The inverse gamma distribution and its use in YADA and Dynare: a short note

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Ingvar Strid
Sveriges Riksbank

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Abstract

This note explains how the inverse gamma distribution is specified in YADA and Dynare. A simple example is provided of how to specify the inverse gamma prior in YADA such that it is identical to the prior already specified in Dynare.
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*The views, analysis, and conclusions in this paper are those of the authors and not necessarily those of the Riksbank's staff or executive board.

Contents

1. Introduction

The gamma or inverse gamma distributions are commonly used as the prior distribution for DSGE model parameters that are bounded from below. The inverse gamma-1 distribution is typically used as the prior distribution for the standard deviation of an innovation to a shock, while the inverse gamma-2 distribution is used for the variance. The density can be parameterised in different ways and, furthermore, different software programs require different inputs in specifying the density. The purpose of this note is to explain how the distribution is used with Dynare and YADA. Both software programs incorporate the inverse gamma-1 distribution (but not the inverse gamma-2) which means that the DSGE model should be parameterised in terms of the standard deviation of an innovation (rather than the variance).

2. The inverse gamma distribution

The inverse Gamma-2 distribution is often used as the prior distribution for the variance of an innovation in a DSGE model. We write

$$\sigma^2 \sim IG_2\left(s, v\right),$$

with density function

$$f_{I_g}\left(\sigma^2|v,s\right) = C_g^{-1}\left(\frac{v}{2},\frac{2}{s}\right)\left(\sigma^2\right)^{-\frac{1}{2}(v+2)}\exp\left(-\frac{s}{2\sigma^2}\right), \tag{2.1}$$

where

$$C_g\left(\frac{v}{2},\frac{2}{s}\right) = \Gamma\left(\frac{v}{2}\right)\left(\frac{2}{s}\right)^{\frac{v}{2}},$$

and where Γ is the gamma function, see [3] (also see 'gamma' and related functions in Matlab). The parameter s is a location parameter and v is a scale parameter (degrees of freedom). The mean and variance of $X = \sigma^2$ is

$$E(X) = \frac{s}{v-2},$$

and

$$V(X) = \frac{2}{v-4}E(X)^2 = \frac{2}{v-4}\left(\frac{s}{v-2}\right)^2,$$

see [2]. The log density is given by

$$\ln f_{I_g}\left(\sigma^2|v,s\right) = -\ln\Gamma\left(\frac{v}{2}\right) - \frac{v}{2}\ln\left(\frac{2}{s}\right) - \frac{1}{2}\left(v+2\right)\ln\left(\sigma^2\right) - \frac{s}{2\sigma^2}, \tag{2.2}$$

and in practical work one typically works with logs, i.e. 2.2 is used when the prior is evaluated for a specific value of the parameter, σ^2.

If the variance σ^2 is inverse Gamma-2 distributed , then the standard deviation σ is inverse gamma-1 distributed (or simply 'inverse gamma')

$$\sigma = \left(\sigma^2\right)^{1/2} \sim IG_1\left(s, v\right)$$

The density function of the inverse gamma-1 can be obtained by change-of-variables. If

$$Y = X^{1/2},$$

then the density of Y is given by

$$f(y) = \left|\frac{d}{dy}\left(g^{-1}(y)\right)\right| f_X\left(g^{-1}(y)\right),$$

where

$$\frac{d}{dy}\left(g^{-1}(y)\right) = \frac{d}{dy}y^2 = 2y.$$

So the density function of the inverse gamma-1 is

$$f_{I_g}\left(\sigma^2|v, s\right) = C_g^{-1}\left(\frac{v}{2}, \frac{2}{s}\right)\left(\sigma^2\right)^{-\frac{1}{2}(v+2)} \exp\left(-\frac{s}{2\sigma^2}\right) 2\sigma,$$

i.e.

$$f_{I_g}\left(\sigma^2|v, s\right) = 2C_g^{-1}\left(\frac{v}{2}, \frac{2}{s}\right)\sigma^{-(v+1)} \exp\left(-\frac{s}{2\sigma^2}\right) \tag{2.3}$$

since

$$\left(\sigma^2\right)^{-\frac{1}{2}(v+2)}\sigma = \sigma^{-(v+2)}\sigma = \sigma^{-(v+1)}.$$

