$$Y_t(i)=\left[u_{i,t}K_{i,t-1}^{E}(j)\right]^{\alpha}\left[\left(1-\omega\right)H_{t-1}^{P}(i)\right]^{\mu}\left(L_t(i)\right)^{1-\alpha-\mu}-\Phi$$

Intermediate production $Y_t(i)$ Capitla utilization ratio $u_{i,t}$

House $H_{t-1}^{P}(i)$ Labor $L_t(i)$

House rent rate R_t^h House price quantity Q_h

Wage $W_t(i)$ Marginal cost mc

Fixed cost Φ

Minimize production cost:

$$Min\ r_{i,t}^{k}(j)\left(u_{i,t}K_{i,t}^{E}(j)\right)+W_t(i)L_t(i)+R_t^h\left(1-\mu\right)Q_hH_t^P(i)$$

Optimal equations:

$$r_{i,t}^{k}(j)=\alpha\cdot mc_t\cdot(u_{i,t}K_{i,t}^{E}(j))^{\alpha-1}\left[\left(1-\omega\right)H_{t-1}^{P}(i)\right]^{\mu}\left(L_t(i)\right)^{1-\alpha-\mu}$$

$$R_t^h\cdot Q_{t,h}=\mu\cdot mc_t\cdot(u_{i,t}K_{i,t}^{E}(j))^{\alpha}\left[\left(1-\omega\right)H_{t-1}^{P}(i)\right]^{\mu-1}\left(L_t(i)\right)^{1-\alpha-\mu}$$

$$W_t(i)=(1-\alpha-\mu)\cdot mc_t\cdot(u_{i,t}K_{i,t}^{E}(j))^{\alpha}\left[\left(1-\omega\right)H_{t-1}^{P}(i)\right]^{\mu}\left(L_t(i)\right)^{1-\alpha-\mu-1}$$

So , Fixed cost $\Phi=\left[u_{i,t}K_{i,t-1}^{E}(j)\right]^{\alpha}\left[\left(1-\omega\right)H_{t-1}^{P}(i)\right]^{\mu}\left(L_t(i)\right)^{1-\alpha-\mu}$

$$-\left[r_{i,t}^{k}(j)\left(u_{i,t}K_{i,t}^{E}(j)\right)+W_t(i)L_t(i)+R_t^h\left(1-\mu\right)Q_hH_t^P(i)\right]$$

$$=\left(1-mc_t\right)\left[u_{i,t}K_{i,t-1}^{E}(j)\right]^{\alpha}\left[\left(1-\omega\right)H_{t-1}^{P}(i)\right]^{\mu}\left(L_t(i)\right)^{1-\alpha-\mu}$$

So, in steady state, $Y(i)=mc\left[u_iK^E(j)\right]^{\alpha}\left[\left(1-\omega\right)H^P(i)\right]^{\mu}\left(L(i)\right)^{1-\alpha-\mu}$

So,

$$r^k(j)=\alpha\cdot\frac{Y(i)}{u_iK^E(j)}$$

$$R_t\cdot Q_t=\eta\cdot\frac{Y(i)}{\left(1-\omega\right)H^P(i)}$$

$$W(i)=\frac{Y(i)}{L(i)}$$