American Economic Review 101 (October 2011): 2530–2561
http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.6.2530

Risk Matters: The Real Effects of Volatility Shocks†

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We show how changes in the volatility of the real interest rate at which small open emerging economies borrow have an important effect on variables like output, consumption, investment, and hours. We start by documenting the strong evidence of time-varying volatility in the real interest rates faced by four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. We estimate a stochastic volatility process for real interest rates. Then, we feed this process in a standard small open economy business cycle model. We find that an increase in real interest rate volatility triggers a fall in output, consumption, investment, hours, and debt. (*JEL* E13, E20, E32, E43, F32, F43, 011)

This paper shows how changes in the volatility of the real interest rate at which emerging economies borrow have a substantial effect on real variables like output, consumption, investment, and hours worked. These effects appear even when the realized real interest rate itself remains constant. We argue that, consequently, the time-varying volatility of real interest rates is an important force behind the distinctive size and pattern of business cycle fluctuations of emerging economies.

To prove our case, this paper makes two points. First, we document the strong evidence of time-varying volatility in the real interest rates faced by four emerging small open economies: Argentina, Brazil, Ecuador, and Venezuela. We postulate a stochastic volatility process for real interest rates and estimate it using T-bill rates and country spreads with the help of the particle filter and Bayesian methods. We uncover large movements in the volatility of real interest rates and a systematic relation of those movements with output, consumption, and investment. Second, we feed the estimated stochastic volatility process for real interest rates in a standard small open economy business cycle model calibrated to match data from our set of countries. We find that

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† To view additional materials, visit the article page at
http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.6.2530.

an increase in real interest rate volatility triggers a fall in output, consumption, investment, and hours worked, and a notable change in the current account. The effects are more salient for Argentina and Ecuador and milder for Venezuela and Brazil.

We think of our exercise as capturing the following sequence of events. Prior to period t, households live in an environment characterized by the average standard deviation of real interest rates. At time t, the standard deviation of the innovation to the country's spread increases by one standard deviation, while the real interest rate itself remains constant. Then, agents optimally adjust their consumption, labor, investment, and savings decisions to face the new degree of risk of real interest rates.

The intuition for our result is as follows. Small open economies rely on foreign debt to smooth consumption and to hedge against idiosyncratic productivity shocks. When the volatility of real interest rates rises, debt becomes riskier as the economy becomes exposed to potentially fast fluctuations in the real interest rate and their associated and unpleasant movements in marginal utility. To reduce this exposure, the economy lowers its outstanding debt by cutting consumption. Moreover, since debt is suddenly a worse hedge for the productivity shocks that drive returns to physical capital, investment falls. A lower investment reduces output and, through a fall in the marginal productivity of labor, hours worked.

To strengthen our argument, we perform a battery of robustness checks. First, we highlight that movements in the volatility of real interest rates are highly correlated with variations in levels. We reestimate our stochastic volatility model while allowing for this correlation and recompute the model with the new processes. The conclusion that changes in risk affect real variables remains unchallenged. If anything, our results are reinforced. In addition, in an online Appendix, we extend the model to incorporate working capital, we explore the consequences of imposing different priors in our estimation, and we check whether the results from Argentina depend on the Corralito data and the partial default of 2001. We find that, again, our results are basically unaltered.

Our investigation begets a number of riveting additional points. First, due to the nonlinear nature of stochastic volatility, we apply the particle filter to evaluate the likelihood function of the process driving the real interest rates (see a description of the particle filter applied to economics in Fernández-Villaverde and Rubio-Ramírez 2007). By doing so, we introduce a new technique that can have many applications in international finance where nonlinearities abound (sudden stops, exchange rate regime switches, large devaluations, etc.).

Second, capturing time-varying volatility creates a computational challenge. Since we are interested in the implications of a volatility increase while keeping the real interest rate constant, we have to consider a third-order Taylor expansion of the solution of the model. In a first-order approximation, stochastic volatility would not even play a role, since the policy rules of the representative agent follow a certainty equivalence principle. In the second-order approximation, only the product of the two innovations appears in the policy function. Only in the third-order approximation do the innovations to volatility play a role by themselves.

Third, we document that time-varying volatility moves the ergodic distribution of the endogenous variables of the model away from their deterministic steady state. This is crucial for the empirical implementation of the model. Thus, we calibrate it according to that ergodic distribution and not, as commonly done, to match steady-state values.

Our paper does not offer a theory of why real interest rate volatility evolves over time. Instead, we take it as an exogenously given process. By doing so, we join an old tradition in macroeconomics, starting with Finn Kydland and Edward C. Prescott (1982), who took their productivity shocks as exogenous, then to Enrique G. Mendoza (1995), who did the same with his terms of trade shocks, or Pablo A. Neumeyer and Fabrizio Perri (2005), who consider country spread shocks as given. Part of the reason is that an exogenous process for volatility sharply concentrates our attention on the mechanism through which real interest rate risk shapes the trade-offs of agents in small open economies. More pointedly, the literature has not developed, even at the prototype stage, an equilibrium model to endogenize volatility shocks. If we had tried to build such a model in this paper, simultaneously with our empirical documentation of volatility and the measurement of its effects, we would lose focus and insight in exchange for a most uncertain reward. In comparison, a thorough understanding of the effects of volatility changes per se will be a solid foundation for more elaborated theories of time-dependent variances.[1]

Fortunately, our strategy is justified empirically by the findings of Martin Uribe and Vivian Yue (2006) and Francis A. Longstaff et al. (2007). Uribe and Yue estimate a vector autoregression (VAR) with panel data from emerging economies to investigate how much of the country spreads are driven by domestic factors and how much by international conditions. They conclude that at least two-thirds of the movements in country spreads are explained by innovations that are exogenous to domestic conditions. Longstaff et al. look at CD spreads for sovereign debt of 26 open economies, and document that country spreads are driven much more by global financial market variables and global risk premia than by local forces. The evidence in these two papers supports the view that a substantial component of changes in volatility is exogenous to the country.

Uribe and Yue's result should not be a surprise because the aim of the literature on financial contagion is to understand phenomena that distinctively look like exogenous shocks to small open economies (Graciela L. Kaminsky, Carmen M. Reinhart, and Carlos A. Vegh 2003). For instance, after Russia defaulted on its sovereign debt in the summer of 1998, Argentina, Brazil, and Hong Kong (countries that have little, if anything, in common with Russia or Russian fundamentals besides appearing in the same table in the back pages of *The Economist* as an emerging market) suffered a significant increase in the volatility of the real interest rates at which they borrowed. At a first pass, thinking about those volatility spikes as exogenous events and tracing their consequences within the framework of a standard business cycle model seem empirically plausible and worthwhile.

Our paper is linked with three literatures. First, our work is related to the literature on time-varying volatility in finance and macroeconomics. While the effects of time-varying volatility have been widely studied in finance (Neil Shephard 2008), the issue has been nearly neglected in macroeconomics. Alejandro Justiniano and Giorgio E. Primiceri (2008) and Fernández-Villaverde and Rubio-Ramírez (2007) estimate dynamic equilibrium models with heteroskedastic shocks. Both papers

[1] We have the additional obstacle of data limitations on real aggregate variables. For the countries in our dataset, it is even difficult to compute the evolution of total factor productivity (TFP). Since we have to use high-frequency data for volatility, the problem becomes more acute.

conclude that time-varying volatility helps to explain the reduction observed in the standard deviation of output growth and other macroeconomic variables between 1984 and 2007. These papers also show, however, that for the US economy, stochastic volatility mainly affects the second moments of the variables with little effect on their first moments. In comparison, Nicholas Bloom (2009) exploits firm-level data to estimate a model where a spike in uncertainty affects real variables by freezing hiring and investment. Also, for the United States, Bloom, Nir Jaimovich, and Max Floetotto (2008) find that uncertainty (as measured by several proxies) is countercyclical. Then, they show how, in an augmented version of the neoclassical growth model, a rise in uncertainty decreases investment and employment. Our paper complements this line of work by offering an alternative mechanism through which time-varying volatility has a first-order impact.

Second, we have many points of contact with the literature that studies the relation between growth and volatility. The empirical evidence shows that countries with higher volatility have lower growth rates, as documented by Garey Ramey and Valery A. Ramey (1995). To link our findings with those of Ramey and Ramey, we could modify our model by introducing mechanisms through which the short-run fluctuations may have long-run impacts. Irreversible capital or investment in research and development are natural candidates for such extensions.