The log of the inverse Gamma-1 density function is given by

$$\ln f_{I_g}\left(\sigma^2|v, s\right) = \ln(2) - \ln\Gamma\left(\frac{v}{2}\right) - \frac{v}{2}\ln\left(\frac{2}{s}\right) - (v+1)\ln\left(\sigma\right) - \frac{s}{2\sigma^2}$$

The mean and variance of $Y = \sigma$ is

$$E(Y) = \sqrt{\frac{s}{2}}\frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)},\ v > 1, \tag{2.4}$$

and

$$V(Y) = \frac{s}{v-2} - E(Y)^2,\ v > 2. \tag{2.5}$$

2.1. An alternative parameterisation (e.g. Wikipedia) and the mode

The inverse gamma-2 distribution can alternatively be parameterised as

$$f_{I_g}\left(x|\alpha,\beta\right) = \frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)} x^{-(\alpha+1)} \exp(-\beta/x),$$

The parameters α and β are related to the parameters s and v through

$$\alpha = v/2,\ \beta = s/2$$

Substituting, we get

$$f_{I_g}\left(x|v,s\right) = C_g^{-1}\left(\frac{v}{2},\frac{2}{s}\right) x^{-\frac{1}{2}(v+2)} \exp(-s/2x).$$

The mode of the inverse gamma-2 density is given by

$$M(x) = \frac{\beta}{\alpha+1} = \frac{s/2}{v/2+1}.$$

Note that this density is referred to simply as 'inverse gamma' in Wikipedia, while we call it inverse gamma-2 here.

3. The inverse gamma in YADA and Dynare

3.1. Parameterisation of the inverse gamma distribution in YADA

In YADA the inverse gamma density is parameterised in the following way

$$f\left(\sigma|\tilde{s},\tilde{q}\right) = \frac{2}{\Gamma\left(\frac{\tilde{q}}{2}\right)}\left(\frac{\tilde{q}\tilde{s}^2}{2}\right)^{\tilde{q}/2} \sigma^{-(\tilde{q}+1)} \exp\left(-\frac{\tilde{q}\tilde{s}^2}{2\sigma^2}\right) \tag{3.1}$$

which is the parameterisation used in [1] (see [3], p. 46). Here $\tilde{s}$ is a location parameter and $\tilde{q}$ is the degrees of freedom. Comparing this with the expression for the density function above, 2.3, the relation between the parameters are

$$\tilde{q} = v, \tag{3.2}$$

and

$$s = \tilde{q}\tilde{s}^2 \text{ (YADA -> standard)} \tag{3.3}$$

or

$$\tilde{s} = \left(\frac{s}{v}\right)^{1/2} \text{ (standard -> YADA)} \tag{3.4}$$

We then have the mean and variance expressed in terms of $\tilde{q}$ and $\tilde{s}$ as

$$E(\sigma) = \sqrt{\frac{s}{2}}\frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} =$$

$$= \sqrt{\frac{\tilde{q}\tilde{s}^2}{2}} \frac{\Gamma\left(\frac{\tilde{q}-1}{2}\right)}{\Gamma\left(\frac{\tilde{q}}{2}\right)} = \tilde{s}\sqrt{\frac{\tilde{q}}{2}} \frac{\Gamma\left(\frac{\tilde{q}-1}{2}\right)}{\Gamma\left(\frac{\tilde{q}}{2}\right)},$$

and

$$V(\sigma) = \frac{s}{v-2} - E(\sigma)^2 =$$
$$= \frac{\tilde{q}\tilde{s}^2}{\tilde{q}-2} - E(\sigma)^2.$$

The mode of the density 3.1 is given by

$$M(\sigma) = \tilde{s}\sqrt{\frac{\tilde{q}}{\tilde{q}+1}} \tag{3.5}$$

such that

$$M(\sigma) = \sqrt{\frac{s}{v}}\sqrt{\frac{v}{v+1}} = \sqrt{\frac{s}{v+1}}, \tag{3.6}$$

when the density is parameterised in terms of s and v (see *logInvertedGammaPDF.m* in the YADA code).

The inverse gamma-2 density does not appear to be specified in YADA. Therefore, the DSGE model should be parameterised in terms of the standard deviation of an innovation, rather than the variance.

