

Here are the full derivations to formulas 49 and 50 in Graham & Wright's linearized model (2007, p.29):

For the first equation, which comes from the optimization problem for the nominal value  $Z_{t+1}$  of a new credit contract in a financial institution, linearized around the steady state, one gets

$$z_{t+1} - E_t p_{t+1} = E_t \left\{ \frac{[1 - B(1)]F}{B(F)} \pi_{t+1} \right\}$$

where  $F$  is the forwardshift operator (that is,  $F^i x_t = x_{t+i}$ ) and  $B(F) = 1 - \beta_1(1 - \phi)F$ , for parameters  $\beta_1$  and  $\phi$ .

From this, one sees that

$$\begin{aligned} B(1) &= 1 - \beta_1(1 - \phi)(1) \\ &= 1 - \beta_1 + \beta_1\phi \end{aligned}$$

so that

$$\begin{aligned} 1 - B(1) &= 1 - (1 - \beta_1 + \beta_1\phi) \\ &= \beta_1 - \beta_1\phi \\ &= \beta_1(1 - \phi) \end{aligned}$$

Hence, multiplying both sides of the original equation by  $B(F)$  –as indicated to us by Graham himself– and, using the previous result,

$$\begin{aligned} B(F)[z_{t+1} - E_t p_{t+1}] &= E_t \{ ([1 - B(1)]F) \pi_{t+1} \} \\ (1 - \beta_1(1 - \phi)F)[z_{t+1} - E_t p_{t+1}] &= E_t \{ [\beta_1(1 - \phi)F] \pi_{t+1} \} \\ &= E_t \{ \beta_1(1 - \phi) \pi_{t+2} \} \\ (1 - \beta_1[1 - \phi]F)z_{t+1} - (1 - \beta_1[1 - \phi]F)E_t p_{t+1} &= E_t \{ \beta_1(1 - \phi)[p_{t+2} - p_{t+1}] \} \\ (1 - \beta_1[1 - \phi]F)z_{t+1} - E_t p_{t+1} + \beta_1[1 - \phi]E_t p_{t+2} &= E_t \{ \beta_1(1 - \phi)[p_{t+2} - p_{t+1}] \} \\ (1 - \beta_1[1 - \phi]F)z_{t+1} - E_t p_{t+1} &= E_t \{ -\beta_1(1 - \phi)p_{t+1} \} \\ (1 - \beta_1[1 - \phi]F)z_{t+1} &= E_t \{ [1 - \beta_1(1 - \phi)]p_{t+1} \} \\ z_{t+1} - \beta_1[1 - \phi]z_{t+2} &= E_t \{ [1 - \beta_1(1 - \phi)][\pi_{t+1} - \pi_t] \} \end{aligned}$$

This last result is the one we used while solving the model in Dynare.

One can proceed analogously in deriving the second equation, which gives the aggregate value  $D_t$  of debt in terms of the individual debt contracts and the price level  $P_t$ , also linearized around the steady state:

$$d_{t+1} + p_{t+1} = \frac{A(1)}{A(L)} z_{t+1}$$

where  $L$  is the backshift operator (that is,  $L^i x_t = x_{t-i}$ ) y  $A(L) = 1 - (1 - \phi)L$ .

From this, one sees that

$$\begin{aligned} A(1) &= 1 - (1 - \phi)(1) \\ &= \phi \end{aligned}$$

so that, multiplying both sides by  $A(L)$ , and using this last result,

$$\begin{aligned} A(L)[d_{t+1} + p_{t+1}] &= A(1)z_{t+1} \\ (1 - (1 - \phi)L)[d_{t+1} + p_{t+1}] &= \phi z_{t+1} \\ (1 - (1 - \phi)L)d_{t+1} + (1 - (1 - \phi)L)p_{t+1} &= \phi z_{t+1} \\ d_{t+1} - (1 - \phi)d_t + p_{t+1} - (1 - \phi)p_t &= \phi z_{t+1} \\ d_{t+1} - (1 - \phi)d_t + \pi_{t+1} + \phi(\pi_t - \pi_{t-1}) &= \phi z_{t+1} \end{aligned}$$

This last result is the one we used in Dynare.

In summary,

$$z_{t+1} - E_t p_{t+1} = E_t \left\{ \frac{[1 - B(1)]F}{B(F)} \pi_{t+1} \right\} \iff z_{t+1} - \beta_1 [1 - \phi] z_{t+2} = E_t \{ [1 - \beta_1 (1 - \phi)] [\pi_{t+1} - \pi_t] \}$$

and

$$d_{t+1} + p_{t+1} = \frac{A(1)}{A(L)} z_{t+1} \iff d_{t+1} - (1 - \phi)d_t + \pi_{t+1} + \phi(\pi_t - \pi_{t-1}) = \phi z_{t+1}$$