Third, we engage in the discussion of why the business cycles of emerging economies present characteristics that diverge from the pattern of business cycle fluctuations in developed small open economies (Mark Aguiar and Gita Gopinath 2007; Neumeyer and Perri 2005; Uribe and Yue 2006, among others). Our paper suggests that the higher time-varying volatility of the real interest rate faced by Argentina in comparison, let's say, with Canada is an important source of differences. Stochastic volatility may help explain, for example, why consumption is more volatile than output in emerging economies. We do not, however, postulate time-varying volatility of the real interest rate as a substitute for any of the theories proposed by previous authors. Instead, we see it as a complement, as many of the channels explored by the literature may become stronger in its presence. We document that this is precisely the case for the real interest rate shocks that are the focus of Neumeyer and Perri (2005): real interest rate shocks and volatility shocks reinforce each other. Our paper also suggests that volatility shocks are a possible explanation for the relatively low cross-country business cycle correlations.

The rest of the paper is organized as follows. Section I presents our econometric exercise. Section II lays down our benchmark small open economy model and explains how to calibrate and compute it. Section III discusses our results, and section IV extends the implications of our analysis with respect to the origins and consequences of volatility. Section V concludes. An online Appendix extends our discussion of several aspects of the paper.

I. Estimating the Law of Motion for Real Interest Rates

In this section, we estimate the law of motion for the evolution of real interest rates in four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. We select our countries based on data availability and because they represent a relatively coherent set of South American economies. We build the real interest rate faced by each country as the sum of the international risk-free real rate and a country-specific spread. Next, we estimate the law of motion of the international risk-free real rate, which is common across countries, and the law of motion of the country spread, one for each economy. Therefore, this section plays a dual role. First, it documents that changes in the volatility of real interest rates are quantitatively significant. Second, it provides us with the processes that we feed into our calibrated model.

A. *Data on Interest Rates*

For any given country, we decompose the real interest rate, r_t, it faces on loans denominated in US dollars as the international risk-free real rate plus a country-specific spread. We use the T-bill rate as a measure of the international risk-free nominal interest rate. This is a standard convention in the literature. We build the international risk-free real rate by subtracting expected inflation from the T-bill rate. Following Neumeyer and Perri (2005), we compute expected inflation as the average US consumer price index (CPI) inflation in the current month and in the 11 preceding months. This assumption is motivated by the observation that inflation in the United States is well approximated by a random walk (Andrew Atkeson and Lee E. Ohanian 2001).[2] Both the T-bill rate and the inflation series are obtained from the St. Louis Federal Reserve FRED database. We use monthly rather than the more popular quarterly data because monthly data are more appropriate for capturing the volatility of interest rates as required by our investigation. Otherwise, quarterly means would smooth out much of the variation in interest rates.

For data on country spreads, we use the Emerging Markets Bond Index Plus (EMBI+) spread reported by J. P. Morgan at a monthly frequency. This index tracks secondary market prices of actively traded emerging market bonds denominated in US dollars. Neumeyer and Perri (2005) explain in detail the advantages of EMBI data in comparison with the existing alternatives. Unfortunately, except for Brazil, EMBI is available only from 1998. Thus, our sample misses the Tequila crisis and the early stages of the Asian crisis. Yet the sample is large enough to cover the 2000–2001 equity price correction in the United States and the Argentinean crisis of 2001–2002. The EMBI data coverage is as follows: Argentina 1997:12–2008:02; Ecuador 1997:12–2008:02; Brazil 1994:04–2008:02; and Venezuela 1997:12–2008:02.

We plot our data in Figure 1. We use annualized rates in percentage points to facilitate comparison with the most commonly quoted rates. The international risk-free real rate is low (with negative interest rates in 2002–2006) and relatively stable over the sample. In comparison, all country spreads are large and volatile. The spreads are nearly always larger than the real T-bill rate itself and fluctuate at least an order of magnitude more. The most prominent case is Argentina, where the 2001–2002 crisis raised the country spreads to 70 percentage points. In the figure, we also see the problems of Ecuador in 1998–1999 and the turbulence in all four countries during the virulent international turmoil of 1998.

[2] We checked that more sophisticated methods to back up expected inflation, such as the IMA(1,1) process proposed by James H. Stock and Mark W. Watson (2007), deliver results that are nearly identical. We computed that the consequences of using these alternative processes for expected inflation, given the size of the changes in country-spreads that we deal with later in Section II, are completely irrelevant from a quantitative perspective.

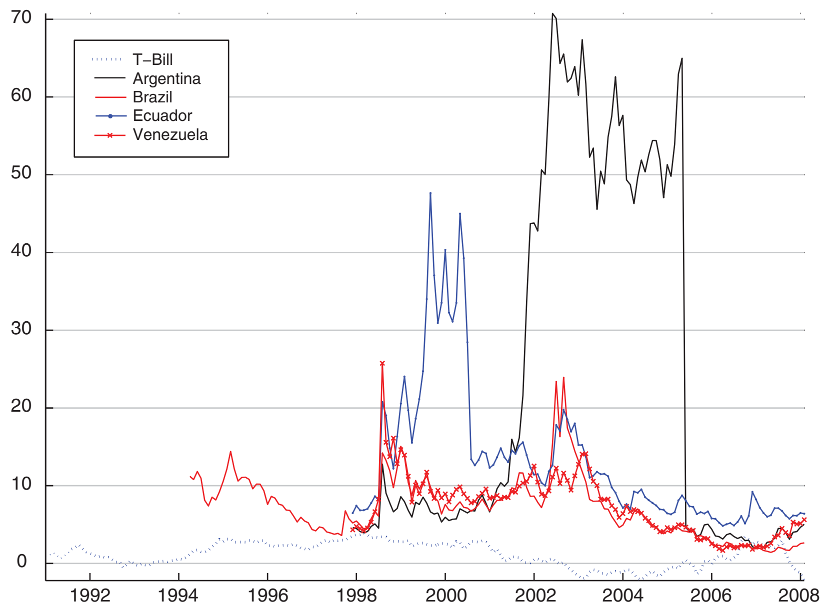

Figure 1. Country Spreads and T-Bill Real Rate

B. *The Law of Motion for Real Interest Rates*

We write the real interest rate faced by domestic residents in international markets at time t as $r_t = r + \varepsilon_{tb,t} + \varepsilon_{r,t}$. In this equation, r is the mean of the international risk-free real rate plus the mean of the country spread. The term $\varepsilon_{tb,t}$ equals the international risk-free real rate subtracted from its mean, and $\varepsilon_{r,t}$ equals the country spread subtracted from its mean. To ease notation, we omit a subindex for the country-specific variables and parameters.

We specify that both $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow AR(1) processes described by

$$\varepsilon_{tb,t} = \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}}u_{tb,t}, \tag{1}$$

$$\varepsilon_{r,t} = \rho_r\varepsilon_{r,t-1} + e^{\sigma_{r,t}}u_{r,t}, \tag{2}$$

where both $u_{r,t}$ and $u_{tb,t}$ are normally distributed random variables with mean zero and unit variance. The main feature of our process is that the standard deviations $\sigma_{tb,t}$ and $\sigma_{r,t}$ are not constant, as commonly assumed, but follow an AR(1) process:

$$\sigma_{tb,t} = (1 - \rho_{\sigma_{tb}})\sigma_{tb} + \rho_{\sigma_{tb}}\sigma_{tb,t-1} + \eta_{tb}u_{\sigma_{tb},t}, \tag{3}$$

$$\sigma_{r,t} = (1 - \rho_{\sigma_r})\sigma_r + \rho_{\sigma_r}\sigma_{r,t-1} + \eta_r u_{\sigma_r,t}, \tag{4}$$

where both $u_{\sigma_r,t}$ and $u_{\sigma_{tb},t}$ are normally distributed random variables with mean zero and unit variance. Thus, our process for interest rates displays stochastic volatility. The parameters σ_{tb} and η_{tb} control the degree of mean volatility and stochastic volatility in the international risk-free real rate: a high σ_{tb} implies a high mean volatility of the international risk-free real rate, and a high η_{tb} implies a high degree of stochastic volatility. The same can be said about σ_r and η_r and the mean volatility and stochastic volatility in the country spread.

Two innovations affect each of the components of the real interest rate. For instance, $\varepsilon_{tb,t}$ is hit by $u_{tb,t}$ and $u_{\sigma_{tb},t}$. The first innovation, $u_{tb,t}$, changes the rate, while the second innovation, $u_{\sigma_{tb},t}$, affects the standard deviation of $u_{tb,t}$. The innovations $u_{r,t}$ and $u_{\sigma_r,t}$ have a similar reading. We call $u_{tb,t}$ and $u_{r,t}$ innovations to the international risk-free real rate and the country-spread, respectively.[3] We call $u_{\sigma_{tb},t}$ and $u_{\sigma_r,t}$ innovations to the volatility of the international risk-free real rate and the country spread, respectively. Sometimes, for simplicity, we call this second type of innovation a stochastic volatility shock. In Section V, we discuss different interpretations of the possible origins of volatility shocks.

