Based on BGG(1999)

Gertler (2007) External Constraints on Monetary Policy and the Financial Accelerator,

1.Households

C_t be a composite of consumption goods.,following CES index defines household preferences over home consumption,C_{Ht} , and foreign consumption, C_{Ft} :

$$C_t = [\gamma^{\frac{1}{\rho}} (C_t^h)^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}} (C_t^f)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}$$

Solving the above problem, an expression linking the relative prices of domestic and foreign goods with their quantities as well as a consumer price index are obtained.:

$$\frac{C_t^h}{C_t^f} = (\frac{\gamma}{1-\gamma})(\frac{P_t^h}{P_t^f})^{-\rho}$$

The corresponding consumer price index (CPI), P_t, is given by:

$$P_t = [\gamma (P_t^h)^{1-\rho} + (1-\gamma)(P_t^f)^{1-\rho}]^{\frac{1}{1-\rho}}$$

Households maximize their discounted utility:

$$\max E_t \sum_{k=0}^{\infty} \beta^k [\ln(C_{t+k} - bC_{t+k-1}) + \zeta \ln(\frac{M_{t+k}}{P_{t+k}}) + \xi \ln(1-H_{t+k})]$$

s.t.:

$$C_t = \frac{W_t}{P_t} H_t + \prod{}^R{}_t - T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{B_{t+1} - (1+rn_t)B_t}{P_t} - \frac{S_t B_{t+1}^* - (1+rn^*{}_t)B_t^*}{P_t}$$

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First Order Conditions:

$$\lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta \frac{b}{C_{t+1} - bC_t}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} (1+rn_{t+1}) \frac{P_t}{P_{t+1}} \right\}$$

$$\lambda_t \frac{W_t}{P_t} = \xi \frac{1}{1-H_t}$$

$$E_t \left\{ \lambda_{t+1} \frac{P_t}{P_{t+1}} [(1+rn_{t+1}) - (1+rn_{t+1}^*) \frac{S_{t+1}}{S_t}] \right\} = 0$$

$$\lambda_t \frac{M_t}{P_t} (1 - \frac{1}{1+rn_t}) = \zeta$$

Let $P_{FW,t}$ denote the wholesale price of foreign goods in domestic currency, and P_{F*t} the foreign currency price of such goods. The law of one price then implies:：

$$P_{w,t}^{f} = S_t P_t^{f*}$$

Log-linearisation:

$$c_t = \gamma c_t^h + (1-\gamma) c_t^f \quad 1$$

$$c_t^h = c_t^f - \rho(p_t^h - p_t^f) \quad 2$$

$$p_t = \gamma p_t^h + (1-\gamma) p_t^f \quad 3$$

$$\hat{\lambda}_t = \frac{1}{(1-b)}[-\hat{c}_t + b\hat{c}_{t-1}] \quad 4$$

$$\hat{\lambda}_{t+1} + rn_{t+1} - \pi_{t+1} = \hat{\lambda}_t \quad 5$$

$$w_t - p_t = -\hat{\lambda}_t + \eta^{-1} h_t \qquad \eta^{-1} = \frac{H}{1-H} \quad 6$$

$$rn_t = s_{t+1} - s_t + rn_t^* \quad 7$$

$$m_t = -\hat{\lambda}_t - \frac{1}{1/\beta - 1} rn_t \quad 8$$

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$$p_{w,t}^f = s_t + p_t^{f*} \quad 9$$

foreign demand for the home tradable good, C_{h*}

$$C_t^{h*} = \left(\frac{P_t^{h*}}{P_t^*}\right)^{-\rho} Y_t^*$$

$$c_t^{h*} = -\rho(p_t^{h*} - p_t^*) + y_t^* \quad 10$$

2. *Firms*

2.1 capital producers：

the index of total investment combines goods produced domestically and imported from abroad.：

$$I_t = [\gamma_I^{\frac{1}{\rho_I}} (I_t^h)^{\frac{\rho_I - 1}{\rho_I}} + (1-\gamma_I)^{\frac{1}{\rho_I}} (I_t^f)^{\frac{\rho_I - 1}{\rho_I}}]^{\frac{\rho_I}{\rho_I - 1}}$$

link between relative prices and quantities of domestic and imported goods is obtained

$$\frac{I_t^h}{I_t^f} = \left(\frac{\gamma_I}{1-\gamma_I}\right)\left(\frac{P_t^h}{P_t^f}\right)^{-\rho_I}$$

$$P_{I,t} = [\gamma_I (P_t^h)^{1-\rho_I} + (1-\gamma_I)(P_t^f)^{1-\rho_I}]^{\frac{1}{1-\rho_I}}$$

