TWO-COUNTRY EQUILIBRIUM

EULER EQUATIONS

$$\frac{1}{1+r_t}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{1+\tau_{VAT,t}}{1+\tau_{VAT,t+1}}\right)\frac{1}{\left(1+\pi_{t+1}\right)}\right]$$ (1) Home country Euler equation

$$\frac{1}{1+r_t}=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\left(\frac{1+\tau^*_{VAT,t}}{1+\tau^*_{VAT,t+1}}\right)\frac{1}{1+\pi^*_{t+1}}\right]$$ (2) Foreign country Euler equation

LABOR-LEISURE TRADEOFF EQUATIONS

$$L_t=n^{1+\frac{\sigma}{\eta}}\left(\frac{w_t}{(1+\tau_{VAT,t})}\right)^{\frac{1}{\eta}}C_t^{-\frac{\sigma}{\eta}}$$ (3) Home country labor-leisure tradeoff equation

$$L^*_t=(1-n)^{1+\frac{\sigma}{\eta}}\left(\frac{w^*_t}{(1+\tau^*_{VAT,t})}\right)^{\frac{1}{\eta}}C_t^{*-\frac{\sigma}{\eta}}$$ (4) Foreign country labor-leisure tradeoff equation

ALLOCATION OF CONSUMPTION ACROSS GOODS

$$C_{N,t}=(1-\lambda_T)\left(p_{N,t}\right)^{-\omega}C_t$$ (5) Home consumption of non-traded goods

$$C_{T,t}=\lambda_T\left(p_{T,t}\right)^{-\omega}C_t$$ (6) Home consumption of traded goods

$$C^*_{M,t}=(1-\lambda^*_T)\left(p^*_{M,t}\right)^{-\omega}C^*_t$$ (7) Foreign consumption of non-traded goods

$$C^*_{T,t}=\lambda^*_T\left(p^*_{T,t}\right)^{-\omega}C^*_t$$ (8) Foreign consumption of traded goods

$$C_{H,t}=(1-\alpha)\left(p_{HT,t}\right)^{-\gamma}C_{T,t}$$ (9) Home consumption of home-produced traded goods

$$C_{F,t}=\alpha\left(p^*_{FT,t}S_t\right)^{-\gamma}C_{T,t}$$ (10) Home consumption of foreign-produced traded goods

$$C^*_{H,t}=\alpha^*\left(p_{HT,t}\frac{1}{S_t}\right)^{-\gamma}C^*_{T,t}$$ (11) Foreign consumption of home-produced traded goods

$$C^*_{F,t}=\left(1-\alpha^*\right)\left(p^*_{FT,t}\right)^{-\gamma}C^*_{T,t}$$ (12) Foreign consumption of foreign-produced traded goods

DEFINITION OF MARGINAL COST

$$MC^R_{N|t,t}=\frac{(1+\tau^P_t)w_t}{p_{N,t}}$$ (13) Real marginal cost of home-produced non-traded goods

$$MC^R_{H|t,t}=\frac{(1+\tau^P_t)w_t}{p_{T,t}p_{HT,t}}$$ (14) Real marginal cost of home-produced traded goods

$$MC^{R*}_{M|t,t} = \frac{(1+\tau^{P*}_t)w^*_t}{p^*_{M,t}}$$ (15) Real marginal cost of foreign-produced non-traded goods

$$MC^{R*}_{F|t,t} = \frac{(1+\tau^{P*}_t)w^*_t}{p^*_{T,t}\,p^*_{FT,t}}$$ (16) Real marginal cost of foreign-produced traded goods

PRODUCTION FUNCTIONS

$$\Lambda_{N,t}Y_{N,t} = L_{N,t}$$ (17) Production function home-produced non-traded goods

$$\Lambda_{H,t}Y_{H,t} = L_{H,t}$$ (18) Production function home-produced traded goods

$$\Lambda^*_{M,t}Y^*_{M,t} = L^*_{M,t}$$ (19) Production function foreign-produced non-traded goods

$$\Lambda^*_{F,t}Y^*_{F,t} = L^*_{F,t}$$ (20) Production function foreign-produced traded goods