Our specification is parsimonious yet powerful enough to capture some salient peculiarities of the data (Shephard 2008). Alternative specifications are less useful for us. For example, estimating realized volatility is difficult because we do not have intraday data and because we need a parametric law of motion for volatility to feed into the equilibrium model of Section III. Similarly, a generalized autoregressive conditional heteroskedasticity (GARCH) specification does not sharply distinguish between innovations to the rate and to the volatility: higher volatilities are triggered only by innovations to the rate. In comparison, in stochastic volatility, as we mentioned above, we have two clearly different shocks.

As our benchmark exercise, we assume that $u_{tb,t}$, $u_{r,t}$, $u_{\sigma_{tb},t}$, and $u_{\sigma_r,t}$ are independent of each other. How strong is this assumption? We checked that $u_{tb,t}$ and $u_{r,t}$ are uncorrelated in our data. This result confirms the findings of Neumeyer and Perri (2005). At the same time, we will report below that (i) the pair $u_{tb,t}$ and $u_{\sigma_{tb},t}$ is strongly correlated, and (ii) the pair $u_{r,t}$ and $u_{\sigma_r,t}$ is strongly correlated as well. Motivated by this evidence, we will reestimate our stochastic volatility process allowing for correlation. We keep the case without correlation as our benchmark, however, because it more neatly separates changes to levels from changes to volatility.

C. *Estimation*

We estimate the parameters of the process in equations (1) to (4) with a likelihood-based approach. The likelihood of these processes is challenging to evaluate because of the presence of two innovations, the innovation to levels and to volatility, that interact in a nonlinear way. We address this problem using the particle filter. This filter is a sequential Monte Carlo algorithm that allows for the evaluation of the likelihood given some parameter values through resampling simulation methods. The online Appendix offers further details. We follow a Bayesian approach to inference by combining the likelihood function with a prior. In our context, Bayesian inference is convenient because we have short samples that can be complemented with presample information.

[3] Strictly speaking, they are shocks to the deviation of the real interest rate with respect to its mean due to the international risk-free rate and the country spreads. Hereafter, to facilitate exposition, we omit the word "deviation" where we do not risk ambiguity.

TABLE 1—PRIORS

	ρ_r	σ_r	ρ_{σ_r}	η_r
Argentina	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-5.30, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$
Brazil	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-6.60, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$
Ecuador	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-5.80, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$
Venezuela	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-6.50, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$

Notes: $\mathcal{B}$, $\mathcal{N}$, and $\mathcal{N}^+$ stand for Beta, Normal, and truncated Normal distributions. Mean and standard deviation in parentheses.

Priors.—We now elicit our priors. We start by concentrating on the priors for the parameters driving the law of motion of the country spread. Then, we analyze the priors for the parameters of the process for international risk-free real rate.

Table 1 reports our priors for the parameters of the processes corresponding to each of the four countries' spreads. Except for σ_r, we adopt the same prior for all four countries. This facilitates the comparison of the posteriors. For ρ_r and ρ_{σ_r} we choose a Beta prior with mean 0.9 and a moderate standard deviation, 0.02, for ρ_r, and a fairly large one, 0.1 , for ρ_{σ_r}. These priors reflect our view that there is a mild persistence in interest rates (since we have a monthly model, a monthly value of 0.9 is equivalent to a quarterly value of 0.73). The small standard deviation for ρ_r pushes the posterior toward lower values of the parameter. Otherwise, the median of the posterior would become virtually identical to one, exacerbating the effects of stochastic volatility. Hence, our choice is conservative in the sense that it biases the results *against* our hypothesis that stochastic volatility is quantitatively relevant. The value of 0.1 for the standard deviation for ρ_{σ_r} embodies our relative ignorance regarding the persistence of the shock to volatility.

For η_r, we pick a truncated normal (to ensure that the parameter is positive). The mean of the prior for η_r implies that, on average, the standard deviation of the innovation to the country spread increases by a factor of roughly 1.7 after a positive stochastic volatility shock of one standard deviation ($\exp(0.5) = 1.6487$). This rise is modest compared to the large swings in interest rate volatility displayed in Figure 1. For the case of Argentina, the standard deviation of the country spread is seven times larger in the period 2002–2005 compared to that in 1998–2002. The standard deviation of 0.3 allows the posterior to move away from the mean of the prior. Last, σ_r is chosen to be a country-specific normal distribution. At the prior mean, the unconditional variance of $\varepsilon_{r,t}$ matches that of the data if we assume no stochastic volatility shocks. The standard deviation of the mean is fixed to be sufficiently high to give flexibility to the posterior. Thus, our priors capture the observation that Argentina and Ecuador have larger country spread variances than Brazil and Venezuela.

Overall, our priors are sufficiently loose to accommodate all countries in our sample. We found that increasing the standard deviation of the priors for σ_r, ρ_{σ_r}, and η_r had no significant impact on our results, while increasing the standard deviation of the prior for ρ_r favors our case. We further elaborate on the effects of the priors on our quantitative results in Section IV.

The priors for the parameters of the law of motion of the international risk-free real rate are chosen following an approach identical to that for the country-specific spreads. Thus, the justifications we provided before for these priors also hold here. We

Table 2—Mean of Real Interest Rate

	Argentina	Ecuador	Venezuela	Brazil
r	0.268	0.140	0.087	0.087

Table 3—Posterior Medians
(*95 percent set in brackets*)

	Argentina	Ecuador	Venezuela	Brazil		T-Bill
ρ_r	0.97 [0.96, 0.98]	0.95 [0.93, 0.97]	0.94 [0.91, 0.96]	0.95 [0.93, 0.96]	ρ_{tb}	0.95 [0.93, 0.97]
σ_r	−5.71 [−6.39, −4.89]	−6.06 [−6.73, −5.27]	−6.88 [−7.40, −6.22]	−6.97 [−7.49, −6.19]	σ_{tb}	−8.05 [−8.44, −7.55]
$\rho_{\sigma r}$	0.94 [0.83, 0.99]	0.96 [0.87, 0.99]	0.91 [0.77, 0.98]	0.95 [0.84, 0.99]	$\rho_{\sigma tb}$	0.94 [0.76, 0.97]
η_r	0.46 [0.33, 0.63]	0.35 [0.23, 0.52]	0.32 [0.19, 0.47]	0.28 [0.18, 0.40]	η_{tb}	0.13 [0.04, 0.29]

choose Beta priors for ρ_{tb} and $\rho_{\sigma_{tb}}$ with mean 0.9 and standard deviations of 0.02 and 0.1, respectively. For η_{tb}, we picked a truncated normal with mean 0.5 and standard deviation 0.3. Finally, σ_{tb} is such that, at the prior mean, -8, the unconditional variance of $\varepsilon_{tb,t}$ matches the one observed in the data without stochastic volatility shocks. The standard deviation of the prior of σ_{tb} is 0.4, 5 percent of the mean.

Posterior Estimates.—We draw 20,000 times from the posterior of each of the five processes that we estimate (one for the international risk-free real rate and one for each country spread) using a random walk Metropolis-Hastings. The draw was run after an exhaustive search for appropriate initial conditions and an additional 5,000 burn-in draws. We select the scaling matrix of the proposal density to induce the appropriate acceptance ratio of proposals. Each evaluation of the likelihood is performed with 2,000 particles. We implemented standard tests of convergence of the simulations, both of the Metropolis-Hastings and of the particle filter. Given the low dimensionality of the problem, even a relatively short draw like ours quickly converges.

The sample mean for the real return of the T-bill, our measure of the international risk-free real interest rate (in annualized terms), is 0.012, a number that coincides, for example, with John Y. Campbell (2003). Table 2 presents the mean of the (annualized) real interest rate for each country, r. Each of them pays a considerable risk premium, from the 0.007 of Brazil and Venezuela to the 0.02 of Argentina.

Table 3 reports the posterior medians of the parameters for the law of motion of the country spread. First, for the case of Argentina and Ecuador (and for Brazil and Venezuela to a lesser degree), the average standard deviation of an innovation to the country spread, σ_r, is large. This finding reveals a large degree of volatility in the country-spread data. Moreover, the posterior is tightly concentrated. Second, for all four countries, there is a substantial presence of stochastic volatility in the country-spread series (a large η_r). The shocks to the level and standard deviation of the country spread are highly persistent (large ρ_r and ρ_{σ_r}). The standard deviation of the posteriors of ρ_r is small (the 95 percent probability sets are entirely above 0.9). The standard deviation of the posteriors of ρ_{σ_r} is larger, but even at the 2.5 percentile, the persistence of the process is in the range of 0.77 to 0.99.

