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Optimal Monetary Policy in an Operational Medium-Sized DSGE Model: Technical Appendix*

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Abstract

Technical appendix to "Optimal Monetary Policy in an Operational Medium-Sized DSGE Model"

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This technical appendix of Adolfson, Laséen, Lindé, and Svensson [1] contains the detailed specification of the matrices A, B, C, and H in the model, as well as the detailed specification of the measurement equation and the related matrices $\bar{D}_0$, $\bar{D}$, and $\bar{D}_s$.

1. Definition of the matrices A, B, C, D, and H and the instrument rule $[f_X \quad f_x]$

We want to identify the submatrices in the model,

$$X_{t+1} = A_{11}X_t + A_{12}x_t + B_1 i_t + C\varepsilon_{t+1},$$

$$Hx_{t+1|t} = A_{22}x_t + A_{21}X_t + B_2 i_t.$$

We specify the predetermined variables $X_t = (X_t^{\mathrm{ex}\prime}, X_t^{\mathrm{pd}\prime})'$, the forward-looking variables x_t, the policy instrument i_t, and the shock vector ε_t as follows (the number of the elements in each vector

*All remaining errors are ours. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank.

is also listed):

$$X_t^{\mathrm{ex}} \equiv \left[\begin{array}{cccc} \widehat{\epsilon}_t & 1 & \hat{\tau}_t^y & 21 \\ \widehat{\epsilon}_{t-1} & 2 & \hat{\tau}_t^c & 22 \\ \hat{\mu}_{zt} & 3 & \hat{\tau}_t^w & 23 \\ \hat{\mu}_{z,t-1} & 4 & \hat{g}_t & 24 \\ \hat{\nu}_t & 5 & \hat{\tau}_{t-1}^k & 25 \\ \hat{\zeta}_t^c & 6 & \hat{\tau}_{t-1}^y & 26 \\ \hat{\zeta}_t^h & 7 & \hat{\tau}_{t-1}^c & 27 \\ \hat{\zeta}_t^q & 8 & \hat{\tau}_{t-1}^w & 28 \\ \hat{\lambda}_t^f & 9 & \hat{g}_{t-1} & 29 \\ \hat{\lambda}_t^{mc} & 10 & \hat{\pi}_t^* & 30 \\ \hat{\lambda}_t^{mi} & 11 & \hat{y}_t^* & 31 \\ \widehat{\tilde{\phi}}_t & 12 & \hat{R}_t^* & 32 \\ \hat{\Upsilon}_t & 13 & \hat{\pi}_{t-1}^* & 33 \\ \widehat{\tilde{z}}_t^* & 14 & \hat{y}_{t-1}^* & 34 \\ \widehat{\tilde{z}}_{t-1}^* & 15 & \hat{R}_{t-1}^* & 35 \\ \hat{\lambda}_{xt} & 16 & \hat{\pi}_{t-2}^* & 36 \\ \hat{\varepsilon}_{Rt} & 17 & \hat{y}_{t-2}^* & 37 \\ \widehat{\bar{\pi}}_t^c & 18 & \hat{R}_{t-2}^* & 38 \\ \widehat{\bar{\pi}}_{t-1}^c & 19 & \hat{\pi}_{t-3}^* & 39 \\ \hat{\tau}_t^k & 20 & \hat{y}_{t-3}^* & 40 \\ & & \hat{R}_{t-3}^* & 41 \end{array}\right], \quad X_t^{\mathrm{pd}} \equiv \left[\begin{array}{cccccc} \widehat{\bar{k}}_t & 1 & 42 & \hat{k}_{t-1} & 16 & 57 \\ \widehat{\bar{m}}_t & 2 & 43 & \hat{q}_{t-1} & 17 & 58 \\ \hat{R}_{t-1} & 3 & 44 & \hat{\mu}_{t-1} & 18 & 59 \\ \hat{\pi}_{t-1}^d & 4 & 45 & \hat{a}_{t-1} & 19 & 60 \\ \hat{\pi}_{t-1}^{mc} & 5 & 46 & \hat{\gamma}_{t-1}^{mcd} & 20 & 61 \\ \hat{\pi}_{t-1}^{mi} & 6 & 47 & \hat{\gamma}_{t-1}^{mid} & 21 & 62 \\ \hat{y}_{t-1} & 7 & 48 & \hat{\gamma}_{t-1}^{x*} & 22 & 63 \\ \hat{\pi}_{t-1}^x & 8 & 49 & \widehat{\tilde{x}}_{t-1} & 23 & 64 \\ \widehat{\bar{w}}_{t-1} & 9 & 50 & \widehat{\mathrm{mc}}_{t-1}^x & 24 & 65 \\ \hat{c}_{t-1} & 10 & 51 & \widehat{\bar{k}}_{t-1} & 25 & 66 \\ \widehat{\bar{\imath}}_{t-1} & 11 & 52 & \hat{u}_{t-1} & 26 & 67 \\ \hat{\psi}_{z,t-1} & 12 & 53 & \hat{\pi}_{t-2}^d & 27 & 68 \\ \hat{P}_{k',t-1} & 13 & 54 & \hat{\pi}_{t-2}^{mc} & 28 & 69 \\ \Delta\hat{S}_{t-1} & 14 & 55 & \hat{\pi}_{t-3}^d & 29 & 70 \\ \hat{H}_{t-1} & 15 & 56 & \hat{\pi}_{t-3}^{mc} & 30 & 71 \end{array}\right],$$

$$x_t \equiv \left[\begin{array}{cccc} \hat{\pi}_t^d & 1 & & \\ \hat{\pi}_t^{mc} & 2 & \hat{k}_t & 13 \\ \hat{\pi}_t^{mi} & 3 & \hat{q}_t & 14 \\ \hat{y}_t & 4 & \hat{\mu}_t & 15 \\ \hat{\pi}_t^x & 5 & \hat{a}_t & 16 \\ \widehat{\bar{w}}_t & 6 & \hat{\gamma}_t^{mcd} & 17 \\ \hat{c}_t & 7 & \hat{\gamma}_t^{mid} & 18 \\ \widehat{\bar{\imath}}_t & 8 & \hat{\gamma}_t^{x*} & 19 \\ \hat{\psi}_{zt} & 9 & \widehat{\tilde{x}}_t & 20 \\ \hat{P}_{k't} & 10 & \widehat{\mathrm{mc}}_t^x & 21 \\ \Delta\hat{S}_t & 11 & \widehat{\bar{k}}_{t+1} & 22 \\ \hat{H}_t & 12 & \widehat{\bar{m}}_{t+1} & 23 \end{array}\right], \quad i_t \equiv \hat{R}_t,$$

$$\varepsilon_t \equiv \left[\begin{array}{cccccccccc} \varepsilon_{\epsilon t} & \varepsilon_{zt} & \varepsilon_{\nu t} & \varepsilon_{\zeta^c t} & \varepsilon_{\zeta^h t} & \varepsilon_{\zeta^q t} & \varepsilon_{\lambda t} & \varepsilon_{\lambda^{mc} t} & \varepsilon_{\lambda^{mi} t} & \cdots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \cdots \\ \varepsilon_{\tilde{\phi} t} & \varepsilon_{\Upsilon t} & \varepsilon_{\tilde{z}^* t} & \varepsilon_{\lambda_x t} & \varepsilon_{\varepsilon_R t} & \varepsilon_{\bar{\pi} t} & \varepsilon_{\tau t}' & \varepsilon_{x^* t}' & & \\ 10 & 11 & 12 & 13 & 14 & 15 & 16:20 & 21:23 & & \end{array}\right]'.$$

1.1. Predetermined variables

First we specify the exogenous variables and the shocks, then the endogenous predetermined variables.

Row 1 - Stationary technology shock

$$\hat{\epsilon}_{t+1} = \rho_\epsilon \hat{\epsilon}_t + \sigma_{\varepsilon_\epsilon} \varepsilon_{\epsilon,t+1}.$$

$$\begin{aligned} \hat{\epsilon}_t &: \quad A_{11}(1,1) = \rho_\epsilon, \\ \varepsilon_{\epsilon,t+1} &: \quad C(1,1) = \sigma_{\varepsilon_\epsilon}. \end{aligned}$$

The notation means that the value of the persistence of the technology shock, ρ_ϵ, is placed in the first row and first column of the matrix A_{11}. The notation "$\hat{\epsilon}_t$: " means that the parameter value pertains to that specific variable.

Row 2 - Lagged stationary technology shock

$$\hat{\epsilon}_t = \hat{\epsilon}_t.$$

$$\hat{\epsilon}_t : A_{11}(2,1) = 1.$$

Row 3 - Permanent technology shock

$$\hat{\mu}_{z,t+1} = \rho_{\mu_z} \hat{\mu}_{zt} + \sigma_{\varepsilon_z} \varepsilon_{z,t+1}.$$

$$\begin{aligned} \hat{\mu}_{zt} &: \quad A_{11}(3,3) = \rho_\epsilon, \\ \varepsilon_{z,t+1} &: \quad C(3,2) = \sigma_{\varepsilon_\epsilon}. \end{aligned}$$

Row 4 - Lagged permanent technology shock

$$\hat{\mu}_{zt} = \hat{\mu}_{zt}.$$

$$\hat{\mu}_{zt} : A_{11}(4,3) = 1.$$

Row 5 - Firm money demand shock

$$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \sigma_{\varepsilon_\nu} \varepsilon_{\nu,t+1}.$$

$$\begin{aligned} \hat{\nu}_t &: \quad A_{11}(5,5) = \rho_\nu, \\ \varepsilon_{\nu,t+1} &: \quad C(5,3) = \sigma_{\varepsilon_\nu}. \end{aligned}$$

Row 6 - Consumption preference shock

$$\hat{\zeta}^c_{t+1} = \rho_{\zeta^c} \hat{\zeta}^c_t + \sigma_{\varepsilon_{\zeta^c}} \varepsilon_{\zeta^c,t+1}.$$

$$\begin{aligned} \hat{\zeta}^c_t &: \quad A_{11}(6,6) = \rho_{\zeta^c}, \\ \varepsilon_{\zeta^c,t+1} &: \quad C(6,4) = \sigma_{\varepsilon_{\zeta^c}}. \end{aligned}$$

Row 7 - Labor supply shock

$$\hat{\zeta}^h_{t+1} = \rho_{\zeta^h}\hat{\zeta}^h_t + \sigma_{\varepsilon_{\zeta^h}}\varepsilon_{\zeta^h,t+1}.$$

$$\begin{aligned} \hat{\zeta}^h_t &: \quad A_{11}(7,7) = \rho_{\zeta^h}, \\ \varepsilon_{\zeta^h,t+1} &: \quad C(7,5) = \sigma_{\varepsilon_{\zeta^h}}. \end{aligned}$$

Row 8 - Household money demand shock

$$\hat{\zeta}^q_{t+1} = \rho_{\zeta^q}\hat{\zeta}^h_t + \sigma_{\varepsilon_{\zeta^q}}\varepsilon_{\zeta^q,t+1}.$$

$$\begin{aligned} \hat{\zeta}^q_t &: \quad A_{11}(8,8) = \rho_{\zeta^q}, \\ \varepsilon_{\zeta^q,t+1} &: \quad C(8,6) = \sigma_{\varepsilon_{\zeta^q}}. \end{aligned}$$

Row 9 - Markup shock - domestic firms

$$\hat{\lambda}^d_{t+1} = \rho_{\lambda}\hat{\lambda}^d_t + \sigma_{\varepsilon_{\lambda}}\varepsilon_{\lambda,t+1}.$$

$$\begin{aligned} \hat{\lambda}^d_t &: \quad A_{11}(9,9) = \rho_{\lambda}, \\ \varepsilon_{\lambda,t+1} &: \quad C(9,7) = \sigma_{\varepsilon_{\lambda}}. \end{aligned}$$

Row 10 - Shock to substitution elasticity between domestic and foreign consumption goods

$$\hat{\lambda}^{mc}_{t+1} = \rho_{\lambda^{mc}}\hat{\lambda}^{mc}_t + \sigma_{\varepsilon_{\lambda^{mc}}}\varepsilon_{\lambda^{mc},t+1}.$$

$$\begin{aligned} \hat{\lambda}^{mc}_t &: \quad A_{11}(10,10) = \rho_{\eta^{mc}}, \\ \varepsilon_{\lambda^{mc},t+1} &: \quad C(10,8) = \sigma_{\varepsilon_{\lambda^{mc}}}. \end{aligned}$$

Row 11 - Shock to substitution elasticity between domestic and foreign investment goods

$$\hat{\lambda}^{mi}_{t+1} = \rho_{\lambda^{mi}}\hat{\lambda}^{mi}_t + \sigma_{\varepsilon_{\lambda^{mi}}}\varepsilon_{\lambda^{mi},t+1}.$$

$$\begin{aligned} \hat{\lambda}^{mi}_t &: \quad A_{11}(11,11) = \rho_{\eta^{mi}}, \\ \varepsilon_{\lambda^{mi},t+1} &: \quad C(11,9) = \sigma_{\varepsilon_{\eta^{mi}}}. \end{aligned}$$

Row 12 - Risk premium shock

$$\hat{\tilde{\phi}}_{t+1} = \rho_{\tilde{\phi}}\hat{\tilde{\phi}}_t + \sigma_{\varepsilon_{\tilde{\phi}}}\varepsilon_{\tilde{\phi},t+1}.$$

$$\begin{aligned} \hat{\tilde{\phi}}_t &: \quad A_{11}(12,12) = \rho_{\tilde{\phi}}, \\ \varepsilon_{\tilde{\phi},t+1} &: \quad C(12,10) = \sigma_{\varepsilon_{\tilde{\phi}}}. \end{aligned}$$

Row 13 - Investment specific technology shock

$$\hat{\Upsilon}_{t+1} = \rho_{\Upsilon}\hat{\Upsilon}_t + \sigma_{\varepsilon_{\Upsilon}}\varepsilon_{\Upsilon,t+1}.$$

$$\begin{aligned} \hat{\Upsilon}_t &: A_{11}(13,13) = \rho_{\Upsilon}, \\ \varepsilon_{\Upsilon,t+1} &: C(13,11) = \sigma_{\varepsilon_{\Upsilon}}. \end{aligned}$$

Row 14 - Relative (asymmetric) technology shock (between foreign and domestic)

$$\widehat{\tilde{z}}^*_{t+1} = \rho_{\tilde{z}^*}\widehat{\tilde{z}}^*_t + \sigma_{\varepsilon_{\tilde{z}^*}}\varepsilon_{\tilde{z}^*,t+1}.$$

$$\begin{aligned} \widehat{\tilde{z}}^*_t &: A_{11}(14,14) = \rho_{\tilde{z}^*}, \\ \varepsilon_{\tilde{z}^*,t+1} &: C(14,12) = \sigma_{\varepsilon_{\tilde{z}^*}}. \end{aligned}$$

Row 15 - Lagged relative (asymmetric) technology shock (between foreign and domestic)

$$\widehat{\tilde{z}}^*_t = \widehat{\tilde{z}}^*_t.$$

$$\widehat{\tilde{z}}^*_t : A_{11}(15,14) = 1.$$

Row 16 - Markup shock - exporting firms

$$\hat{\lambda}_{x,t+1} = \rho_{\lambda_x}\hat{\lambda}_{xt} + \sigma_{\varepsilon_{\lambda_x}}\varepsilon_{\lambda_x,t+1}.$$

$$\begin{aligned} \hat{\lambda}_{xt} &: A_{11}(16,16) = \rho_{\lambda_x}, \\ \varepsilon_{\lambda_x,t+1} &: C(16,13) = \sigma_{\varepsilon_{\lambda_x}}. \end{aligned}$$

Row 17 - Monetary policy shock

$$\epsilon_{R,t+1} = \rho_{\varepsilon_R}\epsilon_{Rt} + \sigma_{\varepsilon_{\varepsilon_R}}\varepsilon_{\varepsilon_R,t+1}.$$

$$\begin{aligned} \epsilon_{Rt} &: A_{11}(17,17) = \rho_{\varepsilon_R}, \\ \varepsilon_{\varepsilon_R,t+1} &: C(17,14) = \sigma_{\varepsilon_{\varepsilon_R}}. \end{aligned}$$