PRICE DISPERSION INDICES

$$\Lambda_{N,t} = \left(1-\mu_N\right)\left(1+\pi^O_{N,t}\right)^{-\varepsilon}\left(1+\pi_{N,t}\right)^{\varepsilon} + \left(1+\pi_{N,t}\right)^{\varepsilon}\mu_N\Lambda_{N,t-1}$$ (21) Law of motion of price dispersion index

$$\Lambda_{H,t} = \left(1-\mu_H\right)\left(1+\pi^O_{H,t}\right)^{-\varepsilon}\left(1+\pi_{H,t}\right)^{\varepsilon} + \left(1+\pi_{H,t}\right)^{\varepsilon}\mu_H\Lambda_{H,t-1}$$ (22) Law of motion of price dispersion index

$$\Lambda^*_{M,t} = \left(1-\mu^*_M\right)\left(1+\pi^{O*}_{M,t}\right)^{-\varepsilon}\left(1+\pi^*_{M,t}\right)^{\varepsilon} + \left(1+\pi^*_{M,t}\right)^{\varepsilon}\mu^*_M\Lambda^*_{M,t-1}$$ (23) Law of motion of price dispersion index

$$\Lambda^*_{F,t} = \left(1-\mu^*_F\right)\left(1+\pi^{O*}_{F,t}\right)^{-\varepsilon}\left(1+\pi^*_{F,t}\right)^{\varepsilon} + \left(1+\pi^*_{F,t}\right)^{\varepsilon}\mu^*_F\Lambda^*_{F,t-1}$$ (24) Law of motion of price dispersion indexe

PRICING EQUATIONS

$$\left(1+\pi^O_{N,t}\right) = \frac{\varepsilon}{\varepsilon-1}\left(1+\pi_{N,t}\right)\frac{x^1_{N,t}}{x^2_{N,t}}$$ (25) Optimal pricing equation

$$\left(1+\pi^O_{H,t}\right) = \frac{\varepsilon}{\varepsilon-1}\left(1+\pi_{H,t}\right)\frac{x^1_{H,t}}{x^2_{H,t}}$$ (26) Optimal pricing equation

$$\left(1+\pi^{O*}_{M,t}\right) = \frac{\varepsilon}{\varepsilon-1}\left(1+\pi^*_{M,t}\right)\frac{x^{1*}_{M,t}}{x^{2*}_{M,t}}$$ (27) Optimal pricing equation

$$\left(1+\pi^{O*}_{F,t}\right) = \frac{\varepsilon}{\varepsilon-1}\left(1+\pi^*_{F,t}\right)\frac{x^{1*}_{F,t}}{x^{2*}_{F,t}}$$ (28) Optimal pricing equation

AUXILIARY VARIABLES FOR PRICING EQUATIONS

$$x^1_{N,t} = Y_{N,t}MC^R_{N,t} + \left(\mu_N\frac{1}{1+r_t}\right)E_t\left(1+\pi_{N,t+1}\right)^{1+\varepsilon}x^1_{N,t+1}$$ (29)

$$x_{N,t}^{2} = Y_{N,t} + \left(\mu_N \frac{1}{1+r_t} \right) E_t x_{N,t+1}^{2} \left(1+\pi_{N,t+1}\right)^{\varepsilon} \quad (30)$$

$$x_{M,t}^{1*} = Y_{M,t}^{*} MC_{M,t}^{R*} + \left(\mu_M^{*} \frac{1}{1+r_t} \right) E_t x_{M,t+1}^{1*} \left(1+\pi_{M,t+1}^{*}\right)^{1+\varepsilon} \quad (31)$$

$$x_{M,t}^{2*} = Y_{M,t}^{*} + \left(\mu_M^{*} \frac{1}{1+r_t} \right) E_t \left(1+\pi_{M,t+1}^{*}\right)^{\varepsilon} x_{M,t+1}^{2*} \quad (32)$$

$$x_{H,t}^{1} = Y_{H,t} MC_{H,t}^{R} + \left(\mu_H \frac{1}{1+r_t} \right) E_t x_{H,t+1}^{1} \left(1+\pi_{H,t+1}\right)^{1+\varepsilon} \quad (33)$$

$$x_{H,t}^{2} = Y_{H,t} + \left(\mu_H \frac{1}{1+r_t} \right) E_t \left(1+\pi_{H,t+1}\right)^{\varepsilon} x_{H,t+1}^{2} \quad (34)$$

$$x_{F,t}^{1*} = Y_{F,t}^{*} MC_{F,t}^{R} + \left(\mu_F^{*} \frac{1}{1+r_t} \right) E_t \left(1+\pi_{F,t+1}^{*}\right)^{1+\varepsilon} x_{F,t+1}^{1*} \quad (35)$$

$$x_{F,t}^{2*} = Y_{F,t}^{*} + \left(\mu_F^{*} \frac{1}{1+r_t} \right) E_t \left(1+\pi_{F,t+1}^{*}\right)^{\varepsilon} x_{F,t+1}^{2*} \quad (36)$$

ALLOCATION OF GOVERNMENT EXPENDITURE ACROSS GOODS

$G_{N,t} = (1-\lambda_T)\left(\frac{p_{N,t}}{p_{G,t}}\right)^{-\omega} G_t$ (37) Home government consumption of non-traded goods