We now examine each country in particular. We start with Argentina, the most volatile country in our sample. The estimated value of σ_r implies that the innovation to the spread has an average annualized standard deviation of 398 basis points $(= 120{,}000 \exp(\sigma_r))$, where the loading factor of 120,000 transforms the estimate into annualized basis points. A positive volatility shock of one standard deviation magnifies the standard deviation of the innovation to the spread by a factor of 1.58 $(= \exp(\eta_r))$. Thus, a combined positive shock to both the level and volatility would raise Argentina's spread by 629 basis points $(= 120{,}000 \exp(\sigma_r + \eta_r))$. In the online Appendix, we show that our findings for Argentina are not dependent on the effects of the Corralito.[4] For example, without the Corralito data, the medians of the posteriors for the stochastic volatility parameters, ρ_σ and η_r, are 0.95 and 0.47, nearly the same as 0.94 and 0.46 in the case with the Corralito. In the Appendix, we also document how the results for all four countries are robust to a wide range of different priors.

We turn now to Brazil, the country with the least volatility. Its innovation to the spread has a mean standard deviation of 113 annual basis points. Furthermore, a positive volatility shock amplifies the effects of a level shock by a factor of 1.32, indicating that a combined positive shock to both the level and volatility would raise Brazil's spread by 149 basis points. Ecuador and Venezuela lay in the middle of our sample. Ecuador has an average standard deviation of 280 basis points, and a combination of positive shocks increases the spread by 398 basis points. These results put Ecuador in line with Argentina. Venezuela's numbers are closer to Brazil's. It has an average standard deviation of 123 basis points, and a combined positive shock increases the interest rate spread by 170 basis points.

In comparison with the country spread, the international risk-free real rate has both lower average standard deviation of its innovation (σ_{tb} is smaller than σ_r for all four countries) and less stochastic volatility (η_{tb} is smaller than η_r for all four countries). The posterior median for σ_{tb} equals -8.05 and for η_{tb} equals 0.13. Thus, the innovation to the international risk-free real rate has an average annualized standard deviation of only 38 basis points, and when combined with a positive shock to volatility, the international risk-free real rate increases to 44 basis points. The persistence ρ_{tb}, 0.95, is in line with other estimates in the literature (Neumeyer and Perri 2005 find a persistence of 0.81 at a quarterly rate). The persistence of the volatility shocks, $\rho_{\sigma_{tb}}$, is also high.

D. *Empirical Regularities*

We exploit the output from our econometric exercise to document several empirical regularities about business cycles and country-spread volatility in our four economies. The objective is to analyze the correlations between country spreads, output, investment, and consumption with country-spread volatility. The challenge is that the country-spread volatility, $\sigma_{r,t}$, is not an observable variable but a latent one. We can, however, take advantage of our model for country spreads, specified by equations (2) and (4), and the particle filter to smooth the distribution of country-spread volatilities

[4] The Corralito was the drastic measures undertaken by the Argentinean government in 2001 to freeze bank accounts and, later, to forcefully transform dollar-denominated deposits into peso-denominated deposits at an artificial exchange rate.

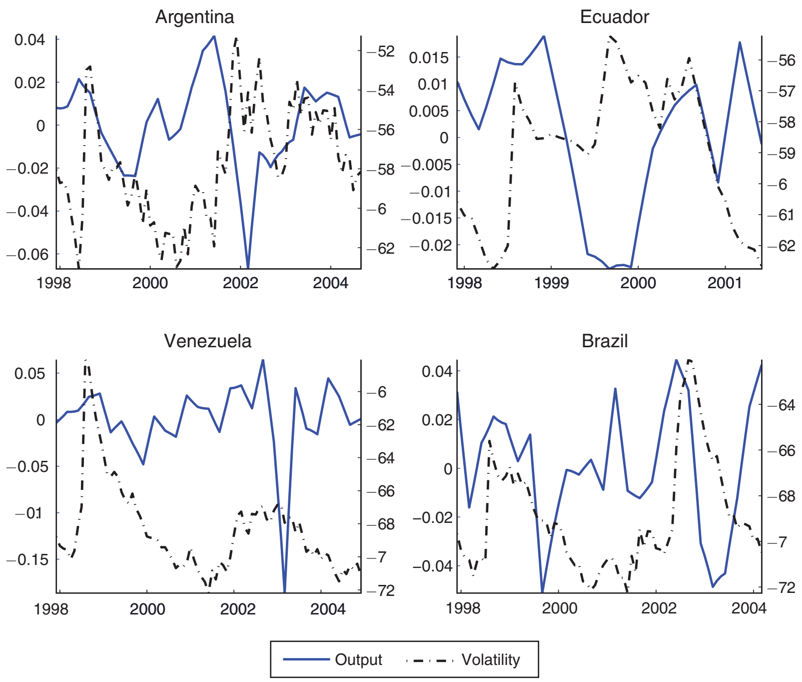

Figure 2. Detrended Output and Smoothed Country Spread Volatility

conditional on our whole sample. In what follows, we refer to the average smoothed volatility conditional on the median of the posterior of the parameters as the country spread volatility. Since we use monthly data for interest rates and quarterly data for aggregate variables, we linearly interpolate output, investment, and consumption.

A first step is to plot, in Figure 2, the time series of output and the country-spread volatility in annualized basis points. Figure 2 indicates a negative correlation between output and country-spread volatility. For all four countries, times of high volatility are times of low output. A similar picture would emerge if we plotted volatility against consumption or investment.

This negative correlation also appears in Figure 3, which plots the cross-correlation between output and country-spread volatility at different lags for our four countries. Country-spread volatility is countercyclical and leads the cycle by about five months. The contemporaneous correlation between output and volatility ranges from around zero in Brazil to $-0.3/-0.4$ in Argentina or Ecuador. The average contemporaneous correlation is -0.17. Figure 3 also plots the cross-correlation between investment and country-spread volatility and consumption and country-spread volatility. As before, country-spread volatility leads the cycle with respect to investment and consumption. For the case of consumption, the contemporaneous correlation varies from slightly below zero for Brazil to -0.43 in Ecuador. The average is around -0.2. For the case of investment, the contemporaneous correlation moves from roughly 0 for Brazil to -0.23 in Ecuador.

Figure 4 plots the time series of country spread and the country-spread volatility. Figure 4 reveals a positive comovement between country spread and country-spread volatility. Hence, periods of high country spreads are associated with periods of high

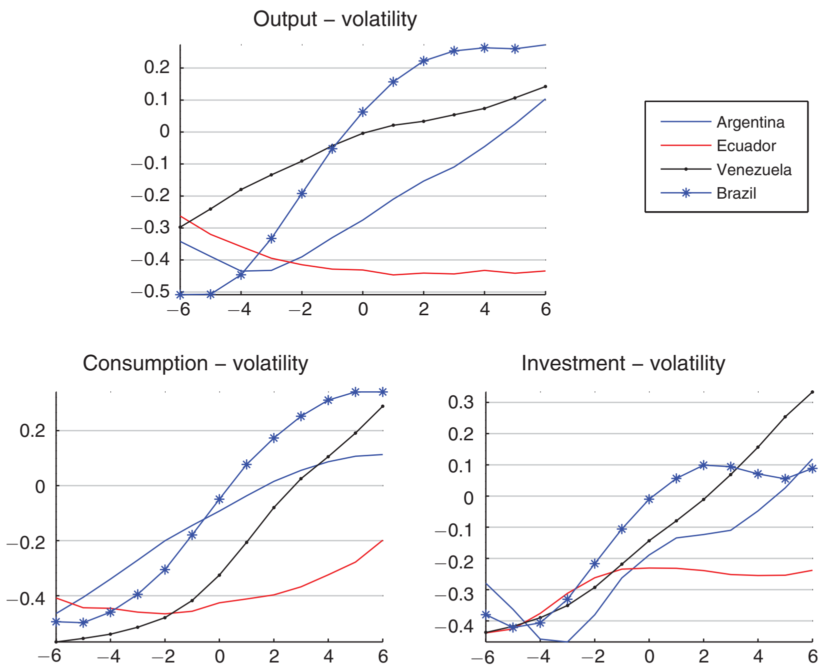

Figure 3. Cross-Correlations: Output-Volatility, Consumption-Volatility, Investment-Volatility

country-spread volatility. This suggests that we need to relax our assumption that the innovations to the country spread and its volatility are uncorrelated.

E. *Reestimating the Processes with Correlation of Shocks*

Motivated by the evidence in Figure 4, we repeat our estimation assuming that the innovations come from a multivariate normal:

$$\begin{pmatrix} u_{r,t} \\ u_{\sigma_r,t} \end{pmatrix} \sim \mathcal{N}\begin{pmatrix} 0, & 1 & \kappa \\ 0 & \kappa & 1 \end{pmatrix}.$$

The parameter κ controls the strength of the correlation and, therefore, the size of the "leverage effect" of level shocks on volatility shocks. We do not correlate the innovations with the international risk-free real rate and its volatility, since their empirical size is small and they would not play a quantitatively significant role in the simulation of the model. We impose a uniform prior for κ in $(-1, 1)$ to reflect a roughly neutral stand on the size of the correlation.