Row 18 - Inflation target

$$\widehat{\bar{\pi}}^c_{t+1} = \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}^c_t + \sigma_{\varepsilon_{\widehat{\bar{\pi}}^c}}\varepsilon_{\widehat{\bar{\pi}}^c,t+1}.$$

$$\begin{aligned} \widehat{\bar{\pi}}^c_t &: A_{11}(18,18) = \rho_{\widehat{\bar{\pi}}^c}, \\ \varepsilon_{\widehat{\bar{\pi}}^c,t+1} &: C(18,15) = \sigma_{\varepsilon_{\widehat{\bar{\pi}}^c}}. \end{aligned}$$

Row 19 - Lagged inflation target

$$\widehat{\bar{\pi}}^c_t = \widehat{\bar{\pi}}^c_t.$$

$$\widehat{\bar{\pi}}^c_t : A_{11}(19,18) = 1.$$

Row 20–29 - Fiscal VAR(2)

$$\tau_{t+1} = \tilde{\Theta}_1 \tau_t + \tilde{\Theta}_2 \tau_{t-1} + e_{\tau,t+1}, \tag{1.1}$$

where $\tilde{\Theta}_1 = \Theta_0^{-1}\Theta_1$, $\tilde{\Theta}_2 = \Theta_0^{-1}\Theta_2$ and $e_{\tau t} = \Theta_0^{-1} S_\tau \varepsilon_{\tau t} \sim N(0, \Sigma_\tau)$. Rewrite (1.1) as VAR(1):

$$\begin{bmatrix} \hat{\tau}_{t+1} \\ \hat{\tau}_t \end{bmatrix} = \begin{bmatrix} \tilde{\Theta}_1 & \tilde{\Theta}_2 \\ I & 0 \end{bmatrix} \begin{bmatrix} \hat{\tau}_t \\ \hat{\tau}_{t-1} \end{bmatrix} + \begin{bmatrix} e_{\tau,t+1} \\ 0 \end{bmatrix}$$

$$\begin{aligned}
\hat{\tau}_t &: \quad A_{11}(20:24, 20:24) = \tilde{\Theta}_1, \\
\hat{\tau}_{t-1} &: \quad A_{11}(20:24, 25:29) = \tilde{\Theta}_2, \\
\hat{\tau}_t &: \quad A_{11}(25{:}29, 20:24) = I, \\
\varepsilon_{\tau,t+1} &: \quad C(20:24, 16:20) = \text{chol}\left(\Theta_0^{-1} S_\tau \left(\Theta_0^{-1} S_\tau\right)'\right)'.
\end{aligned}$$

Row 30–41 - Foreign VAR(4)

$$X^*_{t+1} = \tilde{\Phi}_1 X^*_t + \tilde{\Phi}_2 X^*_{t-1} + \tilde{\Phi}_3 X^*_{t-2} + \tilde{\Phi}_4 X^*_{t-3} + e_{X^*,t+1}, . \tag{1.2}$$

where $\tilde{\Phi}_1 = \Phi_0^{-1}\Phi_1$, $\tilde{\Phi}_2 = \Phi_0^{-1}\Phi_2$, etc., and $e_{X^*t} = \Phi_0^{-1} S_{x^*} \varepsilon_{x^*t} \sim N(0, \Sigma_{x^*})$. Rewrite (1.2) as VAR(1):

$$\begin{bmatrix} X^*_{t+1} \\ X^*_t \\ X^*_{t-1} \\ X^*_{t-2} \end{bmatrix} = \begin{bmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 & \tilde{\Phi}_3 & \tilde{\Phi}_4 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} X^*_t \\ X^*_{t-1} \\ X^*_{t-2} \\ X^*_{t-3} \end{bmatrix} + \begin{bmatrix} e_{X^*,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
X^*_t &: \quad A_{11}(30:32, 30:32) = \tilde{\Phi}_1, \\
X^*_{t-1} &: \quad A_{11}(30:32, 33:35) = \tilde{\Phi}_2, \\
X^*_{t-2} &: \quad A_{11}(30:32, 36:38) = \tilde{\Phi}_3, \\
X^*_{t-3} &: \quad A_{11}(30:32, 39:41) = \tilde{\Phi}_4, \\
X^*_t &: \quad A_{11}(33:35, 30:32) = I, \\
X^*_{t-1} &: \quad A_{11}(36:38, 33:35) = I, \\
X^*_{t-2} &: \quad A_{11}(39:41, 36:38) = I, \\
\varepsilon_{x^*,t+1} &: \quad C(30:32, 21:23) = \text{chol}\left(\Phi_0^{-1} S_{x^*} \left(\Phi_0^{-1} S_{x^*}\right)'\right)'.
\end{aligned}$$

Row 42 - Physical capital stock (note that $\widehat{\bar{k}}_{t+1}$ is predetermined variable no. 42 in period $t+1$ and forward-looking variable no. 22 in period t):

$$\widehat{\bar{k}}_{t+1} = \widehat{\bar{k}}_{t+1},$$

$$\widehat{\bar{k}}_t : A_{12}(42, 22) = 1.$$

Row 43 - Financial assets (note that $\widehat{\bar{m}}_{t+1}$ is predetermined variable no. 43 in period $t+1$ and forward-looking variable no. 23 in period t):

$$\widehat{\bar{m}}_{t+1} = \widehat{\bar{m}}_{t+1},$$

$$\widehat{\bar{m}}_{t+1} : A_{12}(43, 23) = 1.$$

Row 44–66 - Lagged forward-looking variables

$$\hat{R}_t \quad : \quad B_1\,(44,1) = 1$$

$$\begin{aligned}
\hat{\pi}_t^d &\quad : \quad A_{12}\,(45,1) = 1 \\
\hat{\pi}_t^{mc} &\quad : \quad A_{12}\,(46,2) = 1 \\
\hat{\pi}_t^{mi} &\quad : \quad A_{12}\,(47,3) = 1 \\
\hat{y}_t &\quad : \quad A_{12}\,(48,4) = 1 \\
\hat{\pi}_t^x &\quad : \quad A_{12}\,(49,5) = 1 \\
\widehat{\bar{w}}_t &\quad : \quad A_{12}\,(50,6) = 1 \\
\hat{c}_t &\quad : \quad A_{12}\,(51,7) = 1 \\
\widehat{\bar{\imath}}_t &\quad : \quad A_{12}\,(52,8) = 1
\end{aligned}$$

$$\begin{aligned}
\hat{\psi}_{zt} &\quad : \quad A_{12}\,(53,9) = 1 \\
\hat{P}_{k't} &\quad : \quad A_{12}\,(54,10) = 1 \\
\Delta\hat{S}_t &\quad : \quad A_{12}\,(55,11) = 1 \\
\hat{H}_t &\quad : \quad A_{12}\,(56,12) = 1 \\
\hat{u}_t &\quad : \quad A_{12}\,(57,13) = 1 \\
\hat{q}_t &\quad : \quad A_{12}\,(58,14) = 1 \\
\hat{\mu}_t &\quad : \quad A_{12}\,(59,15) = 1 \\
\hat{a}_t &\quad : \quad A_{12}\,(60,16) = 1 \\
\hat{\gamma}_t^{mcd} &\quad : \quad A_{12}\,(61,17) = 1 \\
\hat{\gamma}_t^{mid} &\quad : \quad A_{12}\,(62,18) = 1 \\
\hat{\gamma}_t^{x*} &\quad : \quad A_{12}\,(63,19) = 1 \\
\widehat{\tilde{x}}_t &\quad : \quad A_{12}\,(64,20) = 1 \\
\widehat{\mathrm{mc}}_t^x &\quad : \quad A_{12}\,(65,21) = 1
\end{aligned}$$

Row 66 - Lagged physical capital stock

$$\widehat{\bar{k}}_{t-1} : A_{11}\,(66,42) = 1$$

Row 67 - Lagged capacity utilization

$$\hat{u}_{t-1} \equiv \hat{k}_{t-1} - \widehat{\bar{k}}_{t-1}$$

$$\begin{aligned}
\hat{k}_{t-1} &\quad : \quad A_{11}(67,57) = 1 \\
\widehat{\bar{k}}_{t-1} &\quad : \quad A_{11}(67,66) = -1
\end{aligned}$$

Rows 68-71 - Lagged inflation

$$\begin{aligned}
\hat{\pi}^{d}_{t-1} &: A_{11}(68,45)=1\\
\hat{\pi}^{mc}_{t-1} &: A_{11}(69,46)=1\\
\hat{\pi}^{d}_{t-2} &: A_{11}(70,68)=1\\
\hat{\pi}^{mc}_{t-2} &: A_{11}(71,69)=1
\end{aligned}$$

1.2. Forward-looking variables

Row 1 - Domestic inflation

$$\begin{aligned}
\hat{\pi}^{d}_{t}-\widehat{\bar{\pi}}^{c}_{t} &= \frac{\beta}{1+\kappa_d\beta}\left(\hat{\pi}^{d}_{t+1|t}-\rho_{\bar{\pi}^c}\widehat{\bar{\pi}}^{c}_{t}\right)+\frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}^{d}_{t-1}-\widehat{\bar{\pi}}^{c}_{t}\right)\\
&\quad-\frac{\kappa_d\beta\,(1-\rho_{\bar{\pi}^c})}{1+\kappa_d\beta}\widehat{\bar{\pi}}^{c}_{t}+\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\left(\widehat{\mathrm{mc}}^{d}_{t}+\hat{\lambda}^{d}_{t}\right),\\
\widehat{\mathrm{mc}}^{d}_{t} &\equiv \alpha\left(\hat{\mu}_{zt}+\hat{H}_t-\hat{k}_t\right)+\widehat{\bar{w}}_t+\hat{R}^{f}_{t}-\hat{\varepsilon}_t,\\
\hat{R}^{f}_{t} &= \frac{\nu R}{v(R-1)+1}\hat{R}_{t-1}+\frac{\nu(R-1)}{v(R-1)+1}\hat{\nu}_t
\end{aligned}$$

$$\widehat{\mathrm{mc}}^{d}_{t}\equiv\alpha\hat{H}_t-\alpha\hat{k}_t+\widehat{\bar{w}}_t+\frac{\nu R}{v(R-1)+1}\hat{R}_{t-1}+\frac{\nu(R-1)}{v(R-1)+1}\hat{\nu}_t-\hat{\varepsilon}_t+\alpha\hat{\mu}_{zt}$$

Rewrite the above equations as:

$$\begin{aligned}
-\frac{\beta}{1+\kappa_d\beta}\hat{\pi}^{d}_{t+1|t} &= -\hat{\pi}^{d}_{t}+\frac{\kappa_d}{1+\kappa_d\beta}\hat{\pi}^{d}_{t-1}+\\
&\quad+\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\alpha\hat{H}_t-\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\alpha\hat{k}_t\\
&\quad+\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\widehat{\bar{w}}_t+\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\frac{\nu R}{v(R-1)+1}\hat{R}_{t-1}+\\
&\quad+\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\frac{\nu(R-1)}{v(R-1)+1}\hat{\nu}_t+\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\alpha\hat{\mu}_{zt}\\
&\quad-\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\hat{\varepsilon}_t+\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\,(1+\kappa_d\beta)}\hat{\lambda}^{d}_{t}\\
&\quad-\frac{(1-\kappa_d)\,(\beta\rho_{\bar{\pi}^c}-1)}{1+\kappa_d\beta}\widehat{\bar{\pi}}^{c}_{t}.
\end{aligned}$$

$$
\begin{aligned}
\hat{\pi}^d_{t+1|t} &: H(1,1) = -\frac{\beta}{1+\kappa_d\beta} \\
\hat{\pi}^d_t &: A_{22}(1,1) = -1 \\
\hat{\pi}^d_{t-1} &: A_{21}(1,45) = \frac{\kappa_d}{1+\kappa_d\beta} \\
\widehat{\bar{\pi}}^c_t &: A_{21}(1,18) = 1 - \frac{\beta}{1+\kappa_d\beta}\rho_{\bar{\pi}^c} - \frac{\kappa_d}{1+\kappa_d\beta} - \frac{\kappa_d\beta\left(1-\rho_{\bar{\pi}^c}\right)}{1+\kappa_d\beta} \\
\hat{\lambda}^d_t &: \left[A_{21}(1,9) = \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)}\right] \\
\hat{\lambda}^d_t &: A_{21}(1,9) = 1 \quad \text{(rescaled)} \\
\text{Marginal cost} &: \\
\widehat{\bar{w}}_t &: A_{22}(1,6) = \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)} \\
\hat{H}_t &: A_{22}(1,12) = \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)}\alpha \\
\hat{k}_t &: A_{22}(1,13) = -\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)}\alpha \\
\hat{\varepsilon}_t &: A_{21}(1,1) = -\frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)} \\
\hat{\mu}_{zt} &: A_{21}(1,3) = \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)}\alpha \\
\hat{\nu}_t &: A_{21}(1,5) = \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)}\frac{\nu(R-1)}{v(R-1)+1} \\
\hat{R}_{t-1} &: A_{21}(1,44) = \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d\left(1+\kappa_d\beta\right)}\frac{\nu R}{v(R-1)+1}
\end{aligned}
$$

Row 2 - Imported consumption inflation

$$
\begin{aligned}
\hat{\pi}^{mc}_t - \widehat{\bar{\pi}}^c_t &= \frac{\beta}{1+\kappa_{mc}\beta}\left(\hat{\pi}^{mc}_{t+1|t} - \rho_{\bar{\pi}^c}\widehat{\bar{\pi}}^c_t\right) + \frac{\kappa_{mc}}{1+\kappa_{mc}\beta}\left(\hat{\pi}^{mc}_{t-1} - \widehat{\bar{\pi}}^c_t\right) \\
&\quad - \frac{\kappa_{mc}\beta\left(1-\rho_{\bar{\pi}^c}\right)}{1+\kappa_{mc}\beta}\widehat{\bar{\pi}}^c_t + \frac{(1-\xi_{mc})(1-\beta\xi_{mc})}{\xi_{mc}\left(1+\kappa_{mc}\beta\right)}\left(\widehat{\mathrm{mc}}^{mc}_t + \hat{\lambda}^{mc}_t\right), \\
\widehat{\mathrm{mc}}^{mc}_t &\equiv -\widehat{\mathrm{mc}}^x_t - \hat{\gamma}^{x*}_t - \hat{\gamma}^{mcd}_t
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}_{t+1|t}^{mc} &: H(2,2) = -\frac{\beta}{1+\kappa_{mc}\beta} \\
\hat{\pi}_{t}^{mc} &: A_{22}(2,2) = -1 \\
\hat{\pi}_{t-1}^{mc} &: A_{21}(2,46) = \frac{\kappa_{mc}}{1+\kappa_{mc}\beta} \\
\widehat{\bar{\pi}}_{t}^{c} &: A_{21}(2,18) = 1 - \frac{\beta}{1+\kappa_{mc}\beta}\rho_{\bar{\pi}^c} - \frac{\kappa_{mc}}{1+\kappa_{mc}\beta} - \frac{\kappa_{mc}\beta\left(1-\rho_{\bar{\pi}^c}\right)}{1+\kappa_{mc}\beta} \\
&\left[\hat{\lambda}_t^{mc} : A_{21}(2,10) = \frac{(1-\xi_{mc})(1-\beta\xi_{mc})}{\xi_{mc}\left(1+\kappa_{mc}\beta\right)}\right] \\
\hat{\lambda}_t^{mc} &: A_{21}(2,10) = 1 \quad \text{(rescaled)} \\
\text{Marginal cost} &: \\
\widehat{\mathrm{mc}}_t^{x} &: A_{22}(2,21) = -\frac{(1-\xi_{mc})(1-\beta\xi_{mc})}{\xi_{mc}\left(1+\kappa_{mc}\beta\right)} \\
\hat{\gamma}_t^{x*} &: A_{22}(2,19) = -\frac{(1-\xi_{mc})(1-\beta\xi_{mc})}{\xi_{mc}\left(1+\kappa_{mc}\beta\right)} \\
\hat{\gamma}_t^{mcd} &: A_{22}(2,17) = -\frac{(1-\xi_{mc})(1-\beta\xi_{mc})}{\xi_{mc}\left(1+\kappa_{mc}\beta\right)}
\end{aligned}
$$

