subjective discount factor, function $u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right)$ is well-behaved and increasing in consumption c_t and money holdings m_t^h, while decreasing in hours worked h_t^s. Preferences display habit in consumption levels, measured by the parameter b.

There is a continuum of final goods indexed by $i \in [0,1]$, that are aggregated in the usual CES consumption bundle c_t

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \tag{12}$$

where the parameter η indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good i, given by $c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t$, where P_t is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$.

There is a continuum of labour services h_{jt}, $j \in [0,1]$, that are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj\right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}}, \tag{13}$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. The standard cost minimization problem for the firms yield the labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\tilde{\eta}} h_t^d$, where W_{jt} is the wage paid to labor type j and W_t is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di\right]^{\frac{1}{1-\tilde{\eta}}}$. The total labor supply is found by integrating labour-specific demand functions, to obtain h_t^s

$$h_t^s \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\eta}} dj. \tag{14}$$

Agents owns physical capital k_t that depreciates at rate δ. The capital accumulation equation is

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right], \tag{15}$$

where the function S introduce the adjustment cost on investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$. The model features also variable capacity utilization of physical capital, denoted by u_t,. The cost of capital then depends on the degree of utilization and it is given by $a(u_t)$. Agents rent capital to firms at a real interest rate r_t^k and decide also over the utilization rate. There are complete markets for state contingent assets, such that all agents choose the same level of consumption.

Household first order conditions are hence given by

$$u_{c_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h \right) + u_{c_t}\left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h \right) = \lambda_t \tag{16}$$

$$u_{h_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h \right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \tag{17}$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - a(u_{t+1}) \right] \tag{18}$$

$$q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) - \left[S_i \left(\frac{i_t}{i_{t-1}} \right) \right] i_t \right] - \beta q_{t+1} \lambda_{t+1} S_i \left(\frac{i_{t+1}}{i_t} \right) i_{t+1} = \lambda_t \tag{19}$$

$$a_{u_t}(u_t) = r_t^k \tag{20}$$

$$u_{m_t^h}\left(c_t - bc_{t-1}; h_t^s; m_t^h \right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \tag{21}$$

Wages are sticky a la Calvo, and $1 - \tilde{\alpha}$ is the probability of being able to reset wages next period. If wages can not be re-optimized, the CEE model assumes that wage are anyway updated according to past inflation, such that: $w_{j,t+1} = w_{j,t} \pi_t^{\tilde{\chi}}$ where $\tilde{\chi}$ is the degree of indexation to past inflation. Define $\tilde{w}_t$ as the optimal wage set every period t. The union chooses the optimal wage maximizing its the utility function given by equation (12), subject to demand of labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\eta}} h_t^d$ and the probability of not being able to re-optimize in future periods. The resulting first order condition is

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \tag{22}$$

All the reset optimal wages are identical in all labour markets.

Firms

Each good is produced by a firm which monopolistically supply its own variety using a production technology of the form

$$z_t F(k_{it}, h_{it}) - \psi,$$

where z_t is an aggregate technology factor common across firms, and ψ represents a fixed cost of production. The production function $F(k_{it}, h_{it})$ is well-behaved and it's the same across firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{23}$$

where

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{24}$$

Firms rent capital from the households on a competitive market, and must pay a fraction ν of wages at the beginning of the period by cash. Therefore their money demand function is

$$m_{it}^f = \nu w_t h_{it} \tag{25}$$

The firms' problem is then to maximize the expected value of future profits, under their demand function (23) and the cash-in-advance constraint (25). The first order conditions with respect to capital and labour services are

$$\begin{aligned} mc_{it} z_t F_{k_{it}}(k_{it}, h_{it}) &= r_t^k && (26) \\ mc_{it} z_t F_{h_{it}}(k_{it}, h_{it}) &= w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. && (27) \end{aligned}$$

Since F is homogeneous of degree one, equation (26) and equation (27) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky a la Calvo. Every period each firm can choose a new price of its own good with a probability $1-\alpha$. As for wages, also the prices that can not be resetted optimally, are automatically updated according to past inflation, such that: $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$, where χ is the degree of price indexation. The first order condition for the optimal price is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi^{\chi}_{t+k-1}}\right)^{\eta} \left[\frac{\eta-1}{\eta}\frac{\tilde{P}_t}{P_t}\prod_{k=1}^{s}\left(\frac{\pi^{\chi}_{t+k-1}}{\pi_{t+k}}\right) - mc_{i,t+s}\right] = 0. \tag{28}$$

Again, all the reset optimal prices are identical for all goods.

The Government

Government expenditure is financed through lump-sum taxes and seigniorage

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}. \tag{29}$$

where m_t denotes real money balances, and $\pi_t \equiv P_t/P_{t-1}$ is the (gross) inflation rate at time t. Government minimizes the costs of acquiring the composite good, hence given public expenditure, government's absorption of a single type of good is $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$.

To close the model we postulate the monetary policy uses the simple non-linear nominal interest rate rule as described in the paper.

Equilibrium

The model equilibrium conditions are

Money market : $m_t = m_t^h + m_t^f$

Labor market : $h_t^s = \int_0^1 h_{it}^d di$

Capital market : $\int_0^1 k_{it} di = u_t k_t$

Good i market : $z_t F\left(k_{it}, h_{it}\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right)\left(\frac{P_{it}}{P_t}\right)^{-\eta}$

Aggregate Goods market : $z_t h_t^d F\left(\frac{u_t k_t}{h_t^d}, 1\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di$

where $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$ is the price dispersion generated by price staggering, causing a wedge between aggregate supply and aggregate absorption. Similarly wage staggering gives rise to wage dispersion, given by $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$, see (14).

Functional forms and calibration

As in Schmitt-Grohé and Uribe (2005), we assume the following functional forms:

$$\begin{aligned} u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\ F\left(u_t k_t, h_t^d\right) &= \left(u_t k_t\right)^{\theta} \left(h_t^d\right)^{1-\theta} \\ S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\ a\left(u_t\right) &= \gamma_1 \left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2 . \end{aligned}$$

Calibration is also as in Schmitt-Grohe and Uribe (2005), that follows CEE's estimation results. The parameters values are listed in the Table 3.

Parameter	Value	Description
β	$1.03^{-0.25}$	Time discount rate
θ	0.36	Share of capital
ψ	0.5827	Fixed cost (guarantee zero profits in steady state)
δ	0.025	Depreciation of capital
ν	1	Fraction of wage bill subject to CIA constraint
η	6	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	21	Elasticity of substitution of labour services
α	0.6	Probability of not setting a new price each period
$\tilde{\alpha}$	0.64	Probability of not setting a new wage each period
b	0.65	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Intertemporal elasticity of money
κ	2.48	Investment adjustment cost parameter
χ	1	Price indexation
$\tilde{\chi}$	1	Wage indexation
γ_1	0.0324	Capital utilization cost function parameter
γ_2	0.000324	Capital utilization cost function parameter
z	1	Steady state value of technology shock

Table 3: Calibration of parameters in the Christiano, Eichenbaum and Evans (2005).