Table 4 reports our posterior. The median values of the posterior of the parameters ρ_r, σ_r, ρ_{σ_r}, and η_r for each of the four countries are close to our benchmark estimates. Thus, the quantitative patterns of Figures 2 to 4, redone with the new process, remain virtually identical and we do not include them to save space. The new parameter, κ, is estimated to be high, between 0.69 and 0.89. When we simulate the model, the clustering of the innovations will reinforce our case because

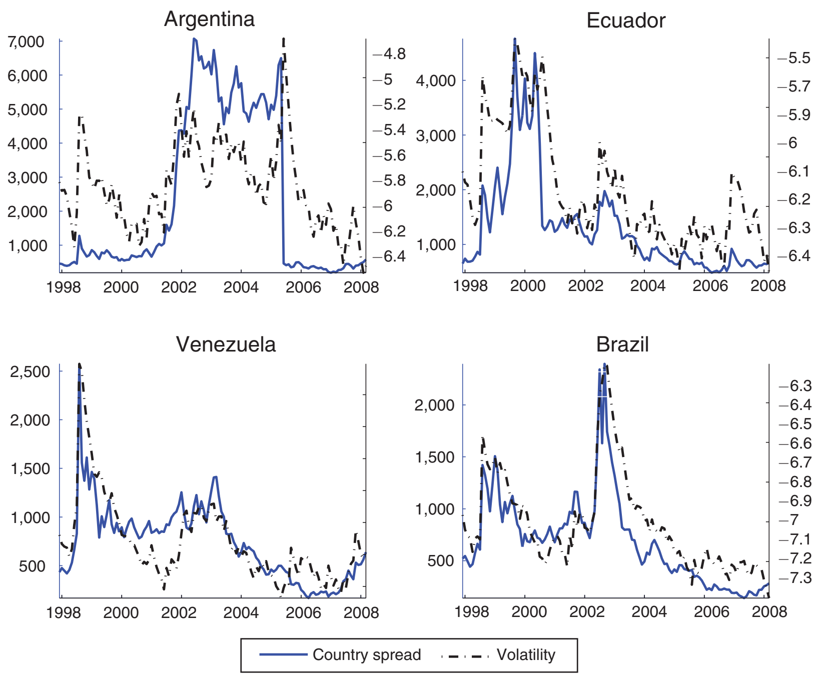

FIGURE 4. COUNTRY SPREAD AND VOLATILITY

both affect the economy in the same direction. By keeping the situation without correlation as our benchmark, we isolate more clearly the direct effects of stochastic volatility. At the same time, for completeness, we will also report the case when the shocks are correlated.

F. *Summary of Empirical Results*

In this section, we have estimated the law of motion for country spreads and international risk-free rates for the four countries in our sample. We have reached four conclusions. First, the average standard deviation of an innovation to the country spread is large. Second, there is substantial stochastic volatility in the country spreads. Third, international risk-free rates have both less mean volatility and less stochastic volatility than the country spread for any of the four countries. Fourth, country-spread volatility is countercyclical and leads the cycle with respect to output, investment, and consumption. Given these findings, we move to use a canonical small open economy model to measure the business cycle implications of the large degree of volatility and stochastic volatility that we find in the country spreads.

II. The Model

We formulate a prototypical small open economy model with incomplete asset markets in the spirit of Mendoza (1991), Neumeyer and Perri (2005), and Uribe and

TABLE 4—POSTERIOR MEDIANS WITH CORRELATION
(95 percent set in brackets)

	Argentina	Ecuador	Venezuela	Brazil		T-bill
ρ_r	0.97 [0.96, 0.98]	0.95 [0.92, 0.96]	0.95 [0.93, 0.97]	0.96 [0.94, 0.97]	ρ_{tb}	0.95 [0.93, 0.97]
σ_r	−5.80 [−6.28, −5.28]	−5.93 [−6.32, −5.50]	−6.61 [−7.00, −6.02]	−6.57 [−6.88, −6.26]	σ_{tb}	−8.05 [−8.44, −7.55]
ρ_{σ_r}	0.90 [0.79, 0.97]	0.89 [0.83, 0.95]	0.92 [0.81, 0.96]	0.91 [0.85, 0.94]	$\rho_{\sigma_{tb}}$	0.94 [0.76, 0.97]
η_r	0.45 [0.28, 0.65]	0.34 [0.23, 0.48]	0.32 [0.21, 0.47]	0.28 [0.22, 0.38]	η_{tb}	0.13 [0.04, 0.29]
κ	0.69 [0.39, 0.89]	0.89 [0.75, 96]	0.75 [0.53, 0.89]	0.89 [0.76, 0.95]		

Yue (2006). The small open economy is populated by a representative household whose preferences are captured by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-v}}{1 - v} - \omega \frac{H_t^{1+\eta}}{1 + \eta} \right). \tag{5}$$

Here, $\mathbb{E}_0$ is the conditional expectations operator, C_t denotes consumption, H_t stands for hours worked, and $\beta \in (0,1)$ corresponds to the discount factor.

The real interest rate r_t faced by domestic residents in financial markets follows equations (1) to (4) specified in Section II. This assumption, motivated by our empirical evidence, is the main difference of our model with respect to the standard small open economy business cycle model.

The household can invest in two types of assets: the stock of physical capital, K_t, and an internationally traded bond, D_t. We maintain the convention that positive values of D_t denote debt. Then, the household's budget constraint is given by

$$\frac{D_{t+1}}{1 + r_t} = D_t - W_t H_t - R_t K_t + C_t + I_t + \frac{\Phi_D}{2}(D_{t+1} - D)^2, \tag{6}$$

where W_t represents the real wage, R_t stands for the real rental rate of capital, I_t is our notation for gross domestic investment, $\Phi_D > 0$ is a parameter that controls the costs of holding a net foreign asset position, and D is a parameter that determines debt in the deterministic steady state. The cost is paid to some foreign international institution (for example, an investment bank that handles the issuing of bonds for the representative household).

We highlight two points about (6). First, the household has access to a one-period, uncontingent bond. This reflects the extremely limited ability of the countries in our sample to issue debt at long horizons; when they do so, it is accepted by the market only at steep discounts. For a theoretical investigation of why this is so, see Laura Alfaro and Fabio Kanczuk (2009) or Fernando Broner, Guido Lorenzoni, and Sergio L. Schmukler (2007). Thus, the household will not have the possibility of structuring its debt maturity to minimize the effects of volatility (or, equivalently, the market for volatility contracts does not exist or it is too small). Second, the household faces this cost of holding a net foreign asset position with the purpose of eliminating possible nonstationarities otherwise built into the dynamics of the small open economy model. These are inconvenient because they make it difficult to analyze transient dynamics. In the working paper version of the paper (Fernández-Villaverde et al. 2009), we quantitatively compared our specification with other ways to close the open economy aspect of the model, and we found that the results were robust (if anything, often bigger).

The stock of capital evolves according to the law of motion with adjustment costs:

$$K_{t+1} = (1 - \delta)K_t + \left(1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)I_t,$$

where δ is the depreciation rate. The parameter $\phi > 0$ controls the size of the adjustment costs. The introduction of these adjustment costs is commonplace in business cycle models of small open economies. They are a convenient and plausible way to avoid excessive investment volatility in response to changes in the real interest rate. The representative household is also subject to the typical no-Ponzi-game condition.

Firms rent capital and labor from households to produce output in a competitive environment according to the technology $Y_t = K_t^{\alpha}(e^{X_t}H_t)^{1-\alpha}$, where X_t corresponds to a labor-augmenting productivity shock that follows an AR(1) process $X_t = \rho_x X_{t-1} + \sigma_X u_{x,t}$ where $u_{x,t}$ is a normally distributed shock with zero mean and variance equal to one.

Firms maximize profits by equating wages and the rental rate of capital to marginal productivities. Thus, we can rewrite equation (6) in terms of net exports NX_t:

$$NX_t = Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1 + r_t} + \frac{\Phi_D}{2}(D_{t+1} - D)^2.$$

Also, we can define the current account as $CA_t = D_t - D_{t+1}$, where the order of the terms is switched from conventional notation because positive values of D_t denote debt.