Row 3 - Imported investment inflation

$$
\begin{aligned}
\hat{\pi}_t^{mi} - \widehat{\bar{\pi}}_t^{c} &= \frac{\beta}{1+\kappa_{mi}\beta}\left(\hat{\pi}_{t+1|t}^{mi} - \rho_{\bar{\pi}^c}\widehat{\bar{\pi}}_t^{c}\right) + \frac{\kappa_{mi}}{1+\kappa_{mi}\beta}\left(\hat{\pi}_{t-1}^{mi} - \widehat{\bar{\pi}}_t^{c}\right) \\
&\quad - \frac{\kappa_{mi}\beta\left(1-\rho_{\bar{\pi}^c}\right)}{1+\kappa_{mi}\beta}\widehat{\bar{\pi}}_t^{c} + \frac{(1-\xi_{mi})(1-\beta\xi_{mi})}{\xi_{mi}\left(1+\kappa_{mi}\beta\right)}\left(\widehat{\mathrm{mc}}_t^{mi} + \hat{\lambda}_t^{mi}\right), \\
\widehat{\mathrm{mc}}_t^{mi} &\equiv -\widehat{\mathrm{mc}}_t^{x} - \hat{\gamma}_t^{x*} - \hat{\gamma}_t^{mi,d}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}_{t+1|t}^{mi} &: H(3,3) = -\frac{\beta}{1+\kappa_{mi}\beta} \\
\hat{\pi}_{t}^{mi} &: A_{22}(3,3) = -1 \\
\hat{\pi}_{t-1}^{mi} &: A_{21}(3,47) = \frac{\kappa_{mi}}{1+\kappa_{mi}\beta} \\
\text{Marginal cost} &: \\
\hat{\gamma}_t^{mid} &: A_{22}(3,18) = -\frac{(1-\xi_{mi})(1-\beta\xi_{mi})}{\xi_{mi}\left(1+\kappa_{mi}\beta\right)} \\
\hat{\gamma}_t^{x*} &: A_{22}(3,19) = -\frac{(1-\xi_{mi})(1-\beta\xi_{mi})}{\xi_{mi}\left(1+\kappa_{mi}\beta\right)} \\
\widehat{\mathrm{mc}}_t^{x} &: A_{22}(3,21) = -\frac{(1-\xi_{mi})(1-\beta\xi_{mi})}{\xi_{mi}\left(1+\kappa_{mi}\beta\right)} \\
&\left[\hat{\lambda}_t^{mi} : A_{21}(3,11) = \frac{(1-\xi_{mi})(1-\beta\xi_{mi})}{\xi_{mi}\left(1+\kappa_{mi}\beta\right)}\right] \\
\hat{\lambda}_t^{mi} &: A_{21}(3,11) = 1 \quad \text{(rescaled)} \\
\widehat{\bar{\pi}}_{t}^{c} &: A_{21}(3,18) = 1 - \frac{\beta}{1+\kappa_{mi}\beta}\rho_{\bar{\pi}^c} - \frac{\kappa_{mi}}{1+\kappa_{mi}\beta} - \frac{\kappa_{mi}\beta\left(1-\rho_{\bar{\pi}^c}\right)}{1+\kappa_{mi}\beta}
\end{aligned}
$$

Row 4 - Output gap

$$0 = -\hat{y}_t + \lambda_f \hat{\epsilon}_t + \lambda_f \alpha \hat{k}_t - \lambda_f \alpha \hat{\mu}_{zt} + \lambda_f (1-\alpha) \hat{H}_t$$

$$\begin{aligned}
\hat{y}_t &: A_{22}(4,4) = -1 \\
\hat{H}_t &: A_{22}(4,12) = \lambda_f (1-\alpha) \\
\hat{k}_t &: A_{22}(4,13) = \lambda_f \alpha \\
\\
\hat{\epsilon}_t &: A_{21}(4,1) = \lambda_f \\
\hat{\mu}_{zt} &: A_{21}(4,3) = -\lambda_f \alpha
\end{aligned}$$

Row 5 - Export price inflation

$$\begin{aligned}
\hat{\pi}_t^x - \widehat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_x\beta}\left(\hat{\pi}_{t+1|t}^x - \rho_{\bar{\pi}^c}\widehat{\bar{\pi}}_t^c\right) + \frac{\kappa_x}{1+\kappa_x\beta}\left(\hat{\pi}_{t-1}^x - \widehat{\bar{\pi}}_t^c\right) \\
&\quad - \frac{\kappa_x\beta(1-\rho_{\bar{\pi}^c})}{1+\kappa_x\beta}\widehat{\bar{\pi}}_t^c + \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\kappa_x\beta)}\left(\widehat{\mathrm{mc}}_t^x + \hat{\lambda}_t^x\right),
\end{aligned}$$

$$\begin{aligned}
\hat{\pi}_{t+1|t}^x &: H(5,5) = -\frac{\beta}{1+\kappa_x\beta} \\
\hat{\pi}_t^x &: A_{22}(5,5) = -1 \\
\hat{\pi}_{t-1}^x &: A_{21}(5,49) = \frac{\kappa_x}{1+\kappa_x\beta} \\
\text{Marginal cost} &: \\
\widehat{\mathrm{mc}}_t^x &: A_{22}(5,21) = \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\kappa_x\beta)} \\
& \left[\hat{\lambda}_t^x : A_{21}(5,16) = \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\kappa_x\beta)}\right] \\
\hat{\lambda}_t^x &: A_{21}(5,16) = 1 \quad \text{(rescaled)} \\
\widehat{\bar{\pi}}_t^c &: A_{21}(5,18) = 1 - \frac{\beta}{1+\kappa_x\beta}\rho_{\bar{\pi}^c} - \frac{\kappa_x}{1+\kappa_x\beta} - \frac{\kappa_x\beta(1-\rho_{\bar{\pi}^c})}{1+\kappa_x\beta}
\end{aligned}$$

Row 6 - Real wage

$$\begin{aligned}
0 &= E_t\left\{\begin{array}{c}
\eta_0\hat{w}_{t-1} + \eta_1\hat{w}_t + \eta_2\hat{w}_{t+1} + \eta_3\left(\hat{\pi}_t^d - \widehat{\bar{\pi}}_t^c\right) + \eta_4\left(\hat{\pi}_{t+1}^d - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\
+\eta_5\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + \eta_6\left(\hat{\pi}_t^c - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\
+\eta_7\hat{\psi}_{zt}^{\tau} + \eta_8\hat{H}_t + \eta_9\hat{\tau}_t^y + \eta_{10}\hat{\tau}_t^w + \eta_{11}\hat{\zeta}_t^h
\end{array}\right\} \\
\widehat{\pi}_t^c &= (1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)}\widehat{\pi}_t^d + \omega_c(\gamma^{cmc})^{-(1-\eta_c)}\widehat{\pi}_t^{mc}
\end{aligned}$$

where

$$\eta \equiv \begin{bmatrix} b_w\xi_w \\ \sigma_L\lambda_w - b_w\left(1+\beta\xi_w^2\right) \\ b_w\beta\xi_w \\ -b_w\xi_w \\ b_w\beta\xi_w \\ b_w\xi_w\kappa_w \\ -b_w\beta\xi_w\kappa_w \\ 1-\lambda_w \\ -(1-\lambda_w)\sigma_L \\ -(1-\lambda_w)\frac{\tau^y}{(1-\tau^y)} \\ -(1-\lambda_w)\frac{\tau^w}{(1+\tau^w)} \\ -(1-\lambda_w) \end{bmatrix} \equiv \begin{bmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \end{bmatrix}. \tag{1.3}$$

Collect terms and use the definition of $\widehat{\pi}_t^c$:

$$\begin{aligned} \eta_2\hat{w}_{t+1|t} + \eta_4\hat{\pi}^d_{t+1|t} \quad = \quad & -\eta_0\hat{w}_{t-1} - \eta_1\hat{w}_t - \left[\eta_3 + \eta_6\left((1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)}\right)\right]\widehat{\pi}_t^d \\ & -\eta_6\omega_c(\gamma^{cmc})^{-(1-\eta_c)}\widehat{\pi}_t^{mc} \\ & -\eta_5\left((1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)}\right)\widehat{\pi}_{t-1}^d - \eta_5\left((\omega_c)(\gamma^{cmc})^{-(1-\eta_c)}\right)\widehat{\pi}_{t-1}^{mc} \\ & -\eta_7\hat{\psi}_{zt}^{\tau} - \eta_8\hat{H}_t - \eta_9\hat{\tau}_t^y - \eta_{10}\hat{\tau}_t^w - \eta_{11}\hat{\zeta}_t^h + \left[\eta_3 + \rho_{\widehat{\bar{\pi}}^c}\eta_4 + \eta_5 + \rho_{\widehat{\bar{\pi}}^c}\eta_6\right]\widehat{\bar{\pi}}_t^c \end{aligned}$$

$$\begin{aligned}
\hat{\pi}^{d}_{t+1|t} &: H(6,1) = \eta_4 \\
\hat{w}_{t+1|t} &: H(6,6) = \eta_2
\end{aligned}$$

$$\begin{aligned}
\widehat{\pi}^{d}_{t} &: A_{22}(6,1) = -\left[\eta_3 + \eta_6\left((1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)}\right)\right] \\
\widehat{\pi}^{mc}_{t} &: A_{22}(6,2) = -\eta_6\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{w}_t &: A_{22}(6,6) = -\eta_1 \\
\hat{\psi}^{\tau}_{zt} &: A_{22}(6,9) = -\eta_7 \\
\hat{H}_t &: A_{22}(6,12) = -\eta_8
\end{aligned}$$

Rescaling the labor supply shock so its coefficient is of the same magnitude as $\hat{w}_t$ $\left[\hat{\zeta}^{h}_{t} : A_{21}(6,7) = -\eta_{11}\right]$

$$\begin{aligned}
\hat{\zeta}^{h}_{t} &: \text{if } \eta_{11}\begin{cases} >0 \Rightarrow A_{21}(6,7) = \eta_1 \\ <0 \Rightarrow A_{21}(6,7) = -\eta_1 \\ =0 \Rightarrow A_{21}(6,7) = \eta_{11}\end{cases} \\
\widehat{\overline{\pi}}^{c}_{t} &: A_{21}(6,18) = (\eta_3 + \rho_{\widehat{\overline{\pi}}^{c}}\eta_4 + \eta_5 + \rho_{\widehat{\overline{\pi}}^{c}}\eta_6) \\
\hat{\tau}^{y}_{t} &: A_{21}(6,21) = -\eta_9 \\
\hat{\tau}^{w}_{t} &: A_{21}(6,23) = -\eta_{10} \\
\widehat{\pi}^{d}_{t-1} &: A_{21}(6,45) = -\eta_5(1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)} \\
\widehat{\pi}^{mc}_{t-1} &: A_{21}(6,46) = -\eta_5\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{w}_{t-1} &: A_{21}(6,50) = -\eta_0
\end{aligned}$$

Row 7 - Consumption

$$E_t\left[\begin{array}{c} -b\beta\mu_z\hat{c}_{t+1} + \left(\mu_z^2 + b^2\beta\right)\hat{c}_t - b\mu_z\hat{c}_{t-1} + b\mu_z\left(\hat{\mu}_{zt} - \beta\hat{\mu}_{z,t+1}\right) + \\ +\left(\mu_z - b\beta\right)\left(\mu_z - b\right)\hat{\psi}_{zt} + \frac{\tau^c}{1+\tau^c}\left(\mu_z - b\beta\right)\left(\mu_z - b\right)\hat{\tau}^{c}_{t} + \left(\mu_z - b\beta\right)\left(\mu_z - b\right)\hat{\gamma}^{cd}_{t} \\ -\left(\mu_z - b\right)\left(\mu_z\hat{\zeta}^{c}_{t} - b\beta\hat{\zeta}^{c}_{t+1}\right)\end{array}\right] = 0$$

$$\begin{aligned}
\widehat{\gamma}^{cd}_{t} &= \omega_c\left(\gamma^{cmc}\right)^{-(1-\eta_c)}\widehat{\gamma}^{mcd}_{t} \\
\hat{\mu}_{z,t+1|t} &= \rho_{\mu_z}\hat{\mu}_{zt} \\
\hat{\zeta}^{c}_{t+1|t} &= \rho_{\zeta^c}\hat{\zeta}^{c}_{t}
\end{aligned}$$

$$\begin{aligned}
b\beta\mu_z\hat{c}_{t+1|t} &= \left(\mu_z^2 + b^2\beta\right)\hat{c}_t - b\mu_z\hat{c}_{t-1} + b\mu_z\left(1 - \beta\rho_{\mu_z}\right)\hat{\mu}_{zt} \\
&\quad + \left(\mu_z - b\beta\right)\left(\mu_z - b\right)\hat{\psi}_{zt} + \frac{\tau^c}{1+\tau^c}\left(\mu_z - b\beta\right)\left(\mu_z - b\right)\hat{\tau}^{c}_{t} \\
&\quad + \left(\mu_z - b\beta\right)\left(\mu_z - b\right)\omega_c\left(\gamma^{cmc}\right)^{-(1-\eta_c)}\widehat{\gamma}^{mcd}_{t} - \left(\mu_z - b\right)\left(\mu_z - b\beta\rho_{\zeta^c}\right)\hat{\zeta}^{c}_{t}
\end{aligned}$$

$$
\begin{aligned}
\hat{c}_{t+1|t} &: H(7,7) = b\beta\mu_z \\
\hat{c}_t &: A_{22}(7,7) = \left(\mu_z^2 + b^2\beta\right) \\
\hat{c}_{t-1} &: A_{21}(7,51) = -b\mu_z
\end{aligned}
$$

$$
\begin{aligned}
\hat{\psi}_{zt} &: A_{22}(7,9) = (\mu_z - b\beta)(\mu_z - b) \\
\widehat{\gamma}_t^{mcd} &: A_{22}(7,17) = (\mu_z - b\beta)(\mu_z - b)\,\omega_c\,(\gamma^{cmc})^{-(1-\eta_c)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mu}_{zt} &: A_{21}(7,3) = b\mu_z\left(1 - \beta\rho_{\mu_z}\right) \\
&\quad \text{Rescaling of } \hat{\zeta}_t^c \text{ to give it the same coefficient as } \hat{c}_t \\
&\quad \left[\hat{\zeta}_t^c : A_{21}(7,6) = -(\mu_z - b)\left(\mu_z - b\beta\rho_{\zeta^c}\right)\right] \\
\hat{\zeta}_t^c &: A_{21}(7,6) = -\left(\mu_z^2 + b^2\beta\right) \\
\hat{\tau}_t^c &: A_{21}(7,22) = \frac{\tau^c}{1+\tau^c}(\mu_z - b\beta)(\mu_z - b)
\end{aligned}
$$