3.2. Parameterisation of the inverse gamma distribution in Dynare

As in YADA the DSGE model should be parameterised in terms of the standard deviation of the innovation, rather than the variance. The inverse gamma density in Dynare is parameterised as in 2.3 (see file: *lpdfig1.m* in Dynare 4.1.0). However, when specifying the prior density in the model file (.mod) the user needs to supply the mean and standard deviation instead of the parameters s and v. The (inverse) transformation from the mean and standard deviation, $E(\sigma)$ and $S(\sigma)$ to s and v is provided by a numerical routine (see *inverse_ gamma_ specification.m* in Dynare 4.1.0).

In the special case when $S(\sigma) = \infty$ we have the transformations

$$v = 2, \tag{3.7}$$

and

$$s = \frac{2E(\sigma)^2}{\pi}. \tag{3.8}$$

The case with $S(\sigma) = \infty$ is quite common. For example, in the Riksbank's model Ramses *all* innovation standard deviations have a prior standard deviation which equals infinity. Note that when $v = 2$ the variance is not defined, see 2.5.

4. Examples

4.1. Going from Dynare to YADA when the variance of the inverse gamma is infinity

In the Riksbank's model Ramses all standard deviations of innovations have a prior variance equal to infinity.To take an example, the innovation to the permanent technology shock in Ramses has the inverse gamma prior with mean and standard deviation

$$E(\sigma) = 0.15,\ S(\sigma) = \infty.$$

(codeline: *inv_gamma_pdf,0.15,inf* in the model file)

Using 3.7 and 3.8 we have

$$v = 2,\ \text{and}\ s = \frac{2E(\sigma)^2}{\pi} = \frac{2 \times 0.15^2}{\pi} = 0.014323944878271.$$

Using 3.2 and 3.4 we then have

$$\tilde{q} = v = 2,\ \text{and}\ \tilde{s} = \left(\frac{s}{v}\right)^{1/2} = \left(\frac{0.014323944878271}{2}\right)^{1/2} = 0.084628437532163.$$

The values for $\tilde{q}$ and $\tilde{s}$ are found in the prior specification file (excel file) in the YADA code. Furthermore, the mode of the density can be computed using 3.6

$$M(\sigma) = \sqrt{\frac{s}{v+1}} = \sqrt{\frac{0.014323944878271}{2+1}} = 0.069098829894268.$$

To check our computations we may use *View-Graph a prior* in the YADA menu. We need to specify $\tilde{s}$ (location) and $\tilde{q}$ (degrees of freedom) and a lower bound (= 0). The density is plotted and the mode is computed.

4.2. Going from Dynare to YADA when the variance of the inverse gamma is finite

When the variance of the inverse gamma distributed variable is finite we need to invert the system 2.4 and 2.5. The input is $E(\sigma)$ and $S(\sigma) = \sqrt{V(\sigma)}$ and we need to i) solve for s and v and then ii) solve for $\tilde{q}$ and $\tilde{s}$. The latter step is easily done, see above. The former step cannot be solved analytically, but here we may use the already existing 'solver' in Dynare - i.e. *inverse_gamma_specification.m* in Dynare 4.1.0. In this function a simple numerical algorithm is applied to solve for s and v given $E(\sigma)$ and (finite) $S(\sigma)$.

5. Concluding remarks

Many parameterisations of the inverse gamma distribution exist, as outlined above. YADA and Dynare supply the inverse gamma-1 distribution (but not the inverse gamma-2) which means that the DSGE model should be parameterised in terms of the standard deviation of the innovation (rather than the variance). The example provided above helps us to swiftly move from Dynare to YADA in specifying priors.

We end with a personal view. Another convenient parameterisation of the inverse gamma distribution is to parameterise it in terms of the parameter v (degrees of freedom) and the mode $M(\sigma)$. We may then solve for s using 3.6, i.e.

$$s = M(\sigma)^2 (v + 1) .$$

6. Bibliography

References

[1] Zellner, Arnold (1971), *An Introduction to Bayesian Inference in Econometrics*, Wiley Classics Library

[2] Bauwens, Luc and Lubrano, Michel and Richard, Jean-Francois (1999), *Bayesian inference in dynamic econometric models*, Oxford University Press.

[3] Warne, Anders (2014), *YADA manual: computational details.*