A competitive equilibrium can be defined in a standard way as a sequence of allocations and prices such that both the representative household and the firm maximize and markets clear. The set of equilibrium conditions is given by

$$C_t^{-v} = \lambda_t, \tag{7}$$

$$\frac{\lambda_t}{1 + r_t} = \lambda_t \Phi_D(D_{t+1} - D) + \beta \mathbb{E}_t \lambda_{t+1}, \tag{8}$$

$$-\varphi_t + \beta \mathbb{E}_t\left[(1 - \delta)\varphi_{t+1} + \alpha \frac{Y_{t+1}}{K_{t+1}}\lambda_{t+1}\right] = 0, \tag{9}$$

$$\omega H_t^{\eta+1} C_t^{v} = (1 - \alpha)Y_t, \tag{10}$$

and

$$\varphi_t\left[1 - \frac{\phi}{2}\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)^2 - \frac{\phi I_t}{I_{t-1}}\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)\right] + \beta \mathbb{E}_t\left[\varphi_{t+1}\phi\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1} - I_t}{I_t}\right)\right] = \lambda_t, \tag{11}$$

together with the resource constraint, the law of motion for capital, the production function, and the stochastic processes for the interest rate. The Lagrangian λ_t is associated with the debt level and the Lagrangian φ_t with physical capital.

A. *Solving the Model*

We solve the model by relying on perturbation methods to approximate the policy functions of the agents and the laws of motion of exogenous variables around the deterministic steady state of our economy. Boragan Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) report that perturbation methods are highly accurate and deliver a fast solution in a closed economy version of the model considered here.[5]

One of the exercises we are keenly interested in is to measure the effects of a volatility increase (a positive shock to either $u_{\sigma_r,t}$ or $u_{\sigma_{tb},t}$) while keeping the real interest rate unchanged (fixing $u_{r,t} = 0$ and $u_{tb,t} = 0$). Consequently, we need to obtain a *third-order* approximation of the policy functions. A first-order approximation to the model would miss all of the dynamics induced by volatility because this approximation is certainty equivalent. Thus, the policy functions would exclusively depend on the normally distributed shocks $u_{tb,t}$, $u_{r,t}$, and $u_{X,t}$. Shocks to volatility, $u_{\sigma_r,t}$ and $u_{\sigma_{tb},t}$, do not appear in this approximation (more precisely, the coefficients in front of these variables are equal to zero). A second-order approximation would capture the volatility effect only indirectly via cross-product terms of the form $u_{r,t}u_{\sigma_r,t}$ and $u_{tb,t}u_{\sigma_{tb},t}$, that is, through the joint interaction of both innovations. Thus, up to the second order, volatility does not have an effect as long as the real interest rate does not change. It is only in a third-order approximation that the innovations to the stochastic volatility shocks, $u_{\sigma,t}$ and $u_{\sigma_{tb},t}$, enter as independent arguments in the policy functions with a coefficient different from zero. Hence, if we want to explore the direct role of volatility, we need to consider cubic terms. Furthermore, given the estimated stochastic volatility processes, the cubic terms in the policy functions are quantitatively significant. This is one of the most relevant findings of our paper. In the online Appendix, we show how the simulation paths of the model are affected by these higher-order terms.

Also, the third-order approximation and our estimated stochastic processes move the mean of the ergodic distributions of the endogenous variables of the model away from their deterministic steady-state values. Thus, our calibration must target the moments of interest generated by the ergodic distributions and not the moments of the deterministic steady state, since those last ones are not representative of the stochastic dynamics.

There are two possible objections to our perturbation solution: first, whether approximating the policy function around the steady state is the best choice; and, second, whether a third-order solution is accurate enough. The first objection can be dealt with by observing that (i) the approximation around the steady state is asymptotically valid (something that cannot be said for certain about other approximation points; see Hehui Jin and Kenneth L. Judd 2002, theorem 6); and that (ii)

[5] Value function iteration or projection methods are too slow to run with the required level of accuracy (we have eight state variables). Moreover, as we will see momentarily, the calibration of the model requires a fair amount of simulations. A slow solution method would make this task too onerous.

the second-order terms include a constant that corrects for precautionary behavior associated with risk. To answer the second objection, we computed a sixth-order approximation to the model. We found that the fourth-, fifth-, and sixth-order terms contributed next to nothing to the dynamics of interest.[6] Once you have the terms on volatility that the third order delivers, fourth- and higher-order terms have extremely small coefficients. Since the additional terms considerably slowed down the solution and limited our ability to simulate and explore the model (in the sixth order we have 1,899,240 terms to compute), we stopped at the third order.

The states of the model are $States_t = (\hat{K}_t, \hat{I}_{t-1}, \hat{D}_t, X_{t-1}, \varepsilon_{r,t-1}, \varepsilon_{tb,t-1}, \sigma_{r,t-1}, \sigma_{tb,t-1}, \Lambda)'$ and the innovations are $\xi_t = (u_{X,t}, u_{r,t}, u_{tb,t}, u_{\sigma_r,t} u_{\sigma tb,t})'$, where $\hat{K}_t$, $\hat{I}_{t-1}$, and $\hat{D}_t$ are deviations of the logs of K_t and I_{t-1}, and the level of D_t with respect to the log of K and I and the level of D (we do not take logs of D because they may be negative). Also, Λ is the perturbation parameter. We take a perturbation solution around $\Lambda = 0$, that is, around the steady state implied when all the variances of the shocks are equal to zero.

B. *Calibration*

We calibrate eight versions of the model, two for each country, one using our benchmark estimates of the law of motion for interest rates (without correlation), and one for the alternative estimates (with correlation). Thereafter, we will call the first version of the model M1, and the second version, where we feed in the processes with correlation, M2.[7] Since the estimated processes for the interest rate are monthly, we calibrate the parameters accordingly and, in the simulation, we build quarters of model data. Thus, all our results below will be on a quarterly basis.

We fix the value of the following five parameters in all eight calibrations: (i) the inverse of the elasticity of intertemporal substitution, $v = 5$; (ii) the parameter that determines the elasticity of labor to wages, $\eta = 1000$; (iii) the depreciation factor, $\delta = 0.014$; (iv) the capital income share, $\alpha = 0.32$; and (v) $\rho_x = 0.95$, the autoregressive of the productivity process.[8] The low Frisch elasticity, 0.001, limits the response of hours to wage changes and helps the model to match the observation that interest rate shocks are followed by reductions in hours worked and not by increases as we would have with a higher elasticity. The capital income share is a conventional value in the literature. The depreciation rate and the inverse of the elasticity of intertemporal substitution are taken from Neumeyer and Perri (2005), who find that a high depreciation value is appropriate for Argentina. The absence of equivalent measures for the other countries forces us to use Argentina's depreciation rate across the eight different versions of our model. The low elasticity of intertemporal substitution might reflect the more limited set of assets available in the countries in our sample. The autoregressive process is more difficult to pin down because of the absence of good data on the Solow residual. Following the suggestion

[6] We want to be careful here. We found that for our calibration and estimated processes, these higher orders were not important. There might exist parameter values for which these orders are relevant.

[7] Ideally, we would like to estimate the structural parameters of the model. The lack of reliable high-frequency data, however, and the nonlinear nature of our solution method make such an enterprise infeasible.

[8] There is one additional parameter, ω, which is irrelevant for the dynamics of the model, since it fixes only the percentage of hours worked in the deterministic steady state.

of Mendoza (1991), we select a value slightly lower than the one commonly chosen for rich economies. We checked that our results are robust to this choice by recalibrating and recomputing the model for values of ρ_x as low as zero without finding much difference in the effects of volatility shocks.

The rest of the parameters differ across each version of the model. First, we set the parameters for the law of motion of the real interest rate equal to the median of the posterior distributions reported in Section II. Second, we set the discount factor equal to the inverse of the gross mean real interest rate of each country $\beta = (1 + r)^{-1}$. Conditional on the previous choices, we pick the last four parameters to match moments of the ergodic distribution of the model with moments of the data. We select four moments in the data: (i) output volatility; (ii) the volatility of consumption relative to the volatility of output; (iii) the volatility of investment with respect to output; and (iv) the ratio of net exports over output. The parameters are (i) σ_x, the standard deviation of productivity shocks; (ii) ϕ, the adjustment cost of capital; (iii) D, the parameter that controls average value debt; and (iv) the holding cost of debt, Φ_D.

If we were using the steady state to calibrate the model, we could pick each parameter to match almost independently each of the four moments of interest in the data (for example, σ_x would nail down output volatility and D would determine the ratio of net exports over output). In the ergodic distribution, in contrast, the moments are all affected by a nonlinear combination of the parameters. Hence, moving one parameter to improve, say, the fit of volatility of consumption relative to the volatility of output might worsen the fit of the volatility of investment with respect to output. We fix this problem by minimizing a quadratic form of the distance of the moments of the model with those of the moments of the data. In addition, to discipline the exercise further, we pick only two Φ_Ds, one for the two most volatile countries, Argentina and Ecuador, and another for Venezuela and Brazil, which is 50 percent of the first value. Our choices for Φ_D are consistent with the values reported in Uribe and Yue (2006). Their small value helps to close the model without significantly affecting its dynamic properties.