Row 8 - Investment

$$
E_t\left\{\hat{P}_{kt} + \hat{\Upsilon}_t - \hat{\gamma}_t^{i,d} - \mu_z^2 S''(\mu_z)\left[\left(\widehat{\imath}_t - \widehat{\imath}_{t-1}\right) - \beta\left(\widehat{\imath}_{t+1} - \widehat{\imath}_t\right) + \hat{\mu}_{zt} - \beta\hat{\mu}_{z,t+1}\right]\right\} = 0
$$

$$
\begin{aligned}
\widehat{\gamma}_t^{i,d} &= \omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)}\widehat{\gamma}_t^{mi,d} \\
\hat{\mu}_{z,t+1|t} &= \rho_{\mu_z}\hat{\mu}_{zt}
\end{aligned}
$$

$$
\begin{aligned}
\mu_z^2 S''(\mu_z)\,\beta\widehat{\imath}_{t+1|t} &= \mu_z^2 S''(\mu_z)(1+\beta)\widehat{\imath}_t - \mu_z^2 S''(\mu_z)\widehat{\imath}_{t-1} - \hat{P}_{kt} \\
&\quad + \omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)}\widehat{\gamma}_t^{mi,d} + \mu_z^2 S''(\mu_z)\left(1 - \beta\rho_{\mu_z}\right)\hat{\mu}_{zt} - \hat{\Upsilon}_t
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\imath}_{t+1|t} &: H(8,8) = \mu_z^2 S''(\mu_z)\,\beta \\
\widehat{\imath}_t &: A_{22}(8,8) = \mu_z^2 S''(\mu_z)(1+\beta) \\
\widehat{\imath}_{t-1} &: A_{21}(8,52) = -\mu_z^2 S''(\mu_z)
\end{aligned}
$$

$$
\begin{aligned}
\hat{P}_{kt} &: A_{22}(8,10) = -1 \\
\widehat{\gamma}_t^{mi,d} &: A_{22}(8,18) = \omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mu}_{zt} &: A_{21}(8,3) = \mu_z^2 S''(\mu_z)\left(1 - \beta\rho_{\mu_z}\right) \\
&\quad \text{Rescaling of } \hat{\Upsilon}_t \text{ to give it the same coefficient as } \widehat{\imath}_t \\
\hat{\Upsilon}_t &: A_{21}(8,13) = -\mu_z^2 S''(\mu_z)(1+\beta)
\end{aligned}
$$

Row 9 - Lagrange multiplier (first order condition with respect to m_{t+1})

$$\begin{aligned} \mathrm{E}_t\left[-\mu\hat{\psi}_{zt}+\mu\hat{\psi}_{z,t+1}-\mu\hat{\pi}^d_{t+1}-\mu\hat{\mu}_{z,t+1}+\left(\mu-\beta\tau^k\right)\hat{R}_t+\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\hat{\tau}^k_{t+1}\right] &= 0 \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z}\hat{\mu}_{zt} \end{aligned}$$

$$\begin{aligned} \mu\hat{\psi}_{z,t+1|t}-\mu\hat{\pi}^d_{t+1|t} &= \mu\hat{\psi}_{zt}-\left(\mu-\beta\tau^k\right)\hat{R}_t+\mu\hat{\mu}_{z,t+1|t}-\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\hat{\tau}^k_{t+1|t} \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z}\hat{\mu}_{zt} \end{aligned}$$

$$\begin{aligned} 0 &= -\mu\hat{\psi}_{zt}+\mu\hat{\psi}_{z,t+1|t}-\mu\hat{\mu}_{z,t+1|t}+\left(\mu-\beta\tau^k\right)\hat{R}_t-\mu\hat{\pi}^d_{t+1|t}+\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\hat{\tau}^k_{t+1|t} \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z}\hat{\mu}_{zt} \\ \hat{\tau}^k_{t+1|t} &= \tilde{\Theta}_1\left(1,1\right)\hat{\tau}^k_t+\tilde{\Theta}_1\left(1,2\right)\hat{\tau}^y_t+\tilde{\Theta}_1\left(1,3\right)\hat{\tau}^c_t+\tilde{\Theta}_1\left(1,4\right)\hat{\tau}^w_t+\tilde{\Theta}_1\left(1,5\right)\hat{g}_t \\ &\quad +\tilde{\Theta}_2\left(1,1\right)\hat{\tau}^k_{t-1}+\tilde{\Theta}_2\left(1,2\right)\hat{\tau}^y_{t-1}+\tilde{\Theta}_2\left(1,3\right)\hat{\tau}^c_{t-1}+\tilde{\Theta}_2\left(1,4\right)\hat{\tau}^w_{t-1}+\tilde{\Theta}_2\left(1,5\right)\hat{g}_{t-1} \end{aligned}$$

$$\begin{aligned} \mu\hat{\psi}_{z,t+1|t}-\mu\hat{\pi}^d_{t+1|t} &= \mu\hat{\psi}_{zt}+\mu\rho_{\mu_z}\hat{\mu}_{zt}-\left(\mu-\beta\tau^k\right)\hat{R}_t \\ &\quad -\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_1\left(1,1\right)\hat{\tau}^k_t-\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_1\left(1,2\right)\hat{\tau}^y_t \\ &\quad -\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_1\left(1,3\right)\hat{\tau}^c_t-\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_1\left(1,4\right)\hat{\tau}^w_t \\ &\quad -\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_1\left(1,5\right)\hat{g}_t-\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_2\left(1,1\right)\hat{\tau}^k_{t-1} \\ &\quad -\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_2\left(1,2\right)\hat{\tau}^y_{t-1}-\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_2\left(1,3\right)\hat{\tau}^c_{t-1} \\ &\quad -\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_2\left(1,4\right)\hat{\tau}^w_{t-1}-\frac{\tau^k}{1-\tau^k}\left(\beta-\mu\right)\tilde{\Theta}_2\left(1,5\right)\hat{g}_{t-1} \end{aligned}$$

$$\begin{aligned} \hat{\pi}^d_{t+1|t} &: H\left(9,1\right)=-\mu \\ \hat{\psi}_{z,t+1|t} &: H\left(9,9\right)=\mu \end{aligned}$$

$$\begin{aligned} \hat{\psi}_{zt} &: A_{22}\left(9,9\right)=\mu \\ \hat{R}_t &: B_2\left(9,1\right)=-\left(\mu-\beta\tau^k\right) \end{aligned}$$

$$\begin{aligned}
\hat{\mu}_{zt} &: A_{21}(9,3) = \mu\rho_{\mu_z} \\
\hat{\tau}_t^k &: A_{21}(9,20) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1,1) \\
\hat{\tau}_t^y &: A_{21}(9,21) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1,2) \\
\hat{\tau}_t^c &: A_{21}(9,22) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1,3) \\
\hat{\tau}_t^w &: A_{21}(9,23) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1,4) \\
\hat{g}_t &: A_{21}(9,24) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1,5) \\
\\
\hat{\tau}_{t-1}^k &: A_{21}(9,25) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1,1) \\
\hat{\tau}_{t-1}^y &: A_{21}(9,26) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1,2) \\
\hat{\tau}_{t-1}^c &: A_{21}(9,27) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1,3) \\
\hat{\tau}_{t-1}^w &: A_{21}(9,28) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1,4) \\
\hat{g}_{t-1} &: A_{21}(9,29) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1,5)
\end{aligned}$$

Row 10 - Price of capital

$$\begin{aligned}
0 &= \hat{\psi}_{zt} + \hat{\mu}_{z,t+1|t} - \hat{\psi}_{z,t+1|t} - \frac{\beta(1-\tilde{\delta})}{\mu_z}\hat{P}_{k,t+1|t} + \hat{P}_{kt} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\hat{r}_{t+1|t}^k \\
&\quad + \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\hat{\tau}_{t+1|t}^k, \\
\hat{r}_{t+1|t}^k &= \rho_{\mu_z}\hat{\mu}_{zt} + \widehat{\overline{w}}_{t+1|t} + \frac{\nu R}{v(R-1)+1}\hat{R}_t + \frac{\nu(R-1)}{v(R-1)+1}\rho_\nu\hat{\nu}_t + \hat{H}_{t+1|t} - \hat{k}_{t+1|t} \\
\hat{\tau}_{t+1|t}^k &= \tilde{\Theta}_1(1,1)\hat{\tau}_t^k + \tilde{\Theta}_1(1,2)\hat{\tau}_t^y + \tilde{\Theta}_1(1,3)\hat{\tau}_t^c + \tilde{\Theta}_1(1,4)\hat{\tau}_t^w + \tilde{\Theta}_1(1,5)\hat{g}_t \\
&\quad + \tilde{\Theta}_2(1,1)\hat{\tau}_{t-1}^k + \tilde{\Theta}_2(1,2)\hat{\tau}_{t-1}^y + \tilde{\Theta}_2(1,3)\hat{\tau}_{t-1}^c + \tilde{\Theta}_2(1,4)\hat{\tau}_{t-1}^w + \tilde{\Theta}_2(1,5)\hat{g}_{t-1}
\end{aligned}$$

$$
\begin{aligned}
& \hat{\psi}_{z,t+1|t} + \frac{\beta(1-\tilde{\delta})}{\mu_z}\hat{P}_{k,t+1|t} + \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\widehat{\bar{w}}_{t+1|t} + \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\hat{H}_{t+1|t} \\
& -\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\hat{k}_{t+1|t} \\
= \; & \hat{\psi}_{zt} + \left[\rho_{\mu_z} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\rho_{\mu_z}\right]\hat{\mu}_{zt} + \hat{P}_{kt} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\frac{\nu R}{v(R-1)+1}\hat{R}_t \\
& -\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\frac{\nu(R-1)}{v(R-1)+1}\rho_\nu\hat{\nu}_t \\
& \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1(1,1)\,\hat{\tau}_t^k + \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1(1,2)\,\hat{\tau}_t^y \\
& +\frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1(1,3)\,\hat{\tau}_t^c + \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1(1,4)\,\hat{\tau}_t^w \\
& +\frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1(1,5)\,\hat{g}_t + \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2(1,1)\,\hat{\tau}_{t-1}^k \\
& +\frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2(1,2)\,\hat{\tau}_{t-1}^y + \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2(1,3)\,\hat{\tau}_{t-1}^c \\
& +\frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2(1,4)\,\hat{\tau}_{t-1}^w + \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2(1,5)\,\hat{g}_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\bar{w}}_{t+1|t} &: H(10,6) = \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \\
\hat{\psi}_{z,t+1|t} &: H(10,9) = 1 \\
\hat{P}_{k,t+1|t} &: H(10,10) = \frac{\beta(1-\tilde{\delta})}{\mu_z} \\
\hat{H}_{t+1|t} &: H(10,12) = \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \\
\hat{u}_{t+1|t} &: H(10,13) = -\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\psi}_{zt} &: A_{22}(10,9) = 1 \\
\hat{\psi}_{zt} &: A_{22}(10,9) = 1 \\
\hat{P}_{kt} &: A_{22}(10,10) = 1
\end{aligned}
$$

$$\hat{R}_t \quad : \quad B_2\left(10,1\right) = -\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\frac{\nu R}{v(R-1)+1}$$

$$\hat{\mu}_{zt} \quad : \quad A_{21}\left(10,3\right) = \rho_{\mu_z} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\rho_{\mu_z}$$

$$\hat{\nu}_t \quad : \quad A_{21}\left(10,5\right) = -\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\frac{\nu(R-1)}{v(R-1)+1}\rho_\nu$$

$$\hat{\tau}_t^k \quad : \quad A_{21}\left(10,20\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1\left(1,1\right)$$

$$\hat{\tau}_t^y \quad : \quad A_{21}\left(10,21\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1\left(1,2\right)$$

$$\hat{\tau}_t^c \quad : \quad A_{21}\left(10,22\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1\left(1,3\right)$$

$$\hat{\tau}_t^w \quad : \quad A_{21}\left(10,23\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1\left(1,4\right)$$

$$\hat{g}_t \quad : \quad A_{21}\left(10,24\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_1\left(1,5\right)$$

$$\hat{\tau}_{t-1}^k \quad : \quad A_{21}\left(10,25\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2\left(1,1\right)$$

$$\hat{\tau}_{t-1}^y \quad : \quad A_{21}\left(10,26\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2\left(1,2\right)$$

$$\hat{\tau}_{t-1}^c \quad : \quad A_{21}\left(10,27\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2\left(1,3\right)$$

$$\hat{\tau}_{t-1}^w \quad : \quad A_{21}\left(10,28\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2\left(1,4\right)$$

$$\hat{g}_{t-1} \quad : \quad A_{21}\left(10,29\right) = \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\tilde{\Theta}_2\left(1,5\right)$$

Row 11 - Change in nominal exchange rate (UIP)

$$\left(1-\tilde{\phi}_s\right)\mathrm{E}_t\Delta\hat{S}_{t+1} = \tilde{\phi}_s\Delta\hat{S}_t + \left(\hat{R}_t - \hat{R}_t^*\right) + \tilde{\phi}_a\hat{a}_t - \widehat{\tilde{\phi}}_t$$

$$\Delta\hat{S}_{t+1} \quad : \quad H\left(11,11\right) = \left(1-\tilde{\phi}_s\right)$$

$$\Delta\hat{S}_t \quad : \quad A_{22}\left(11,11\right) = \tilde{\phi}_s$$

$$\hat{a}_t \quad : \quad A_{22}\left(11,16\right) = \tilde{\phi}_a$$

$$\hat{R}_t \quad : \quad B_2\left(11,1\right) = 1$$

$$\widehat{\tilde{\phi}}_t \quad : \quad A_{21}\left(11,12\right) = -1$$

$$\hat{R}_t^* \quad : \quad A_{21}\left(11,32\right) = -1$$

Row 12 - Hours worked (aggregate resource constraint)

$$(1-\omega_c)\left(\gamma^{cd}\right)^{\eta_c}\frac{c}{y}\left(\hat{c}_t+\eta_c\hat{\gamma}_t^{cd}\right)+(1-\omega_i)\left(\gamma^{id}\right)^{\eta_i}\frac{\tilde{\imath}}{y}\left(\widehat{\tilde{\imath}}_t+\eta_i\hat{\gamma}_t^{id}\right)+\frac{g}{y}\hat{g}_t+\frac{y^*}{y}\left(\hat{y}_t^*-\eta_f\hat{\gamma}_t^{x*}+\widehat{\tilde{z}}_t^*\right)$$
$$=\lambda_d\left[\hat{\varepsilon}_t+\alpha\left(\hat{k}_t-\hat{\mu}_{zt}\right)+(1-\alpha)\hat{H}_t\right]-\left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\left(\hat{k}_t-\widehat{\bar{k}}_t\right)$$

$$\begin{aligned}0 \;=\;& (1-\omega_c)\left(\gamma^{cd}\right)^{\eta_c}\frac{c}{y}\left(\hat{c}_t+\eta_c\hat{\gamma}_t^{cd}\right)+(1-\omega_i)\left(\gamma^{id}\right)^{\eta_i}\frac{\tilde{\imath}}{y}\left(\widehat{\tilde{\imath}}_t+\eta_i\hat{\gamma}_t^{id}\right)+\frac{g}{y}\hat{g}_t+\frac{y^*}{y}\left(\hat{y}_t^*-\eta_f\hat{\gamma}_t^{x*}+\widehat{\tilde{z}}_t^*\right)\\ &-\lambda_d\left[\hat{\varepsilon}_t+\alpha\left(\hat{k}_t-\hat{\mu}_{zt}\right)+(1-\alpha)\hat{H}_t\right]+\left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\left(\hat{k}_t-\widehat{\bar{k}}_t\right)\end{aligned}$$

$$\begin{aligned}\widehat{\gamma}_t^{cd} &= \omega_c\left(\gamma^{cmc}\right)^{-(1-\eta_c)}\widehat{\gamma}_t^{mcd}\\ \widehat{\gamma}_t^{i,d} &= \omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)}\widehat{\gamma}_t^{mi,d}\end{aligned}$$

$$\begin{aligned}0 \;=\;& (1-\omega_c)\left(\gamma^{cd}\right)^{\eta_c}\frac{c}{y}\hat{c}_t+(1-\omega_c)\left(\gamma^{cd}\right)^{\eta_c}\frac{c}{y}\eta_c\omega_c\left(\gamma^{cmc}\right)^{-(1-\eta_c)}\widehat{\gamma}_t^{mcd}+(1-\omega_i)\left(\gamma^{id}\right)^{\eta_i}\frac{\tilde{\imath}}{y}\widehat{\tilde{\imath}}_t\\ &+(1-\omega_i)\left(\gamma^{id}\right)^{\eta_i}\frac{\tilde{\imath}}{y}\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)}\widehat{\gamma}_t^{mi,d}-\frac{y^*}{y}\eta_f\hat{\gamma}_t^{x*}+\frac{g}{y}\hat{g}_t+\frac{y^*}{y}\hat{y}_t^*+\frac{y^*}{y}\widehat{\tilde{z}}_t^*\\ &-\lambda_d\hat{\varepsilon}_t+\lambda_d\alpha\hat{\mu}_{zt}-\lambda_d(1-\alpha)\hat{H}_t-\left[\lambda_d\alpha-\left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\right]\hat{k}_t-\left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\widehat{\bar{k}}_t\end{aligned}$$

$$\begin{aligned}\hat{c}_t &: A_{22}(12,7)=(1-\omega_c)\left(\gamma^{cd}\right)^{\eta_c}\frac{c}{y}\\ \widehat{\tilde{\imath}}_t &: A_{22}(12,8)=(1-\omega_i)\left(\gamma^{id}\right)^{\eta_i}\frac{\tilde{\imath}}{y}\\ \hat{H}_t &: A_{22}(12,12)=-\lambda_d(1-\alpha)\\ \hat{k}_t &: A_{22}(12,13)=-\left[\lambda_d\alpha-\left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\right]\\ \widehat{\gamma}_t^{mcd} &: A_{22}(12,17)=(1-\omega_c)\left(\gamma^{cd}\right)^{\eta_c}\frac{c}{y}\eta_c\omega_c\left(\gamma^{cmc}\right)^{-(1-\eta_c)}\\ \widehat{\gamma}_t^{mi,d} &: A_{22}(12,18)=(1-\omega_i)\left(\gamma^{id}\right)^{\eta_i}\frac{\tilde{\imath}}{y}\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)}\\ \hat{\gamma}_t^{x*} &: A_{22}(12,19)=-\frac{y^*}{y}\eta_f\end{aligned}$$

$$\begin{aligned}\hat{\varepsilon}_t &: A_{21}(12,1)=-\lambda_d\\ \hat{\mu}_{zt} &: A_{21}(12,3)=\lambda_d\alpha\\ \widehat{\tilde{z}}_t^* &: A_{21}(12,14)=\frac{y^*}{y}\\ \hat{g}_t &: A_{21}(12,24)=\frac{g}{y}\\ \hat{y}_t^* &: A_{21}(12,31)=\frac{y^*}{y}\\ \widehat{\bar{k}}_t &: A_{21}(12,42)=-\left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\end{aligned}$$