Capital production technology is described by the following expression：

$$K_{t+1}=\Phi(\frac{I_t}{K_t})K_t+(1\text{-}\delta)K_t$$

The profit realized by capital producers is：

$$Q_t\Phi(\frac{I_t}{K_t})K_t-(\frac{P_{It}}{P_t})I_t$$

FOC：

$$E_{t-1}\left\{Q_t\Phi'(\frac{I_t}{K_t})-(\frac{P_{It}}{P_t})\right\}=0$$

Log-linearisation:

$$i_t=\gamma_I i_t^h+(1-\gamma_I)i_t^f \qquad 11$$

$$i_t^h=-\rho_I(p_t^h-p_t^f)+i_t^f \qquad 12$$

$$p_{I,t}=\gamma_I p_t^h+(1-\gamma_I)p_t^f \qquad 13$$

$$k_{t+1}=\delta i_t+(1-\delta)k_t \qquad 14$$

$$q_t+p_t-p_{I,t}=\varphi(i_t-k_t) \qquad \varphi=\frac{\Phi''(I\ /K\ )}{\Phi'(I\ /K\ )}\frac{I}{K} \qquad 15$$

2.*Entrepreneurs*

$$Y_t=A_tK_t^{\alpha}L_t^{1\text{-}\alpha}$$

$$\ln A_t-\ln(\overline{A})=\rho^A(\ln A_{t-1}-\ln(\overline{A}))+\varepsilon_t^a$$

$$L_t=H_t^{\Omega}(H_t^e)^{1\text{-}\Omega}$$

Entrepreneurs employ labour to equate the marginal productivity and the average wage . Accordingly, labor demand satisfies:

$$(1\text{-}\alpha)\Omega\frac{Y_t}{H_t}=X_t^h\frac{W_t}{P_t}$$

$$(1\text{-}\alpha)(1\text{-}\Omega)\frac{Y_t}{H_t^e}=X_t^h\frac{W_t^e}{P_t}$$

P_t^w is the wholesale price. $\qquad P_t^w=\frac{P_t}{X_t^h}$

The budget constraint for capital accumulation：

$$Q_tK_{t+1}=N_{t+1}+B_{t+1}$$

The expected return on capital $\{1+r_{t+1}^k\}$

$$E_t(1+r_{t+1}^k)=E_t\left\{\frac{\frac{1}{X_{t+1}^h}\frac{\alpha Y_{t+1}}{K_{t+1}}+Q_{t+1}(1\text{-}\delta)}{Q_t}\right\}$$

The risk premium between the expected return and the economy-wide free interest rate depends on how heavily the firm is leveraged.

$$\frac{Q_t K_{t+1}}{N_{t+1}}=\varphi^r\left(\frac{E_t(1+r_{t+1}^k)}{1+r_{t+1}}\right)$$

Fisher equation:

$$1+r_{t+1}=(1+rn_{t+1})\frac{P_t}{P_{t+1}}$$

net worth of the firm:

$$N_{t+1}=\upsilon_t^n\left[\gamma V_t\right]+\frac{W_t^e}{P_t}$$

$$\ln\upsilon_t^n=\rho^\upsilon\ln\upsilon_{t-1}^n+\varepsilon_t^{\upsilon^n}$$

$$N_{t+1}=\upsilon_t^n\gamma\left[R_t^k Q_{t\text{-}1}K_t\text{-}R_t(\mathrm{Q}_{t\text{-}1}K_t\text{-}N_t)\right]+\frac{W_t^e}{P_t}$$