$G_{M,t}^{*} = (1-\lambda_T^{*})\left(\frac{p_{M,t}^{*}}{p_{G,t}^{*}}\right)^{-\omega} G_t^{*}$ (38) Foreign government consumption of non-traded goods

$G_{H,t} = \lambda_T \left(\frac{p_{HT,t}\, p_{T,t}}{p_{G,t}}\right)^{-\omega} G_t$ (39) Home government consumption of home-produced traded goods

$G_{F,t}^{*} = \lambda_T^{*} \left(\frac{p_{FT,t}^{*}\, p_{T,t}^{*}}{p_{G,t}^{*}}\right)^{-\omega} G_t^{*}$ (40) Foreign government consumption of foreign-produced traded goods

GOVERNMENT BUDGET CONSTRAINT

$\tau_{VAT,t} C_t + \tau_t^{P} w_t L_t = p_{G,t} G_t$ (41) Home government budget constraint

$\tau_{VAT,t}^{*} C_t^{*} + \tau_t^{P*} w_t^{*} L_t^{*} = p_G^{*} G_t^{*}$ (42) Foreign government budget constraint

LAW OF GOVERNMENT EXPENDITURE

$G_t = \Delta_G \frac{p_{Y,t}}{p_{G,t}} Y_t$ (43) Law of Home government expenditure

$$G_t^* = \Delta_G^* \frac{p_{Y,t}^*}{p_{G,t}^*} Y_t^*$$ (44) Law of foreign government expenditure

RELATIVE PRICES DEFINITION

$$p_{N,t} = \left[\frac{1-\lambda_T \left(p_{T,t} \right)^{1-\omega}}{(1-\lambda_T)} \right]^{\frac{1}{1-\omega}} \quad (45)$$

$$p_{M,t}^* = \left[\frac{1-\lambda_T^* \left(p_{T,t}^* \right)^{1-\omega}}{(1-\lambda_T^*)} \right]^{\frac{1}{1-\omega}} \quad (46)$$

$$\frac{1}{p_{HT,t}} = \left[(1-\alpha) + \alpha \left(T_t \right)^{1-\omega} \right]^{\frac{1}{1-\omega}} \quad (47)$$

$$\frac{1}{p_{FT,t}^*} = \left[(1-\alpha^*) + \alpha^* \left(\frac{1}{T_t} \right)^{1-\omega} \right]^{\frac{1}{1-\omega}} \quad (48)$$

$$p_{T,t} = \left[(1-\alpha) \left(p_{HT,t} p_{T,t} \right)^{1-\gamma} + \alpha \left(p_{FT,t}^* p_{T,t}^* RER_t \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \quad (49)$$

$$p_{T,t}^* = \left[(1-\alpha^*) \left(p_{FT,t}^* p_{T,t}^* \right)^{1-\gamma} + \alpha^* \left(p_{HT,t} p_{T,t} \frac{1}{RER_t} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \quad (50)$$

$$p_{G,t} = \left[(1-\lambda_T) \left(p_{N,t} \right)^{1-\omega} + \lambda_T \left(p_{HT,t} p_{T,t} \right)^{1-\omega} \right]^{\frac{1}{1-\omega}} \quad (51)$$

$$p_{G,t}^* = \left[(1-\lambda_T^*) \left(p_{M,t}^* \right)^{1-\omega} + \lambda_T^* \left(p_{FT,t}^* p_{T,t}^* \right)^{1-\omega} \right]^{\frac{1}{1-\omega}} \quad (52)$$

EVOLUTION OF PRICES

$$\left(1+\pi_{H,t}\right)^{1-\varepsilon} = \left(1-\mu_H\right)\left(1+\pi_{H,t}^O\right)^{1-\varepsilon} + \mu_H \quad (53)$$

$$\left(1+\pi_{N,t}\right)^{1-\varepsilon} = \left(1-\mu_N\right)\left(1+\pi_{N,t}^O\right)^{1-\varepsilon} + \mu_N \quad (54)$$

$$\left(1+\pi_{F,t}^*\right)^{1-\varepsilon} = \left(1-\mu_F^*\right)\left(1+\pi_{F,t}^{O*}\right)^{1-\varepsilon} + \mu_F^* \quad (55)$$

$$\left(1+\pi_{M,t}^*\right)^{1-\varepsilon} = \left(1-\mu_M^*\right)\left(1+\pi_{M,t}^{O*}\right)^{1-\varepsilon} + \mu_M^* \quad (56)$$