Row 13 - Capital-services flow

$$
\begin{aligned}
\hat{u}_t &= \frac{1}{\sigma_a}\hat{r}_t^k - \frac{1}{\sigma_a}\frac{\tau^k}{(1-\tau^k)}\hat{\tau}_t^k, \\
\hat{u}_t &\equiv \hat{k}_t - \hat{\bar{k}}_t, \\
\hat{r}_t^k &= \hat{\mu}_{zt} + \hat{\bar{w}}_t + \hat{R}_t^f + \hat{H}_t - \hat{k}_t.
\end{aligned}
$$

$$
0 = -(1+\sigma_a)\hat{k}_t + \sigma_a\hat{\bar{k}}_t + \hat{\mu}_{zt} + \hat{\bar{w}}_t + \hat{R}_t^f + \hat{H}_t - \frac{\tau^k}{(1-\tau^k)}\hat{\tau}_t^k
$$

$$
0 = -(1+\sigma_a)\hat{k}_t + \sigma_a\hat{\bar{k}}_t + \hat{\bar{w}}_t + \hat{H}_t + \frac{\nu R}{v(R-1)+1}\hat{R}_{t-1} + \hat{\mu}_{zt} + \frac{\nu(R-1)}{v(R-1)+1}\hat{\nu}_t - \frac{\tau^k}{(1-\tau^k)}\hat{\tau}_t^k
$$

$$
\begin{aligned}
\hat{\bar{w}}_t &: A_{22}(13,6) = 1 \\
\hat{H}_t &: A_{22}(13,12) = 1 \\
\hat{k}_t &: A_{22}(13,13) = -(1+\sigma_a)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mu}_{zt} &: A_{21}(13,3) = 1 \\
\hat{\nu}_t &: A_{21}(13,5) = \frac{\nu(R-1)}{v(R-1)+1} \\
\hat{\tau}_t^k &: A_{21}(13,20) = -\frac{\tau^k}{(1-\tau^k)} \\
\hat{\bar{k}}_t &: A_{21}(13,42) = \sigma_a \\
\hat{R}_{t=1} &: A_{21}(13,44) = \frac{\nu R}{v(R-1)+1}
\end{aligned}
$$

Row 14 - Real money balances

$$
\hat{q}_t = \frac{1}{\sigma_q}\left[\hat{\zeta}_t^q + \frac{\tau^k}{1-\tau^k}\hat{\tau}_t^k - \hat{\psi}_{zt} - \frac{R}{R-1}\hat{R}_{t-1}\right],
$$

$$
0 = -\sigma_q\hat{q}_t - \hat{\psi}_{zt} - \frac{R}{R-1}\hat{R}_{t-1} + \hat{\zeta}_t^q + \frac{\tau^k}{1-\tau^k}\hat{\tau}_t^k,
$$

$$
\begin{aligned}
\hat{\psi}_{zt} &: A_{22}(14,9) = -1 \\
\hat{q}_t &: A_{22}(14,14) = -\sigma_q
\end{aligned}
$$

$$
\begin{aligned}
\hat{\zeta}_t^q &: A_{21}(14,8) = 1 \\
\hat{\tau}_t^k &: A_{21}(14,20) = \frac{\tau^k}{1-\tau^k} \\
\hat{R}_{t-1} &: A_{21}(14,44) = -\frac{R}{R-1}
\end{aligned}
$$

Row 15 - Loan market clearing

$$\nu \bar{w} H \left(\hat{\nu}_t + \widehat{\bar{w}}_t + \hat{H}_t\right) = \frac{\mu \bar{m}}{\pi^d \mu_z}\left(\hat{\mu}_t + \widehat{\bar{m}}_t - \hat{\pi}_t^d - \hat{\mu}_{zt}\right) - q\hat{q}_t,$$

$$0 = -\nu \bar{w} H \hat{\nu}_t - \nu \bar{w} H \widehat{\bar{w}}_t - \nu \bar{w} H \hat{H}_t + \frac{\mu \bar{m}}{\pi^d \mu_z}\hat{\mu}_t + \frac{\mu \bar{m}}{\pi^d \mu_z}\widehat{\bar{m}}_t - \frac{\mu \bar{m}}{\pi^d \mu_z}\hat{\pi}_t^d - \frac{\mu \bar{m}}{\pi^d \mu_z}\hat{\mu}_{zt} - q\hat{q}_t,$$

$$\begin{aligned}
\hat{\pi}_t^d &: \quad A_{22}(15,1) = -\frac{\mu \bar{m}}{\pi^d \mu_z} \\
\widehat{\bar{w}}_t &: \quad A_{22}(15,6) = -\nu \bar{w} H \\
\hat{H}_t &: \quad A_{22}(15,12) = -\nu \bar{w} H \\
\hat{q}_t &: \quad A_{22}(15,14) = -q \\
\hat{\mu}_t &: \quad A_{22}(15,15) = \frac{\mu \bar{m}}{\pi^d \mu_z}
\end{aligned}$$

$$\begin{aligned}
\hat{\nu}_t &: \quad A_{21}(15,5) = -\nu \bar{w} H \\
\hat{\mu}_{zt} &: \quad A_{21}(15,3) = -\frac{\mu \bar{m}}{\pi^d \mu_z} \\
\widehat{\bar{m}}_t &: \quad A_{21}(15,43) = \frac{\mu \bar{m}}{\pi^d \mu_z}
\end{aligned}$$

Row 16 - Net foreign assets

$$\begin{aligned}
\hat{a}_t &= -y^* \widehat{\mathrm{mc}}_t^x - \eta_f y_t^* \hat{\gamma}_t^{x*} + y^* \hat{y}_t^* + y^* \widehat{\tilde{z}}_t^* + (c^m + \tilde{\imath}^m)\hat{\gamma}_t^f \\
&\quad -c^m \hat{c}_t + c^m \eta_c (1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\
&\quad -\tilde{\imath}^m \widehat{\tilde{\imath}}_t + \tilde{\imath}^m \eta_i (1-\omega_i)\left(\gamma^{id}\right)^{-(1-\eta_i)} \hat{\gamma}_t^{mid} + \frac{R}{\pi \mu_z}\hat{a}_{t-1}, \\
\hat{\gamma}_t^f &= \widehat{\mathrm{mc}}_t^x + \hat{\gamma}_t^{x,*}
\end{aligned}$$

$$\begin{aligned}
0 &= -\hat{a}_t - \left[y^* - (c^m + \tilde{\imath}^m)\right]\widehat{\mathrm{mc}}_t^x - \left[\eta_f y_t^* - (c^m + \tilde{\imath}^m)\right]\hat{\gamma}_t^{x,*} + y^* \hat{y}_t^* + y^* \widehat{\tilde{z}}_t^* \\
&\quad -c^m \hat{c}_t + c^m \eta_c (1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\
&\quad -\tilde{\imath}^m \widehat{\tilde{\imath}}_t + \tilde{\imath}^m \eta_i (1-\omega_i)\left(\gamma^{id}\right)^{-(1-\eta_i)} \hat{\gamma}_t^{mid} + \frac{R}{\pi \mu_z}\hat{a}_{t-1},
\end{aligned}$$

$$\hat{c}_t \quad : \quad A_{22}(16,7) = -c^m$$
$$\widehat{\tilde{\imath}}_t \quad : \quad A_{22}(16,8) = -\tilde{\imath}^m$$
$$\hat{a}_t \quad : \quad A_{22}(16,16) = -1$$
$$\hat{\gamma}_t^{mcd} \quad : \quad A_{22}(16,17) = c^m \eta_c (1-\omega_c)\left(\gamma^{cd}\right)^{-(1-\eta_c)}$$
$$\hat{\gamma}_t^{mid} \quad : \quad A_{22}(16,18) = \tilde{\imath}^m \eta_i (1-\omega_i)\left(\gamma^{id}\right)^{-(1-\eta_i)}$$
$$\hat{\gamma}_t^{x*} \quad : \quad A_{22}(16,19) = -\left[\eta_f y_t^* + (c^m + \tilde{\imath}^m)\right]$$
$$\widehat{\mathrm{mc}}_t^x \quad : \quad A_{22}(16,21) = -\left[y^* + (c^m + \tilde{\imath}^m)\right]$$

$$\widehat{\tilde{z}}_t^* \quad : \quad A_{21}(16,14) = y^*$$
$$\hat{y}_t^* \quad : \quad A_{21}(16,31) = y^*$$
$$\hat{a}_{t-1} \quad : \quad A_{21}(16,60) = \frac{R}{\pi \mu_z}$$

Row 17 - Relative price - imported cons vs. domestic

$$0 = -\hat{\gamma}_t^{mcd} + \hat{\gamma}_{t-1}^{mcd} + \hat{\pi}_t^{mc} - \hat{\pi}_t^d$$

$$\hat{\pi}_t^d \quad : \quad A_{22}(17,1) = -1$$
$$\hat{\pi}_t^{mc} \quad : \quad A_{22}(17,2) = 1$$
$$\hat{\gamma}_t^{mcd} \quad : \quad A_{22}(17,17) = -1$$

$$\hat{\gamma}_{t-1}^{mcd} \quad : \quad A_{21}(17,61) = 1$$

Row 18 - Relative price - imported investment vs. domestic

$$0 = -\hat{\gamma}_t^{mid} + \hat{\gamma}_{t-1}^{mid} + \hat{\pi}_t^{mi} - \hat{\pi}_t^d$$

$$\hat{\pi}_t^d \quad : \quad A_{22}(18,1) = -1$$
$$\hat{\pi}_t^{mi} \quad : \quad A_{22}(18,3) = 1$$
$$\hat{\gamma}_t^{mid} \quad : \quad A_{22}(18,18) = -1$$

$$\hat{\gamma}_{t-1}^{mid} \quad : \quad A_{21}(18,62) = 1$$

Row 19 - Relative price - export vs. foreign

$$0 = -\hat{\gamma}_t^{x*} + \hat{\gamma}_{t-1}^{x*} + \hat{\pi}_t^x - \hat{\pi}_t^*$$

$$\hat{\pi}_t^x \quad : \quad A_{22}(19,5) = 1$$
$$\hat{\gamma}_t^{x*} \quad : \quad A_{22}(19,19) = -1$$
$$\hat{\pi}_t^* \quad : \quad A_{21}(19,30) = -1$$
$$\hat{\gamma}_{t-1}^{x*} \quad : \quad A_{21}(19,63) = 1$$

Row 20 - Real exchange rate

$$\begin{aligned}\widehat{\tilde{x}}_t &= -\omega_c(\gamma^{cmc})^{-(1-\eta_c)}\hat{\gamma}_t^{mcd} - \hat{\gamma}_t^{x,*} - \widehat{\mathrm{mc}}_t^x \\ 0 &= -\widehat{\tilde{x}}_t - \omega_c(\gamma^{cmc})^{-(1-\eta_c)}\hat{\gamma}_t^{mcd} - \hat{\gamma}_t^{x,*} - \widehat{\mathrm{mc}}_t^x\end{aligned}$$

$$\begin{aligned}\widehat{\mathrm{mc}}_t^x &= \widehat{\mathrm{mc}}_{t-1}^x + \widehat{\pi}_t^d - \widehat{\pi}_t^x - \Delta\widehat{S}_t \\ \hat{\gamma}_t^{x*} &= \hat{\gamma}_{t-1}^{x*} + \hat{\pi}_t^x - \hat{\pi}_t^* \\ \hat{\gamma}_t^{mcd} &= \hat{\gamma}_{t-1}^{mcd} + \hat{\pi}_t^{mc} - \hat{\pi}_t^d\end{aligned}$$

$$\begin{aligned}\hat{\gamma}_t^{mcd} &: A_{22}(20,17) = -\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\ \hat{\gamma}_t^{x,*} &: A_{22}(20,19) = -1 \\ \widehat{\tilde{x}}_t &: A_{22}(20,20) = -1 \\ \widehat{\mathrm{mc}}_t^x &: A_{22}(20,21) = -1\end{aligned}$$

Row 21 - Marginal cost export

$$0 = -\widehat{\mathrm{mc}}_t^x + \widehat{\mathrm{mc}}_{t-1}^x + \widehat{\pi}_t^d - \widehat{\pi}_t^x - \Delta\widehat{S}_t$$

$$\begin{aligned}\widehat{\pi}_t^d &: A_{22}(21,1) = 1 \\ \widehat{\pi}_t^x &: A_{22}(21,5) = -1 \\ \Delta\widehat{S}_t &: A_{22}(21,11) = -1 \\ \widehat{\mathrm{mc}}_t^x &: A_{22}(21,21) = -1\end{aligned}$$

$$\widehat{\mathrm{mc}}_{t-1}^x : A_{21}(21,65) = 1$$

Row 22 - Physical capital stock (note that $\widehat{\bar{k}}_{t+1}$ is predetermined variable no. 42 in period $t+1$ and forward-looking variable no. 22 in period t):

$$0 = -\widehat{\bar{k}}_{t+1} + (1-\tilde{\delta})\frac{1}{\mu_z}\widehat{\bar{k}}_t - (1-\tilde{\delta})\frac{1}{\mu_z}\hat{\mu}_{zt} + \left[1-(1-\tilde{\delta})\frac{1}{\mu_z}\right]\hat{\Upsilon}_t + \left[1-(1-\tilde{\delta})\frac{1}{\mu_z}\right]\widehat{\imath}_t,$$

$$\begin{aligned}\widehat{\bar{k}}_{t+1} &: A_{22}(22,22) = -1, \\ \widehat{\bar{k}}_t &: A_{21}(22,42) = (1-\tilde{\delta})\frac{1}{\mu_z}, \\ \hat{\mu}_{zt} &: A_{21}(22,3) = -(1-\tilde{\delta})\frac{1}{\mu_z}, \\ \hat{\Upsilon}_t &: \left[A_{21}(22,13) = 1-(1-\tilde{\delta})\frac{1}{\mu_z}\right], \\ \hat{\Upsilon}_t &: A_{21}(22,13) = \tilde{S}''\mu_z^2(1+\beta)\left(1-(1-\tilde{\delta})\frac{1}{\mu_z}\right) \quad \text{(rescaled shock!)} \\ \widehat{\imath}_t &: A_{22}(22,8) = 1-(1-\tilde{\delta})\frac{1}{\mu_z}.\end{aligned}$$