Log-linearisation:

$$y_t=a_t+\alpha k_t+(1-\alpha)\Omega h_t \qquad 16$$

$$y_t-h_t-x_t^h=w_t-p_t \qquad 17$$

$$y_t-x_t^h=w_t^e-p_t \qquad 18$$

$$E_t\left\{r_{t+1}^k\right\}=(1-\chi)(y_{t+1}-k_{t+1}-x_{t+1})+\chi q_{t+1}-q_t,\ \chi=(1-\delta)/R^k \qquad 19$$

$$E_t\left\{r_{t+1}^k\right\}-r_{t+1}=-\chi_r\left[n_{t+1}-(q_t+k_{t+1})\right]\quad \chi_r=\frac{\varphi_r(R^k/R)}{\varphi_r'(R^k/R)}\left(\frac{R^k}{R}\right)^{-1} \qquad 20$$

$$r_{t+1}=i_{t+1}-\pi_{t+1} \qquad 21$$

$$n_{t+1}=\frac{\gamma R^k K}{N}(r_t^k-r_t)+\gamma R(\hat{v}_t^n+r_t+n_t) \qquad 22$$

Retailers, price setting, and inflation

Calvo-style price setting

Let $Y_{Ht}(i)$ be the good sold by retailer z. Final domestic output is a CES composite of individual retail goods:

$$Y_t^h = \left[\int_0^1 Y_t^h(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$Y_t^h(z) = (\frac{P_i^h(i)}{P_t^h})^{-\epsilon} Y_t^h$$

$$P_t^h = \left[\theta (P_{t-1}^h)^{1-\epsilon} + (1-\theta)(P_{t-1}^{ho})^{1-\epsilon} \right]^{1/(1-\epsilon)}$$

$$\max \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,k} \frac{P_t^{ho} - P_{t+k}^w}{P_{t+k}} Y_{t+k}^{ho}(z) \right\}$$

$$P_t^w = \frac{P_t^h}{X_t^h}$$

s inflation rate for domestically produced goods

$$\pi_t^h = \beta E_t \left\{ \pi_{t+1}^h \right\} - \kappa x_t^h \qquad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \qquad 23$$

The inflation rate for foreign goods then satisfies：

$$\pi_t^f = \beta E_t \left\{ \pi_{t+1}^f \right\} - \kappa x_t^f \ ,$$

$$X_t^f = \frac{S_t P_t^{f*}}{P_t^f}$$

We assume that the foreign price level p_t^*, the foreign output y_t^* and the foreign interest rate rn_t^* are exogenous and follow an AR(1)process given in log-linearized form:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Where $z_t = \left\{ p_t^*, y_t^*, rn_t^* \right\}$, ρ_z is an autoregressive coefficient vector and a ector of uncorrelated and normally distributed innovations with zero mean and standard deviation .

3. *Resource Constraint Government Budget Constraint*

$$Y_t^h = C_t^h + C_t^{eh} + C_t^{h*} + I_t^h + G_t^h$$

$$\ln G_t^h - \ln(\overline{G^h}) = \rho^G (\ln G_{t-1}^h - \ln(\overline{G^h})) + \varepsilon_t^g$$

$$\frac{P_t^h}{P_t} G_t^h = \frac{M_t - M_{t-1}}{P_t} + T_t$$

Log-linearisation:

$$y_t^h = \frac{C^h}{Y} c_t^h + \frac{C^{eh}}{Y} c_t^{eh} + \frac{C^{h*}}{Y} c_t^{h*} + \frac{I^h}{Y} i_t^h + \frac{G^h}{Y} g_t^h \qquad 24$$

4. *Exchange Rate Regimes*

(i) a pure fixed exchange rate regime;

$$S_t = \bar{S} \quad \forall t.$$

Log-linearisation:

$$s_t = 0$$

(ii) a floating exchange rate regime, where the central bank manages the nominal interest rate according to a Taylor rule;

Log-linearisation:

$$i_t = \rho_i i_{t-1} + \rho_{i\pi} \pi_t + \rho_{iy} y_t \qquad 25$$

Model Parametrization:

γ	0.7	γ_I	0.7	C^{h*}/Y	0.12
C^h/Y	0.3	I/Y	0.42	G/Y	0.15
β	0.9936	R	1.0064	R^k	1.0264
b	0.65	δ	0.03	α	0.45
K/N	2.5	ϵ	11	ϵ_w	2
θ	0.75	χ_r	0.0215	φ	0.25
Ω	0.99	η	2		