Optimization problem (without non-negativity and transversality conditions):

$$\max_{C_t, L_t, K_{t+1}} \quad \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma^C}}{1-\sigma^C} - \frac{L_t^{1+\frac{1}{\sigma^L}}}{1+\frac{1}{\sigma^L}} \right) \right\} \tag{1a}$$

$$\text{s.t.} \quad Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{1b}$$

$$Y_t = C_t + I_t + G_t, \tag{1c}$$

$$G_t = g^y \bar{Y} e^{\epsilon_{G_t}}, \tag{1d}$$

$$I_t = K_{t+1} - (1-\delta) K_t, \tag{1e}$$

where G_t and A_t obey the following AR(1) processes

$$\log(G_t) = \rho_G \log(G_{t-1}) + \epsilon_{G_t}$$
$$\log(A_t) = \rho_A \log(A_{t-1}) + \epsilon_{A_t}$$

with $\epsilon_{G_t}, \epsilon_{A_t} \sim \mathcal{N}(0, \sigma^2)$ and $\rho_A, \rho_G \in (0,1)$.
Solution:

$$L_t^{1+\frac{1}{\sigma^L}} = (1-\alpha) C_t^{-\sigma^C} Y_t$$
$$1 = \beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma^C} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right) \right\}$$