Row 23 - Financial assets (note that $\widehat{\bar{m}}_{t+1}$ is predetermined variable no. 43 in period $t+1$ and forward-looking variable no. 23 in period t)

$$0 = -\,\widehat{\bar{m}}_{t+1} + \hat{\mu}_t - \hat{\mu}_{zt} - \hat{\pi}_t^d + \widehat{\bar{m}}_t$$

$$\begin{aligned}
\widehat{\bar{m}}_{t+1} &: A_{22}(23,23) = -\,1 \\
\hat{\mu}_{zt} &: A_{21}(23,3) = -1 \\
\widehat{\bar{m}}_t &: A_{21}(23,43) = 1 \\
\hat{\pi}_t^d &: A_{22}(23,1) = -1 \\
\hat{\mu}_t &: A_{22}(23,15) = 1
\end{aligned}$$

1.3. Specifying D

We also specify the target variables, for convenience including some variables of interest which may have zero weight in the loss function. They are the quarterly and four-quarterly CPI inflation gaps relative to steady-state inflation, which equals the official inflation target; the output gap relative to steady-state output (we may also use the gap relative to potential output, meaning the flexprice and flexwage output level); the quarterly interest rate; the quarterly interest-rate differential; and the real exchange rate. The inflation and interest rates are all measured at an annual rate. Then the vector of target variables, Y_t, and the corresponding matrix D satisfy

$$Y_t \equiv [4\hat{\pi}_t^{cpi}, \overline{\hat{\pi}}_t^{cpi}, \hat{y}_t, 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \widehat{\bar{x}}_t]' \equiv D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix},$$

where

$$\overline{\hat{\pi}}_t^{cpi} = \hat{\pi}_t^{cpi} + \hat{\pi}_{t-1}^{cpi} + \hat{\pi}_{t-2}^{cpi} + \hat{\pi}_{t-3}^{cpi}.$$

Quarterly CPI inflation gap:

$$4\hat{\pi}_t^{cpi} = 4[(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}\widehat{\pi}_t^d + \omega_c(\gamma^{cmc})^{-(1-\eta_c)}\widehat{\pi}_t^{mc}], \tag{1.4}$$

$$\begin{aligned}
\widehat{\pi}_t^d &: D(1, n_X+1) = 4(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\widehat{\pi}_t^{mc} &: D(1, n_X+2) = 4\omega_c(\gamma^{cmc})^{-(1-\eta_c)}.
\end{aligned}$$

Four-quarter CPI inflation gap:

$$\overline{\hat{\pi}}_t^{cpi} = \hat{\pi}_t^{cpi} + \hat{\pi}_{t-1}^{cpi} + \hat{\pi}_{t-2}^{cpi} + \hat{\pi}_{t-3}^{cpi}. \tag{1.5}$$

$$\begin{aligned}
\hat{\pi}_t^d &: D(2, n_X+1) = (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\hat{\pi}_{t-1}^d &: D(2,45) = (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\hat{\pi}_{t-2}^d &: D(2,68) = (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\hat{\pi}_{t-3}^d &: D(2,70) = (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\widehat{\pi}_t^{mc} &: D(2, n_X+2) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}, \\
\widehat{\pi}_{t-1}^{mc} &: D(2,46) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}, \\
\widehat{\pi}_{t-2}^{mc} &: D(2,69) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}, \\
\widehat{\pi}_{t-3}^{mc} &: D(2,71) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}.
\end{aligned}$$

Alternatively, we could incorporate the exogenous time-varying inflation target in the inflation gap,[1]

$$Y_t \equiv [4(\hat{\pi}_t^{cpi} - \widehat{\bar{\pi}}_t^c), \bar{\pi}_t^{cpi} - 4\widehat{\bar{\pi}}_t^c, \hat{y}_t, 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \widehat{\bar{x}}_t]' \equiv D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix},$$

in which case

$$\begin{aligned} \widehat{\bar{\pi}}_t^c &: D(1,18) = -4, \\ \widehat{\bar{\pi}}_t^c &: D(2,18) = -4. \end{aligned}$$

Output gap,

$$\hat{y}_t : D(3, n_X + 4) = 1.$$

Interest rate,

$$\hat{R}_t : D(4, n_X + n_x + n_i) = 4.$$

Interest-rate differential,

$$\begin{aligned} \hat{R}_t &: D(5, n_X + n_x + n_i) = 4, \\ \hat{R}_{t-1} &: D(5, 44) = -4. \end{aligned}$$

Real exchange rate,

$$\widehat{\bar{x}}_t : D(6, n_X + 20) = 1.$$

1.4. Defining f_X and f_x - Interest rate rule

$$i_t = f_X \breve{X}_t + f_x \breve{x}_t$$

$$\begin{aligned} \hat{R}_t &= \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\widehat{\bar{\pi}}_t^c + r_\pi\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + r_y \hat{y}_{t-1} + r_x \widehat{\bar{x}}_{t-1}\right] \\ &\quad + r_{\Delta\pi}\left(\hat{\pi}_t^c - \hat{\pi}_{t-1}^c\right) + r_{\Delta y}\left(\hat{y}_t - \hat{y}_{t-1}\right) + \varepsilon_{Rt} \end{aligned}$$

$$\begin{aligned} \hat{R}_t &= \rho_R \hat{R}_{t-1} + (1-\rho_R)\widehat{\bar{\pi}}_t^c + (1-\rho_R) r_\pi \hat{\pi}_{t-1}^c - (1-\rho_R) r_\pi \widehat{\bar{\pi}}_t^c \\ &\quad + (1-\rho_R) r_y \hat{y}_{t-1} + (1-\rho_R) r_x \widehat{\bar{x}}_{t-1} + r_{\Delta\pi}\hat{\pi}_t^c - r_{\Delta\pi}\hat{\pi}_{t-1}^c + r_{\Delta y}\hat{y}_t - r_{\Delta y}\hat{y}_{t-1} + \varepsilon_{Rt} \\ &= \rho_R \hat{R}_{t-1} + (1-\rho_R)[1 - r_\pi]\widehat{\bar{\pi}}_t^c + r_{\Delta\pi}(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}\widehat{\pi}_t^d \\ &\quad + r_{\Delta\pi}(\omega_c)\left(\gamma^{cmc}\right)^{-(1-\eta_c)}\widehat{\pi}_t^{mc} \\ &\quad + [(1-\rho_R) r_\pi - r_{\Delta\pi}](1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}\widehat{\pi}_{t-1}^d \\ &\quad + [(1-\rho_R) r_\pi - r_{\Delta\pi}]\omega_c\left(\gamma^{cmc}\right)^{-(1-\eta_c)}\widehat{\pi}_{t-1}^{mc} \\ &\quad + r_{\Delta y}\hat{y}_t + [(1-\rho_R) r_y - r_{\Delta y}]\hat{y}_{t-1} + (1-\rho_R) r_x \widehat{\bar{x}}_{t-1} + \varepsilon_{Rt} \end{aligned}$$

[1] Note that one could alternative replace $4\widehat{\bar{\pi}}_t^c$ by $\widehat{\bar{\pi}}_t^c + \widehat{\bar{\pi}}_{t-1}^c + \widehat{\bar{\pi}}_{t-2}^c + \widehat{\bar{\pi}}_{t-3}^c$ and consider the latter the time-varying four-quarter inflation target.

$$
\begin{aligned}
\hat{R}_{t-1} &: f_X(1,44) = \rho_R \\
\widehat{\pi}_t^c &: f_X(1,18) = (1-\rho_R)[1-r_\pi] \\
\widehat{\pi}_t^d &: f_x(1,1) = r_{\Delta\pi}(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\widehat{\pi}_t^{mc} &: f_x(1,2) = r_{\Delta\pi}\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\
\widehat{\pi}_{t-1}^d &: f_X(1,45) = [(1-\rho_R)r_\pi - r_{\Delta\pi}](1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\widehat{\pi}_{t-1}^{mc} &: f_X(1,46) = [(1-\rho_R)r_\pi - r_{\Delta\pi}](\omega_c)(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{y}_t &: f_x(1,4) = r_{\Delta y} \\
\hat{y}_t^{flex} &: f_x(1,n_x+4) = -r_{\Delta y} \\
\hat{y}_{t-1} &: f_X(1,48) = (1-\rho_R)r_y - r_{\Delta y} \\
\hat{y}_{t-1}^{flex} &: f_X(1,n_X+48) = -((1-\rho_R)r_y - r_{\Delta y}) \\
\widehat{\tilde{x}}_{t-1} &: f_X(1,64) = (1-\rho_R)r_x \\
\varepsilon_{Rt} &: f_X(1,17) = 1
\end{aligned}
$$

2. Potential output under flexible prices and wages - expanding the state

The flexible price and wage model is parameterized under i) $\xi^d = \xi^{mc} = \xi^{mi} = \xi^x = \xi^w = 0$, ii) setting the four markup as well as the fiscal shocks to zero ($\sigma_{\varepsilon_\lambda} = \sigma_{\varepsilon_{\lambda^{mc}}} = \sigma_{\varepsilon_{\lambda^{mi}}} = \sigma_{\varepsilon_{\lambda^x}} = 0$, and $\Theta_0^{-1}S_\tau = 0_{5x5}$, respectively), and iii) we solve the system under the assumption that $\hat{\pi}^{cpi} = 0$. The flexprice model can be written

$$
\begin{bmatrix} X_{t+1} \\ H^f x_{t+1|t} \end{bmatrix} = A^f \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} + \begin{bmatrix} C \\ 0_{n_x \times n_\varepsilon} \\ 0_{n_i \times n_\varepsilon} \end{bmatrix} \varepsilon_{t+1}, \tag{2.1}
$$

where,

$$
H^f \equiv \begin{bmatrix} H \\ 0_{1\times n_x} \end{bmatrix}, \quad A^f \equiv \begin{bmatrix} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ 0_{1\times n_X} & e_1 & 0 \end{bmatrix},
$$

and

$$
\begin{aligned}
\widehat{\pi}_t^d &: e_1(1,1) = (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\widehat{\pi}_t^{mc} &: e_1(1,2) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}.
\end{aligned}
$$

This implies the following solution

$$
\begin{aligned}
X_t^{flex} &= M^{flex} X_{t-1}^{flex} + C^{flex}\varepsilon_t, \\
\tilde{x}_t^{flex} &= \tilde{F}^{flex}\tilde{X}_t^{flex}, \\
&= \tilde{F}^{flex} M^{flex} X_{t-1}^{flex} + \tilde{F}^{flex} C^{flex}\varepsilon_t,
\end{aligned}
$$

where $\tilde{x}_t^{flex} = \begin{bmatrix} x_t^{flex} \\ i_t^{flex} \end{bmatrix}$ and $\tilde{F}^{flex} = [\ F_x^{flex} \quad F_i^{flex}\ ]'$.

We expand the state of the distorted economy with the predetermined state variables in the flexprice model so that $\dot{X}_t = (X', X_t^{flex\prime})'$, which implies

$$
\dot{X}_{t+1} = \dot{A}_{11}\dot{X}_t + \dot{A}_{12}x_t + \dot{B}_1 i_t + \dot{C}\varepsilon_{t+1},
$$

$$\dot{H}x_{t+1|t} = \dot{A}_{21}\dot{X}_t + \dot{A}_{22}x_t + \dot{B}_2 i_t,$$

where $\dot{A}_{11} = \begin{bmatrix} A_{11} & 0_{nX*nX} \\ 0_{nX*nX} & M^{flex} \end{bmatrix}$, $\dot{A}_{12} = \begin{bmatrix} A_{12} \\ 0_{nX*nx} \end{bmatrix}$,

$\dot{A}_{21} = \begin{bmatrix} A_{21} & 0_{nx*nX} \end{bmatrix}$, $\dot{A}_{22} = A_{22}$, $\dot{B}_1 = \begin{bmatrix} B_1 \\ 0_{nX*ni} \end{bmatrix}$, $\dot{B}_2 = B_2$,

$\dot{C} = \begin{bmatrix} C \\ C^{flex} \end{bmatrix}$, and $\dot{H} = H$.

We also need to change the target variable matrix to include the flexprice output gap

$$Y_t \equiv [4\hat{\pi}_t^{cpi}, \overline{\hat{\pi}}_t^{cpi}, \overline{\hat{\pi}}_t^{d}, (y_t - y_t^{flex}), 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \hat{\overline{x}}_t]' \equiv \dot{D} \begin{bmatrix} X_t \\ X_t^{flex} \\ x_t \\ i_t \end{bmatrix}$$

.

$$\dot{D} = \begin{bmatrix} D_X & D_{X^{flex}} & D_x & D_i \end{bmatrix}',$$

where D above has been partitioned such that $D = \begin{bmatrix} D_X & D_x & D_i \end{bmatrix}'$ and $D_{X^{flex}}$ is defined so that

$$\hat{y}_t^{flex} : D_{X^{flex}}(4, n_X + 4) = -F_{x,4}^{flex}.$$

3. The measurement equation and the definition of the matrices $\bar{D}_0$, $\bar{D}$, and $\bar{D}_s$

Let $s_t \equiv (X_t', \Xi_{t-1}', x_t', i_t')'$ and call s_t the state of the economy (although it includes nonpredetermined variables). The solution to the model under optimal policy under commitment can be written in Klein form as

$$s_{t+1} \equiv \begin{bmatrix} \tilde{X}_{t+1} \\ x_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} M & 0 & 0 \\ f_x M & 0 & 0 \\ f_i M & 0 & 0 \end{bmatrix} s_t + \begin{bmatrix} I \\ f_x \\ f_i \end{bmatrix} \tilde{C}\varepsilon_{t+1}.$$

As noted in Adolfson et al. [1, Appendix C], it can also be written in AIM form as

$$s_{t+1} = \bar{B}s_t + \bar{C}\varepsilon_{t+1}, \tag{3.1}$$

where

$$\bar{C} \equiv \Phi\Psi$$

and the innovation $\bar{C}\varepsilon_t$ has the distribution $N(0, Q)$ with $Q \equiv \mathrm{E}[\bar{C}\varepsilon_{t+1}(\bar{C}\varepsilon_{t+1})'] = \bar{C}\bar{C}'$. Consider the state unobservable, let Z_t denote the n_Z-vector of observable variables, and let the measurement equation be

$$Z_t = \bar{D}_0 + \bar{D}_s s_t + \eta_t, \tag{3.2}$$

where η_t is an n_Z-vector of measurement errors with distribution $N(0, \Sigma_\eta)$ (the vector η_t here should not be confused with vector η of coefficients specified in (1.3)). We assume that the matrix R is diagonal with very small elements.[2] Equations (3.1) and (3.2) correspond to the state-space form for the derivation of the Kalman filter in Hamilton [3].

Note that the matrix $\bar{D}_s$ is related to the matrix $\bar{D} \equiv [\bar{D}_X \; \bar{D}_x \; \bar{D}_i]$ (partitioned conformably with X_t, x_t, and i_t) as

$$\bar{D}_s = [\bar{D}_X \;\; 0 \;\; \bar{D}_x \;\; \bar{D}_i],$$

since s_t includes the vector of predetermined variables Ξ_{t-1}.

[2] The role of the measurement errors in the estimation of the model and the state of the economy is further discussed in Adolfson et al. [2].

3.1. Difference specification

The vector of observable variables is

$$Z_t = \begin{bmatrix} \pi_t^d & \Delta\ln(W_t/P_t) & \Delta\ln C_t & \Delta\ln I_t & \widehat{\tilde{x}}_t & R_t & \hat{H}_t & ... \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & ... \\ \Delta y_t & \Delta\ln\tilde{X}_t & \Delta\ln\tilde{M}_t & \pi_t^{cpi} & \pi_t^{def,i} & \Delta\ln Y_t^* & \pi_t^* & R_t^* \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{bmatrix}' \tag{3.3}$$

and corresponds to the data used in estimating Ramses.