The four empirical moments to be matched are reported in Table 5 and they are based on H-P filtered quarterly data. The row nx/y displays the average of net exports as a percentage point of output. A positive value means that the country is running a trade surplus.

To compute the moments of the ergodic distribution generated by our model, we proceed as follows. First, we simulate the model, starting from the steady state, for 2,096 periods. We disregard the first 2,000 periods as a burn-in and use the last 96 periods to compute the moments of the ergodic distribution. As we mentioned before, since our data come in quarterly frequency, we build quarters of data from the model-simulated variables, and we H-P filter them. We repeat this exercise 200 times to obtain the mean of the moments over the 200 simulations. We checked the stability of our simulations. The country-specific results of our calibration are summarized in Table 6.

Our values for D roughly align with the ratio of net exports to output (a higher ratio signaling a higher foreign debt). Higher values for Φ_D mainly reflect higher volatility of consumption. Higher volatility of output appears in higher values of σ_x. The values of ϕ are more difficult to interpret.

Table 5—Empirical Second Moments

	Argentina	Ecuador	Venezuela	Brazil
σ_y	4.77	2.46	4.72	4.64
σ_c/σ_y	1.31	2.48	0.87	1.10
σ_i/σ_y	3.80	9.32	3.42	1.65
nx/y	1.77	3.86	4.07	0.10

Table 6—Summary Calibration

	Argentina		Ecuador		Venezuela		Brazil	
	M1	M2	M1	M2	M1	M2	M1	M2
β	0.980	0.980	0.989	0.989	0.993	0.993	0.993	0.993
Φ_D	0.001	0.001	0.001	0.001	$5e-4$	$5e-5$	$5e-4$	$5e-5$
D	4	4	13	13	22	22	3	2
ϕ	95	85	35	20	12	25	50	60
σ_x	0.015	0.014	0.0055	0.0058	0.013	0.013	0.013	0.013

III. Results

In this section, we analyze the quantitative implications of our model. First, we report the moments generated by the model and compare them with the data. Second, we look at the impulse response functions (IRFs) of shocks to the country spreads and its volatility. If we compare the volatility shocks to the international risk-free real rate and to the country spreads, $\sigma_{tb,t}$ and $\sigma_{r,t}$, the latter is, on average, between three to ten times larger than the former and has a time-varying component that is between two to four times bigger. These relative sizes justify why we concentrate on the study of IRF shocks to the country spreads and their volatility and forget about shocks to the international risk-free real rate. Third, we decompose the variance of aggregate variables among different shocks and we close by discussing the effect of volatility in cross-country business cycle correlations.

A. *Moments*

Our first exercise is to compute the model-based moments with those of the data. For each country, Table 7 reports the results for both versions of the model (M1 and M2) and the data moments.[9] For both calibrations, the model does a fair job at matching the moments of the data. Even if we have used four of the moments for calibration, the relative success of the model is no small accomplishment, as open economy models often have a tough time accounting for the data for any combination of parameter values. We found it challenging to match simultaneously the volatility of consumption over the volatility of output and the ratio of net exports-to-output.

We highlight two results from Table 7. First, the model roughly reproduces the relative volatility of net exports over output, although it tends to overestimate it. This

[9] In our simulation, to compute moments, we truncated the innovations of the shocks to be less than one standard deviation. Otherwise, the model might wander away from the ergodic distribution for a long time and it is difficult to compute accurate first and second moments.

Table 7—Second Moments

	Argentina			Ecuador		
	Data	M1	M2	Data	M1	M2
σ_y	4.77	5.30	4.83	2.46	2.23	2.15
σ_c/σ_y	1.31	1.54	1.10	2.48	2.13	1.43
σ_i/σ_y	3.81	3.90	3.66	9.32	9.05	9.13
σ_{nx}/σ_y	0.39	0.48	0.54	0.65	1.77	1.68
$\rho_{nx,y}$	−0.76	0.05	0.07	−0.60	−0.04	−0.04
nx/y	1.78	1.75	1.92	3.86	3.95	3.88
	Venezuela			**Brazil**		
	Data	M1	M2	Data	M1	M2
σ_y	4.72	4.56	4.43	4.64	4.52	4.37
σ_c/σ_y	0.87	0.51	0.68	1.10	0.44	0.45
σ_i/σ_y	3.42	3.81	3.64	1.65	1.67	1.68
σ_{nx}/σ_y	0.19	1.60	0.93	0.23	0.60	0.68
$\rho_{nx,y}$	−0.11	−0.10	0.01	−0.26	0.18	0.21
nx/y	4.07	4.14	4.03	0.1	0.52	0.30

finding is relevant because this is a moment that we did not use in the calibration and that small open economy models have difficulty matching. Second, it is interesting that the moments with and without correlated innovations are quite similar.

B. *Impulse Responses*

Our second exercise looks at the IRFs of the model to shocks in the country spreads and volatility. Computing these IRFs in a nonlinear environment is somewhat involved, since the IRFs are not invariant to rescaling and to the previous history of shocks. We refer the reader to the online Appendix for details on how we construct them.

Argentina.—We start by analyzing Argentina. The graphs for the other three countries will follow the same format in the order of presentation. In Figure 5 we plot the IRFs to three shocks (rows) of consumption (first column of panels), investment (second column), output (third column), labor (fourth column), the interest rate (fifth column), and debt (the sixth column). Interest rates are expressed in basis points while all other variables are expressed as percentage deviations from the mean of their ergodic distributions.

The first row of panels plots the IRFs to a one-standard-deviation shock to the Argentinean country spread, $u_{r,t}$ in the M1 version of the model. Following a 385 annual basis point rise (which corresponds to an increase of nearly 33 basis points at a monthly rate) in Argentina's spread, the country experiences a persistent contraction, with consumption dropping 3.20 percent upon impact and investment falling for seven quarters. To match the second moments found in the Argentinean data, our model requires a significant degree of adjustment costs in investment. Consequently, we find that the decline in output is highly persistent: after 16 quarters, output is still falling (at that time it is -1.16 percent below its original level). Labor starts by slightly increasing (due to the negative wealth effects) but later falls (by a very small margin given our preferences) due to the reduction in investment and the subsequent

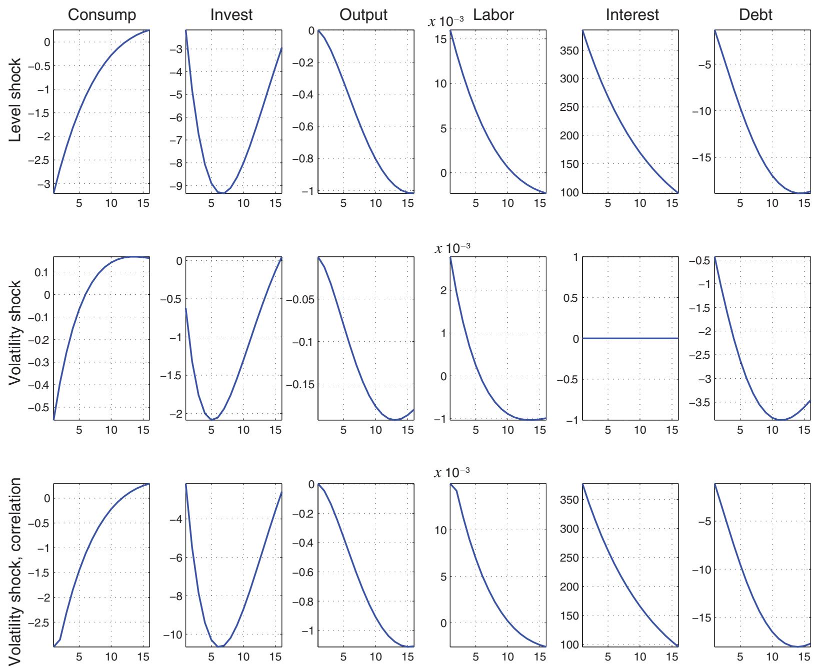

Figure 5. IRFs, Argentina

decrease of marginal productivity. Debt falls for 14 quarters, with a total reduction of nearly 19 percent of the original value of the liability. The intuition for the drop in output, consumption, and investment is well understood (see Neumeyer and Perri 2005). A higher r_t raises the service payment of the debt, reduces consumption, forces a decrease in the level of debt (since now it is more costly to finance it), and lowers investment through a nonarbitrage condition between the returns to physical capital and to foreign assets. We include this exercise to show that our model delivers the same answers as the standard model when hit by equivalent level shocks, and to place in context the size of the IRFs to volatility shocks.