CENTRAL BANK INTEREST RATE RULE

$$\frac{\left(1+r_t\right)}{\left(1+r^T\right)}=\left(\frac{\left(1+r_{t-1}\right)}{\left(1+r^T\right)}\right)^{\phi_R}\left[\left[\left[\left(\frac{\frac{(1+\tau_{VAT,t})}{(1+\tau_{VAT,t-1})}\left(1+\pi_t\right)}{\left(1+\pi_t^T\right)}\right)^n\left(\frac{\frac{(1+\tau_{VAT,t}^*)}{(1+\tau_{VAT,t-1}^*)}\left(1+\pi_t^*\right)}{\left(1+\pi_t^{T*}\right)}\right)^{1-n}\right]^{\phi_\pi}\left[\left(\frac{\frac{Y_t}{Y_{t-1}}}{g_Y}\right)^n\left(\frac{\frac{Y_t^*}{Y_{t-1}^*}}{g_Y^*}\right)^{1-n}\right]^{1-\phi_\pi}\right]^{1-\phi_R}$$

(57)

GOOD AND LABOR MARKET CLEARING

$Y_{N,t}=C_{N,t}+G_{N,t}$ (58)

$Y_{M,t}^*=C_{M,t}^*+G_{M,t}^*$ (59)

$Y_{H,t}=C_{H,t}+C_{H,t}^*+G_{H,t}$ (60)

$Y_{F,t}^*=C_{F,t}^*+C_{F,t}+G_{F,t}^*$ (61)

$L_t=L_{N,t}+L_{H,t}$ (62)

$L_t^*=L_{M,t}^*+L_{F,t}^*$ (63)

INFLATION DEFINITIONS

$$\left(1+\pi_t\right)=\left[(1-\lambda_T)\left(\left(1+\pi_{N,t}\right)p_{N,t-1}\right)^{1-\omega}+\lambda_T\left(\left(1+\pi_{T,t}\right)p_{T,t-1}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} \quad (64)$$

$$\left(1+\pi_t^*\right)=\left[(1-\lambda_T^*)\left(\left(1+\pi_{M,t}^*\right)p_{M,t-1}^*\right)^{1-\omega}+\lambda_T^*\left(\left(1+\pi_{T,t}^*\right)p_{T,t-1}^*\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} \quad (65)$$

$$\left(1+\pi_{T,t}\right)=\left[(1-\alpha)\left(\left(1+\pi_{H,t}\right)p_{HT,t-1}\right)^{1-\gamma}+\alpha\left(\left(1+\pi_{F,t}^*\right)p_{FT,t-1}^*S_{t-1}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \quad (66)$$

$$\left(1+\pi_{T,t}^*\right)=\left[(1-\alpha^*)\left(\left(1+\pi_{F,t}^*\right)p_{FT,t-1}^*\right)^{1-\gamma}+\alpha^*\left(\left(1+\pi_{H,t}\right)p_{HT,t-1}\frac{1}{S_{t-1}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

(67)

$$\left(1+\pi_{G,t}\right)=\left[(1-\lambda_T)\left(\left(1+\pi_{N,t}\right)\frac{p_{N,t-1}}{p_{G,t-1}}\right)^{1-\omega}+\lambda_T\left(\left(1+\pi_{H,t}\right)\frac{p_{HT,t-1}p_{T,t-1}}{p_{G,t-1}}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} \quad (68)$$

$$\left(1+\pi_{G,t}^*\right)=\left[(1-\lambda_T^*)\left(\left(1+\pi_{M,t}^*\right)\frac{p_{M,t-1}^*}{p_{G,t-1}^*}\right)^{1-\omega}+\lambda_T^*\left(\left(1+\pi_{F,t}^*\right)\frac{p_{FT,t-1}^*p_{T,t-1}^*}{p_{G,t-1}^*}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} \quad (69)$$

INTERNATIONAL RELATIVE PRICES

$T_t=\frac{\left(1+\pi_{F,t}^*\right)}{\left(1+\pi_{H,t}\right)}T_{t-1}$ (70) Home terms of trade

$$S_t = \frac{\left(1+\pi^*_{T,t}\right)}{\left(1+\pi_{T,t}\right)} S_{t-1}$$ (71) Relative price of foreign-consumed tradable goods basket

$$RER_t = \frac{\left(1+\pi^*_t\right)}{\left(1+\pi_t\right)} RER_{t-1}$$ (72) Home real exchange rate

TOTAL OUTPUT DEFINITION

$$Y_t = Y_{N,t} + Y_{H,t}$$ (73) Home total output

$$Y^*_t = Y^*_{M,t} + Y^*_{F,t}$$ (74) Foreign total output

IMPLICIT DEFINITION OF OUTPUT PRICE INDEX

$$p_{Y,t} Y_t = p_{N,t} Y_{N,t} + p_{HT,t}\, p_{T,t} Y_{H,t}$$ (75) Implicit definition of Home output relative price index

$$p^*_{Y,t} Y^*_t = p^*_{M,t} Y^*_{M,t} + p^*_{FT,t}\, p^*_{T,t} Y^*_{F,t}$$ (76) Implicit definition of Foreign output relative price index