Next, we need specify how the data corresponds to the model variables:

$$\begin{aligned}
\pi_t^d &= 400(\pi-1)\pi + 4\pi\hat{\pi}_t^d, \\
\Delta\ln(W_t/P_t) &= 100\ln\mu_z + \Delta\widehat{\bar{w}}_t + \hat{\mu}_{zt}, \\
\Delta\ln C_t &= 100\ln\mu_z + \Delta\hat{c}_t + \frac{c}{c^d+c^m}\eta_c\frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right]\Delta\widehat{\gamma}_t^{mcd} + \hat{\mu}_{zt}, \\
\Delta\ln I_t &= 100\ln\mu_z + \Delta\widehat{\tilde{\imath}}_t + \frac{\tilde{\imath}}{\tilde{\imath}^d+\tilde{\imath}^m}\eta_i\frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[\left(\gamma^{i,mi}\right)^{-1} - \left(\gamma^{i,d}\right)^{-1}\right]\Delta\widehat{\gamma}_t^{mi,d} \\
&\quad + \left(1-\tau^k\right)r^k\bar{k}\frac{1}{\mu_z}\Delta\hat{u}_t + \hat{\mu}_{zt}, \\
\widehat{\tilde{x}}_t &= \widehat{\tilde{x}}_t, \\
R_t &= 400(R-1)R + 4R\hat{R}_t, \\
\hat{H}_t &= \hat{H}_t, \\
\Delta y_t &= 100\ln\mu_z + \Delta\hat{y}_t + \hat{\mu}_{zt}, \\
\Delta\ln\tilde{X}_t &= 100\ln\mu_z + \Delta\left[\hat{y}_t^* - \eta_f\widehat{\gamma}_t^{x,*} + \widehat{\tilde{z}}_t^*\right] + \hat{\mu}_{zt}, \\
\Delta\ln\tilde{M}_t &= 100\ln\mu_z + \frac{c^m}{c^m+\tilde{\imath}^m}\Delta\hat{c}_t - \frac{c^m}{c^m+\tilde{\imath}^m}\eta_c(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}\Delta\widehat{\gamma}_t^{mcd} \\
&\quad + \frac{\tilde{\imath}^m}{c^m+\tilde{\imath}^m}\Delta\widehat{\tilde{\imath}}_t - \frac{\tilde{\imath}^m}{c^m+\tilde{\imath}^m}\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\Delta\widehat{\gamma}_t^{mi,d} + \hat{\mu}_{zt}, \\
\pi_t^{cpi} &\equiv 400(\pi-1)\pi + 4\pi\hat{\pi}_t^{cpi} + 4\pi\frac{\tau^c}{1+\tau^c}\Delta\hat{\tau}_t^c \\
&= 400(\pi-1)\pi + 4\pi(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}\widehat{\pi}_t^d \\
&\quad + 4\pi\omega_c(\gamma^{cmc})^{-(1-\eta_c)}\widehat{\pi}_t^{mc} + 4\pi\frac{\tau^c}{1+\tau^c}\Delta\widehat{\tau}_t^c, \\
\pi_t^{def,i} &= 400(\pi-1)\pi + 4\pi\tilde{\imath}_{\pi^d}\hat{\pi}_t^d + 4\pi\tilde{\imath}_{\pi^m}\hat{\pi}_t^{mi} \\
&\quad + 4\pi\begin{bmatrix} \left(\tilde{\imath}_{\pi^d} - \frac{I^d}{I^d+I^m}\right)\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)} \\ -\left(\tilde{\imath}_{\pi^m} - \frac{I^m}{I^d+I^m}\right)\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)} \end{bmatrix}\Delta\widehat{\gamma}_t^{mi,d}, \\
\Delta\ln Y_t^* &= 100\ln\mu_z + \Delta\hat{y}_t^* + \Delta\widehat{\tilde{z}}_t^* + \hat{\mu}_{zt}, \\
\pi_t^* &= 400(\pi-1)\pi + 4\pi\hat{\pi}_t^*, \\
R_t^* &= 400(R-1)R + 4R\hat{R}_t^*.
\end{aligned} \tag{3.4}$$

Domestic inflation

$$\pi_t^d = 400(\pi-1)\pi + 4\pi\hat{\pi}_t^d,$$

$$\begin{aligned}\bar{D}_0(1,1) &= 400(\pi-1)\pi \\ \bar{D}_s\left(1, n_{\tilde{X}}+1\right) &= 4\pi\end{aligned}$$

Change in real wage

$$\Delta \ln(W_t/P_t) = 100 \ln \mu_z + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} + \hat{\mu}_{zt},$$

$$\begin{aligned}\bar{D}_0(2,1) &= 100 \ln \mu_z \\ \widehat{\bar{w}}_t &: \bar{D}_s\left(2, n_{\tilde{X}}+6\right) = 1 \\ \widehat{\bar{w}}_{t-1} &: \bar{D}_s\left(2, 50\right) = -1 \\ \hat{\mu}_{zt} &: \bar{D}_s\left(2, 3\right) = 1\end{aligned}$$

Change in real consumption

$$\begin{aligned}\Delta \ln C_t &= 100 \ln \mu_z + \hat{c}_t - \hat{c}_{t-1} + \frac{c}{c^d+c^m}\eta_c \frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right]\widehat{\gamma}_t^{mcd} \\ &\quad -\frac{c}{c^d+c^m}\eta_c \frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right]\widehat{\gamma}_{t-1}^{mcd} + \hat{\mu}_{zt},\end{aligned}$$

$$\begin{aligned}\bar{D}_0(3,1) &= 100 \ln \mu_z \\ \hat{c}_t &: \bar{D}_s\left(3, n_{\tilde{X}}+7\right) = 1 \\ \hat{c}_{t-1} &: \bar{D}_s\left(3, 51\right) = -1 \\ \widehat{\gamma}_t^{mcd} &: \bar{D}_s\left(3, n_{\tilde{X}}+17\right) = \frac{c}{c^d+c^m}\eta_c \frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right] \\ \widehat{\gamma}_{t-1}^{mcd} &: \bar{D}_s\left(3, 61\right) = -\frac{c}{c^d+c^m}\eta_c \frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right] \\ \hat{\mu}_{zt} &: \bar{D}_s\left(3, 3\right) = 1\end{aligned}$$

Change in real investment

$$\begin{aligned}\Delta \ln I_t &= 100 \ln \mu_z + \widehat{\tilde{\imath}}_t - \widehat{\tilde{\imath}}_{t-1} + \frac{\tilde{\imath}}{\tilde{\imath}^d+\tilde{\imath}^m}\eta_i \frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[(\gamma^{i,mi})^{-1} - \left(\gamma^{i,d}\right)^{-1}\right]\widehat{\gamma}_t^{mi,d} \\ &\quad -\frac{\tilde{\imath}}{\tilde{\imath}^d+\tilde{\imath}^m}\eta_i \frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[(\gamma^{i,mi})^{-1} - \left(\gamma^{i,d}\right)^{-1}\right]\widehat{\gamma}_{t-1}^{mi,d} \\ &\quad + \left(1-\tau^k\right) r^k \bar{k}\frac{1}{\mu_z}\hat{k}_t - \left(1-\tau^k\right) r^k \bar{k}\frac{1}{\mu_z}\widehat{\bar{k}}_t - \left(1-\tau^k\right) r^k \bar{k}\frac{1}{\mu_z}\hat{u}_{t-1} + \hat{\mu}_{zt}, \\ \hat{u}_t &= \hat{k}_t - \widehat{\bar{k}}_t\end{aligned}$$

$$\begin{aligned}
\bar{D}_0(4,1) &= 100\ln\mu_z \\
\widehat{\tilde{\imath}}_t &: \bar{D}_s\left(4, n_{\tilde{X}}+8\right) = 1 \\
\widehat{\tilde{\imath}}_{t-1} &: \bar{D}_s\left(4,52\right) = -1 \\
\widehat{\gamma}_t^{mi,d} &: \bar{D}_s\left(4, n_{\tilde{X}}+18\right) = \frac{\tilde{\imath}}{\tilde{\imath}^d+\tilde{\imath}^m}\eta_i\frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[\left(\gamma^{i,mi}\right)^{-1}-\left(\gamma^{i,d}\right)^{-1}\right] \\
\widehat{\gamma}_{t-1}^{mi,d} &: \bar{D}_s\left(4,62\right) = -\frac{\tilde{\imath}}{\tilde{\imath}^d+\tilde{\imath}^m}\eta_i\frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[\left(\gamma^{i,mi}\right)^{-1}-\left(\gamma^{i,d}\right)^{-1}\right] \\
\hat{k}_t &: \bar{D}_s\left(4, n_{\tilde{X}}+13\right) = \left(1-\tau^k\right)r^k\bar{k}\frac{1}{\mu_z} \\
\widehat{\bar{k}}_t &: \bar{D}_s\left(4,42\right) = -\left(1-\tau^k\right)r^k\bar{k}\frac{1}{\mu_z} \\
\hat{u}_{t-1} &: \bar{D}_s\left(4,67\right) = -\left(1-\tau^k\right)r^k\bar{k}\frac{1}{\mu_z} \\
\hat{\mu}_{zt} &: \bar{D}_s\left(4,3\right) = 1
\end{aligned}$$

Real exchange rate

$$\widehat{\tilde{x}}_t = \widehat{\tilde{x}}_t$$

$$\widehat{\tilde{x}}_t : \bar{D}_s\left(5, n_{\tilde{X}}+20\right) = 1$$

Interest rate

$$R_t = 400(R-1)R + 4R\hat{R}_t,$$

$$\begin{aligned}
\bar{D}_0(6,1) &= 400(R-1)R \\
\hat{R}_t &: \bar{D}_s\left(6, n_{\tilde{X}}+n_x+n_i\right) = 4R
\end{aligned}$$

Hours worked

$$\begin{aligned}
\hat{H}_t &= \hat{H}_t, \\
\hat{H}_t &: \bar{D}_s\left(7, n_{\tilde{X}}+12\right) = 1
\end{aligned}$$

Change in output (GDP)

$$\Delta y_t = 100\ln\mu_z + \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_{zt},$$

$$\begin{aligned}
\bar{D}_0(8,1) &= 100\ln\mu_z \\
\hat{y}_t &: \bar{D}_s\left(8, n_{\tilde{X}}+4\right) = 1 \\
\hat{y}_{t-1} &: \bar{D}_s\left(8,48\right) = -1 \\
\hat{\mu}_{zt} &: \bar{D}_s\left(8,3\right) = 1
\end{aligned}$$

Change in real exports

$$\Delta \ln \tilde{X}_t = 100 \ln \mu_z + \hat{y}_t^* - \hat{y}_{t-1}^* - \eta_f \widehat{\gamma}_t^{x,*} + \eta_f \widehat{\gamma}_{t-1}^{x,*} + \widehat{\tilde{z}}_t^* - \widehat{\tilde{z}}_{t-1}^* + \hat{\mu}_{zt},$$

$$\begin{aligned}
\bar{D}_0(9,1) &= 100 \ln \mu_z \\
\hat{y}_t^* &: \bar{D}_s(9,31) = 1 \\
\hat{y}_{t-1}^* &: \bar{D}_s(9,34) = -1 \\
\widehat{\gamma}_t^{x,*} &: \bar{D}_s\left(9, n_{\tilde{X}} + 19\right) = -\eta_f \\
\widehat{\gamma}_{t-1}^{x,*} &: \bar{D}_s(9,63) = \eta_f \\
\widehat{\tilde{z}}_t^* &: \bar{D}_s(9,14) = 1 \\
\widehat{\tilde{z}}_{t-1}^* &: \bar{D}_s(9,15) = -1 \\
\hat{\mu}_{zt} &: \bar{D}_s(9,3) = 1
\end{aligned}$$

Change in real imports

$$\begin{aligned}
\Delta \ln \tilde{M}_t = {} & 100 \ln \mu_z + \frac{c^m}{c^m + \tilde{\imath}^m} \hat{c}_t - \frac{c^m}{c^m + \tilde{\imath}^m} \hat{c}_{t-1} - \frac{c^m}{c^m + \tilde{\imath}^m} \eta_c (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)} \widehat{\gamma}_t^{mcd} \\
& + \frac{c^m}{c^m + \tilde{\imath}^m} \eta_c (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)} \widehat{\gamma}_{t-1}^{mcd} + \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \widehat{\tilde{\imath}}_t - \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \widehat{\tilde{\imath}}_{t-1} \\
& - \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \eta_i (1-\omega_i) \left(\gamma^{i,d}\right)^{-(1-\eta_i)} \widehat{\gamma}_t^{mi,d} + \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \eta_i (1-\omega_i) \left(\gamma^{i,d}\right)^{-(1-\eta_i)} \widehat{\gamma}_{t-1}^{mi,d} + \hat{\mu}_{zt},
\end{aligned}$$

$$\begin{aligned}
\bar{D}_0(10,1) &= 100 \ln \mu_z \\
\hat{c}_t &: \bar{D}_s\left(10, n_{\tilde{X}} + 7\right) = \frac{c^m}{c^m + \tilde{\imath}^m} \\
\hat{c}_{t-1} &: \bar{D}_s(10,51) = -\frac{c^m}{c^m + \tilde{\imath}^m} \\
\widehat{\gamma}_t^{mcd} &: \bar{D}_s\left(10, n_{\tilde{X}} + 17\right) = -\frac{c^m}{c^m + \tilde{\imath}^m} \eta_c (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\widehat{\gamma}_{t-1}^{mcd} &: \bar{D}_s(10,61) = \frac{c^m}{c^m + \tilde{\imath}^m} \eta_c (1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\widehat{\tilde{\imath}}_t &: \bar{D}_s\left(10, n_{\tilde{X}} + 8\right) = \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \\
\widehat{\tilde{\imath}}_{t-1} &: \bar{D}_s(10,52) = -\frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \\
\widehat{\gamma}_t^{mi,d} &: \bar{D}_s\left(10, n_{\tilde{X}} + 18\right) = -\frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \eta_i (1-\omega_i) \left(\gamma^{i,d}\right)^{-(1-\eta_i)} \\
\widehat{\gamma}_{t-1}^{mi,d} &: \bar{D}_s(10,62) = \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m} \eta_i (1-\omega_i) \left(\gamma^{i,d}\right)^{-(1-\eta_i)} \\
\hat{\mu}_{zt} &: \bar{D}_s(10,3) = 1
\end{aligned}$$

CPI inflation

$$\begin{aligned}
\pi_t^{cpi} = {} & 400(\pi - 1)\pi + 4\pi(1-\omega_c)\widehat{\pi}_t^d \\
& + 4\pi\omega_c \widehat{\pi}_t^{mc} + 4\pi \frac{\tau^c}{1+\tau^c} \hat{\tau}_t^c - 4\pi \frac{\tau^c}{1+\tau^c} \hat{\tau}_{t-1}^c,
\end{aligned}$$

$$
\begin{aligned}
\bar{D}_0(11,1) &= 400(\pi-1)\pi \\
\widehat{\pi}_t^d &: \bar{D}_s\left(11, n_{\tilde{X}}+1\right) = 4\pi(1-\omega_c) \\
\widehat{\pi}_t^{mc} &: \bar{D}_s\left(11, n_{\tilde{X}}+2\right) = 4\pi\omega_c \\
\hat{\tau}_t^c &: \bar{D}_s(11,22) = 4\pi\frac{\tau^c}{1+\tau^c} \\
\hat{\tau}_{t-1}^c &: \bar{D}_s(11,27) = -4\pi\frac{\tau^c}{1+\tau^c}
\end{aligned}
$$

Inflation investment deflator

$$
\begin{aligned}
\widehat{\pi}_t^{def,i} &= \frac{\tilde{\imath}^d}{\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m}\widehat{\pi}_t^d + \frac{c^m}{\left(c^d+\frac{\eta^{mc}}{\eta^{mc}-1}c^m\right)}\frac{\eta^{mc}}{\eta^{mc}-1}\widehat{\pi}_t^{mc} \\
&+ \left[\begin{array}{l}\left(\frac{\tilde{\imath}^d}{\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m} - \frac{\tilde{\imath}^d}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)} \\ -\left(\frac{\tilde{\imath}^m}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)}\frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{\imath}^m}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\end{array}\right]\Delta\widehat{\gamma}_t^{mcd}.
\end{aligned}
$$

$$
\Delta\widehat{\gamma}_t^{mi,d} = \widehat{\pi}_t^{mi} - \widehat{\pi}_t^d
$$

$$
\begin{aligned}
\widehat{\pi}_t^{def,i} &= 400(\pi-1)\pi \\
&+4\pi\left\{\begin{array}{l}\frac{\tilde{\imath}^d}{\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m} - \left(\frac{\tilde{\imath}^d}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)} - \frac{\tilde{\imath}^d}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)} \\ +\left(\frac{\tilde{\imath}^m}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)}\frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{\imath}^m}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\end{array}\right\}\widehat{\pi}_t^d \\
&+4\pi\left\{\begin{array}{l}\frac{\tilde{\imath}^m}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)}\frac{\eta^{mi}}{\eta^{mi}-1} + \left(\frac{\tilde{\imath}^d}{\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m} - \frac{\tilde{\imath}^d}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)} \\ -\left(\frac{\tilde{\imath}^m}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)}\frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{\imath}^m}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\end{array}\right\}\widehat{\pi}_t^{mi}.
\end{aligned}
$$

$$
\begin{aligned}
\bar{D}_0(12,1) &= 400(\pi-1)\pi \\
\widehat{\pi}_t^d &: \bar{D}_s\left(12, n_{\tilde{X}}+1\right) = 4\pi\left\{\begin{array}{l}\frac{\tilde{\imath}^d}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)} \\ -\left(\frac{\tilde{\imath}^d}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)} - \frac{\tilde{\imath}^d}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)} \\ +\left(\frac{\tilde{\imath}^m}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)}\frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{\imath}^m}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\end{array}\right\} \\
\hat{\pi}_t^{mi} &: \bar{D}_s\left(12, n_{\tilde{X}}+3\right) = 4\pi\left\{\begin{array}{l}\frac{\tilde{\imath}^m}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)}\frac{\eta^{mi}}{\eta^{mi}-1} \\ +\left(\frac{\tilde{\imath}^d}{\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m} - \frac{\tilde{\imath}^d}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i\omega_i\left(\gamma^{i,mi}\right)^{-(1-\eta_i)} \\ -\left(\frac{\tilde{\imath}^m}{\left(\tilde{\imath}^d+\frac{\eta^{mi}}{\eta^{mi}-1}\tilde{\imath}^m\right)}\frac{\eta^{mi}}{\eta^{mi}-1} - \frac{\tilde{\imath}^m}{\tilde{\imath}^d+\tilde{\imath}^m}\right)\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\end{array}\right\}
\end{aligned}
$$

Foreign output

$$\Delta \ln Y_t^* = 100 \ln \mu_z + \hat{y}_t^* - \hat{y}_{t-1}^* + \widehat{\tilde{z}}_t^* - \widehat{\tilde{z}}_{t-1}^* + \hat{\mu}_{zt},$$

$$\begin{aligned}
\bar{D}_0(13,1) &= 100 \ln \mu_z \\
\hat{\mu}_{zt} &: \bar{D}_s(13,3) = 1 \\
\widehat{\tilde{z}}_t^* &: \bar{D}_s(13,14) = 1 \\
\widehat{\tilde{z}}_{t-1}^* &: \bar{D}_s(13,15) = -1 \\
\hat{y}_t^* &: \bar{D}_s(13,31) = 1 \\
\hat{y}_{t-1}^* &: \bar{D}_s(13,34) = -1
\end{aligned}$$

Foreign inflation

$$\pi_t^* = 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^*,$$

$$\begin{aligned}
\bar{D}_0(14,1) &= 400(\pi - 1)\pi \\
\hat{\pi}_t^* &: \bar{D}_s(14,30) = 4\pi
\end{aligned}$$

Foreign interest rate

$$R_t^* = 400(R - 1)R + 4R\hat{R}_t^*.$$

$$\begin{aligned}
\bar{D}_0(15,1) &= 400(R - 1)R \\
\hat{\pi}_t^* &: \bar{D}_s(15,32) = 4R
\end{aligned}$$

3.2. Cointegration specification

The vector of observable variables is

$$Z_t = \left[\begin{array}{ccccccccc}
\pi_t^d & \ln(W_t/P_t) - \ln Y_t & \ln C_t - \ln Y_t & \ln I_t - \ln Y_t & \widehat{\tilde{x}}_t & R_t & \hat{H}_t & \ldots \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots \\
\Delta \ln Y_t & \ln \tilde{X}_t - \ln Y_t & \ln \tilde{M}_t - \ln Y_t & \pi_t^{cpi} & \pi_t^{def,i} & \ln Y_t^* - \ln Y_t & \pi_t^* & R_t^* \\
8 & 9 & 10 & 11 & 12 & 13 & 14 & 15
\end{array}\right]' \tag{3.5}$$

and corresponds to the data used in estimating Ramses. However, it should be noted that in the current version of [1] we are only using the difference specification in equation (3.3).

Next, we need to specify how the cointegrated data corresponds to the model variables (see the section above for the specification of the non-cointegrated variables):

$$\begin{aligned}
\ln(W_t/P_t) - \ln Y_t &= \widehat{\bar{w}}_t - \hat{y}_t, \\
\ln C_t - \ln Y_t &= \hat{c}_t + \frac{c}{c^d + c^m}\eta_c \frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right]\widehat{\gamma}_t^{mcd} - \hat{y}_t, \\
\ln I_t - \ln Y_t &= \widehat{\tilde{\imath}}_t + \frac{\tilde{\imath}}{\tilde{\imath}^d + \tilde{\imath}^m}\eta_i \frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[\left(\gamma^{i,mi}\right)^{-1} - \left(\gamma^{i,d}\right)^{-1}\right]\widehat{\gamma}_t^{mi,d} + \left(1 - \tau^k\right) r^k \bar{k} \frac{1}{\mu_z}\hat{u}_t - \hat{y}_t, \\
\ln \tilde{X}_t - \ln Y_t &= \hat{y}_t^* - \eta_f \widehat{\gamma}_t^{x,*} + \widehat{\tilde{z}}_t^* - \hat{y}_t, \\
\ln \tilde{M}_t - \ln Y_t &= \frac{c^m}{c^m + \tilde{\imath}^m}\hat{c}_t - \frac{c^m}{c^m + \tilde{\imath}^m}\eta_c(1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}\widehat{\gamma}_t^{mcd} \\
&\quad + \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m}\widehat{\tilde{\imath}}_t - \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m}\eta_i(1 - \omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\widehat{\gamma}_t^{mi,d} - \hat{y}_t, \qquad (3.6) \\
\ln Y_t^* - \ln Y_t &= \hat{y}_t^* + \widehat{\tilde{z}}_t^* - \hat{y}_t.
\end{aligned}$$

Real wage

$$\ln(W_t/P_t) = \widehat{\bar{w}}_t - \hat{y}_t,$$

$$\begin{aligned}
\widehat{\bar{w}}_t &: \quad \bar{D}_s\left(2, n_{\tilde{X}} + 6\right) = 1, \\
y_t &: \quad \bar{D}_s\left(2, n_{\tilde{X}} + 4\right) = -1.
\end{aligned}$$

Real consumption

$$\ln C_t - y_t = \hat{c}_t + \frac{c}{c^d + c^m}\eta_c \frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right]\widehat{\gamma}_t^{mcd} - y_t,$$

$$\begin{aligned}
\hat{c}_t &: \quad \bar{D}_s\left(3, n_{\tilde{X}} + 7\right) = 1, \\
\widehat{\gamma}_t^{mcd} &: \quad \bar{D}_s\left(3, n_{\tilde{X}} + 17\right) = \frac{c}{c^d + c^m}\eta_c \frac{c^d}{c}\frac{c^m}{c}\left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1}\right], \\
\hat{y}_t &: \quad \bar{D}_s\left(3, n_{\tilde{X}} + 4\right) = -1.
\end{aligned}$$

Real investment

$$\begin{aligned}
\ln I_t - \ln Y_t &= \widehat{\tilde{\imath}}_t + \frac{\tilde{\imath}}{\tilde{\imath}^d + \tilde{\imath}^m}\eta_i \frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[\left(\gamma^{i,mi}\right)^{-1} - \left(\gamma^{i,d}\right)^{-1}\right]\widehat{\gamma}_t^{mi,d} + \left(1 - \tau^k\right) r^k \bar{k}\frac{1}{\mu_z}\hat{k}_t - \left(1 - \tau^k\right) r^k \bar{k}\frac{1}{\mu_z}\widehat{\bar{k}}_t - \hat{y}_t \\
\hat{u}_t &= \hat{k}_t - \widehat{\bar{k}}_t,
\end{aligned}$$

$$\begin{aligned}
\widehat{\tilde{\imath}}_t &: \quad \bar{D}_s\left(4, n_{\tilde{X}} + 8\right) = 1, \\
\widehat{\gamma}_t^{mi,d} &: \quad \bar{D}_s\left(4, n_{\tilde{X}} + 18\right) = \frac{\tilde{\imath}}{\tilde{\imath}^d + \tilde{\imath}^m}\eta_i \frac{\tilde{\imath}^d}{\tilde{\imath}}\frac{\tilde{\imath}^m}{\tilde{\imath}}\left[\left(\gamma^{i,mi}\right)^{-1} - \left(\gamma^{i,d}\right)^{-1}\right], \\
\hat{k}_t &: \quad \bar{D}_s\left(4, n_{\tilde{X}} + 13\right) = \left(1 - \tau^k\right) r^k \bar{k}\frac{1}{\mu_z}, \\
\widehat{\bar{k}}_t &: \quad \bar{D}_s(4, 42) = -\left(1 - \tau^k\right) r^k \bar{k}\frac{1}{\mu_z}, \\
\hat{y}_t &: \quad \bar{D}_s\left(4, n_{\tilde{X}} + 4\right) = -1.
\end{aligned}$$

Real exports

$$\ln \tilde{X}_t - \ln Y_t = \hat{y}_t^* - \eta_f \widehat{\gamma}_t^{x,*} + \widehat{\tilde{z}}_t^* - \hat{y}_t,$$

$$\begin{aligned}
\hat{y}_t^* &: \quad \bar{D}_s(9, 31) = 1, \\
\widehat{\gamma}_t^{x,*} &: \quad \bar{D}_s\left(9, n_{\tilde{X}} + 19\right) = -\eta_f, \\
\widehat{\tilde{z}}_t^* &: \quad \bar{D}_s(9, 14) = 1, \\
\hat{y}_t &: \quad \bar{D}_s\left(9, n_{\tilde{X}} + 4\right) = -1.
\end{aligned}$$

Real imports

$$\begin{aligned}\ln \tilde{M}_t - \ln Y_t &= \frac{c^m}{c^m + \tilde{\imath}^m}\hat{c}_t - \frac{c^m}{c^m + \tilde{\imath}^m}\eta_c(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}\hat{\gamma}_t^{mcd} \\ &+ \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m}\hat{\imath}_t - \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m}\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\hat{\gamma}_t^{mi,d} - \hat{y}_t,\end{aligned}$$

$$\begin{aligned}\hat{c}_t &: \bar{D}_s\left(10, n_{\tilde{X}}+7\right) = \frac{c^m}{c^m + \tilde{\imath}^m}, \\ \hat{\gamma}_t^{mcd} &: \bar{D}_s\left(10, n_{\tilde{X}}+17\right) = -\frac{c^m}{c^m + \tilde{\imath}^m}\eta_c(1-\omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\ \hat{\imath}_t &: \bar{D}_s\left(10, n_{\tilde{X}}+8\right) = \frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m}, \\ \hat{\gamma}_t^{mi,d} &: \bar{D}_s\left(10, n_{\tilde{X}}+18\right) = -\frac{\tilde{\imath}^m}{c^m + \tilde{\imath}^m}\eta_i(1-\omega_i)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}, \\ \hat{y}_t &: \bar{D}_s\left(10, n_{\tilde{X}}+4\right) = -1.\end{aligned}$$

Foreign output

$$\ln Y_t^* - \ln Y_t = \hat{y}_t^* + \hat{\tilde{z}}_t^* - \hat{y}_t,$$

$$\begin{aligned}\hat{\tilde{z}}_t^* &: \bar{D}_s(13,14) = 1, \\ \hat{y}_t^* &: \bar{D}_s(13,31) = 1, \\ \hat{y}_t &: \bar{D}_s\left(13, n_{\tilde{X}}+4\right) = -1.\end{aligned}$$

3.3. Smoothed estimates

The Kalman filter calculates a forecast of the state vector s_t, $s_{t|t-1}$, as a linear function of previous observations,

$$s_{t|t-1} \equiv \mathrm{E}[s_t|\mathcal{Z}_{t-1}], \tag{3.7}$$

where $\mathcal{Z}_{t-1} \equiv (Z_{t-1}, Z_{t-2}, ..., Z_1)$. The matrix $P_{t|t-1}$ represents the MSE of this forecast

$$P_{t|t-1} \equiv \mathrm{E}\left[\left(s_t - s_{t|t-1}\right)\left(s_t - s_{t|t-1}\right)'|\mathcal{Z}_{t-1}\right].$$

The key equations of the Kalman filter are

$$s_{t|t} = s_{t|t-1} + P_{t|t-1}\bar{D}_s'\left(\bar{D}_s P_{t|t-1}\bar{D}_s' + \Sigma_\eta\right)^{-1}\left(Z_t - \bar{D}_0 - \bar{D}_s s_{t|t-1}\right), \tag{3.8}$$

$$s_{t+1|t} = \bar{B}s_{t|t}, \tag{3.9}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}\bar{D}_s'\left(\bar{D}_s P_{t|t-1}\bar{D}_s' + \Sigma_\eta\right)^{-1}\bar{D}_s P_{t|t-1}, \tag{3.10}$$

$$P_{t+1|t} = \bar{B}P_{t|t}\bar{B}' + Q. \tag{3.11}$$

The smoothed estimates of s_t, $s_{t|T}$ for $t \leq T$, are denoted

$$s_{t|T} \equiv \mathrm{E}[s_t|\mathcal{Z}_T]. \tag{3.12}$$

The sequence of smoothed estimates $\left\{s_{t|T}\right\}_{t=1}^{T}$ can be calculated by first calculating the sequences $\left\{s_{t|t}\right\}_{t=1}^{T}$, $\left\{s_{t+1|t}\right\}_{t=1}^{T-1}$, $\left\{P_{t|t}\right\}_{t=1}^{T}$ and $\left\{P_{t+1|t}\right\}_{t=1}^{T-1}$ and then calculate

$$s_{t|T} = s_{t|t} + P_{t|t}\bar{B}'P_{t+1|t}^{-1}\left(s_{t+1|T} - s_{t+1|t}\right) \tag{3.13}$$

by iterating backward through the sample for $t = T-1, T-2, ..., 1$ (Hamilton [3, chapt. 13.6]).

References

[1] Adolfson, Malin, Stefan Laséen, Jesper Lindé, and Lars E.O. Svensson (2008), "Optimal Monetary Policy in an Operational Medium-Sized DSGE Model," NBER Working Paper No. 14092.

[2] Adolfson, Malin, Stefan Laséen, Jesper Lindé, and Mattias Villani (2006), "Evaluating an Estimated New Keynesian Small Open Economy Model", working paper, www.riksbank.se, *Journal of Economic Dynamics and Control*, forthcoming .

[3] Hamilton, James D. (1994), *Time Series Analysis*, Princeton University Press.