The contraction in economic activity may seem large. It is close, however, to the empirical estimates reported by Uribe and Yue (2006). These authors, for instance, estimate that for the increase in the spread that we consider, output will fall a bit less than 0.80 percent. However, Uribe and Yue also find that it takes only about two years for output to reach its lowest level. Their result raises the question of whether our model may overpredict the persistence of output because of the large investment adjustment cost that we need to account for investment volatility.

The second row of panels plots the IRFs to a one-standard-deviation shock to the volatility of the Argentinean country spread, $u_{\sigma,t}$. To put a shock of this size in perspective, our econometric estimates of Section II indicate that the collapse of Long Term Capital Management in 1998 meant a positive volatility shock of 1.5 standard deviations, and that the 2001 financial troubles amounted to two repeated shocks of roughly one standard deviation.

This second row is one of the main points of our paper. First, note that there is no movement on the domestic interest rate faced by Argentina or its expected value.

Second, there is (i) a contraction in consumption (0.56 percent at impact), (ii) a slow decrease of investment (after five quarters it falls 2.09 percent), (iii) a slow fall in output (after four years, it falls 0.19 percent), (iv) a slight increase in labor, which falls later, and (v) a shrinking of debt upon impact, which keeps declining until it reaches its lowest level (-3.88 percent), roughly three and a half years after the shock. These IRFs show how increments in risk have real effects on the economy, even when the real interest rate remains constant.

To understand the economic logic behind this mechanism, we go back to the equilibrium conditions of the model. Our starting point is equation (8), which we can rewrite as

$$\frac{1}{1 + r_t} - \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \Phi_D (D_{t+1} - D). \tag{12}$$

A volatility shock leaves r_t unchanged, but it raises $\mathbb{E}_t \lambda_{t+1}/\lambda_t$. Why? The Lagrangian λ_t is the marginal utility of consumption. A higher real interest rate risk causes more volatile consumption in the future. Our estimate for η_r implies that a typical stochastic volatility shock in Argentina raises the standard deviation of an innovation to the interest rate by a factor of 1.58 ($= \exp(\eta_r)$). Thus, households may face a 608 (1.58×385) basis point surge in the annual interest rates on their debt obligations if a one-standard-deviation shock to the interest rate materializes tomorrow. Since marginal utility is convex, Jensen's inequality tells us that $\mathbb{E}_t \lambda_{t+1}$ rises.[10] The total increment of the ratio $\mathbb{E}_t \lambda_{t+1}/\lambda_t$ is smaller because, as we saw in the IRFs, consumption drops at impact and recovers in the following periods, which increases marginal utility today and λ_t. In our calibration, this second effect is dominated by the dispersion of marginal utilities. Hence, the left-hand side of (12) falls and we can make the equation hold with equality only if D_{t+1} falls as well. The intuition is that holding foreign debt is now riskier than before, and therefore the representative household wants to reduce its exposure to this risk.[11]

How can the representative household reduce its foreign debt? Since the country is not more productive than before, the only way to do so is to increase net exports either by working more or by reducing national absorption (the sum of consumption and investment). The first alternative, working more, is limited by the increase in marginal disutility. Hence, the household must reduce national absorption. This can be done in three different ways: (i) consuming and investing less; (ii) investing more and consuming sufficiently less that national absorption falls; or (iii) consuming more and investing sufficiently less that national absorption falls. Option (iii) does not smooth utility over time for standard parameter values (although there are unrealistic combinations of parameter values where they may be the optimal response).[12] Option (ii) is eliminated because, as we will show below, investment must fall. Option (i) is, therefore, the only alternative.

[10] Third-order terms are determined by the fourth derivative of the utility function, which has to be positive to induce the household to respond to volatility shocks by lowering its debt.

[11] This argument is independent of technology shocks. Even with $\sigma_x = 0$, a volatility shock increases the dispersion of future marginal utilities through more dispersed real interest rate levels.

[12] In the absence of adjustment costs, investment still falls but consumption increases at impact. Without adjustment costs, however, the model does very poorly accounting for the moments of the data.

To further understand why investment falls, we rewrite the Euler equation as

$$\beta \mathbb{E}_t \left[\frac{(1 - \delta) q_{t+1} + R_{t+1}}{q_t} \frac{\lambda_{t+1}}{\lambda_t} \right] = 1,$$

where we have defined the marginal cost of a unit of installed capital K_{t+1} in terms of consumption units as $q_t = \varphi_t / \lambda_t$ and R_t is the rental rate of capital. Then,

$$\beta \mathbb{E}_t \frac{(1 - \delta) q_{t+1} + R_{t+1}}{q_t} \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} + \operatorname{cov} \left(\frac{(1 - \delta) q_{t+1} + R_{t+1}}{q_t}, \frac{\lambda_{t+1}}{\lambda_t} \right) = 1.$$

In this expression, the conditional covariance of the return to capital and the ratio of Lagrangians decreases when volatility rises. Households use debt to smooth productivity shocks. Imagine that we are in a situation with low volatility. Then, after a negative shock to X_t and the subsequent fall in the return to capital, consumption drops by a small amount (and hence the ratio of Lagrangians rises by a small amount) because debt increases to smooth consumption. When volatility is high, however, the household accepts a bigger reduction in consumption after a productivity shock, since increasing the debt level carries a large interest rate risk. At the same time, we just saw that $\mathbb{E}_t \lambda_{t+1} / \lambda_t$ increases only by a small amount because of the interaction of mean-reverting consumption with the increased dispersion of marginal utilities. Therefore, the only term that can change in our previous equation to accommodate the lower covariance is a higher $\mathbb{E}_t \big((1 - \delta) q_{t+1} + R_{t+1} \big) / q_t$. This goal is accomplished with a lower investment today.[13]

A slightly different way to understand the fall in investment after a volatility shock is to note that foreign debt allows the household to hedge against the risk of holding physical capital. This hedging property raises the desired amount of physical capital. The total effect is, however, small because debt also allows the representative household to rely less on physical capital as a self-insurance device. A higher volatility of the real interest rate makes the hedge provided by foreign debt less attractive, it induces the household to reduce its debt, and, hence, it also lowers its holdings of physical capital with a fall in investment. This point is related to the response of precautionary savings to an increase in capital income risk in models with incomplete markets, where it is also the case that an increase in risk lowers investment and capital for empirically plausible parameterizations (see George-Marios Angeletos 2007).

To quantify the debt reduction mechanism, we show in Figure 6 the evolution of debt, current account, and net exports (all linked with debt through the budget constraint). Debt is expressed as a percentage of quarterly output, and the bottom two panels are in percentage points of their ergodic means. After a volatility shock, debt falls for a value equal to 2.9 points of quarterly output after three years, the current account improves 1.29 percent, and net exports rise 1.18 percent, both at impact. This figure suggests that volatility is a potentially substantial factor behind movements in current accounts and net exports in countries like Argentina.

[13] The fall of investment requires either a positive standard deviation of the productivity shock and/or adjustment costs. If none of these mechanisms is present, the return to capital is risk free and the covariance is zero.

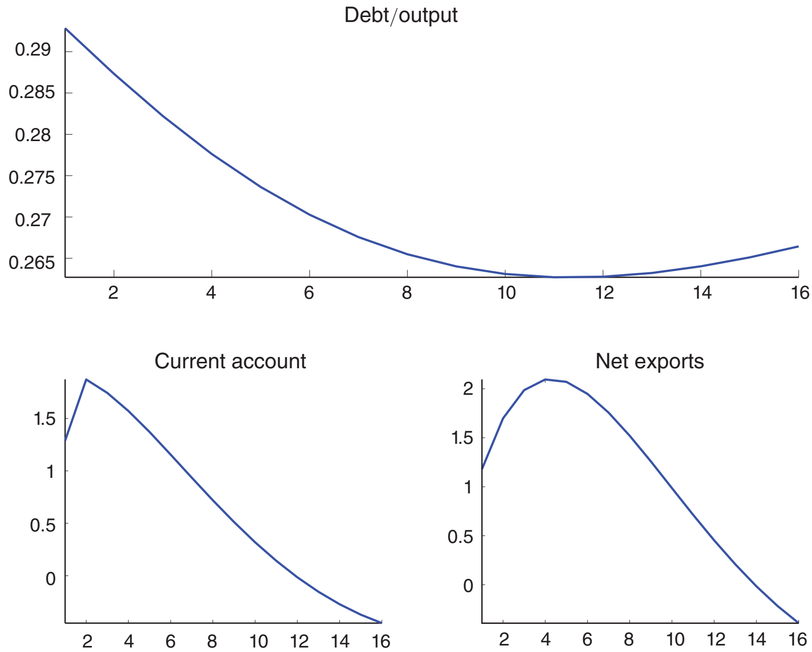

Figure 6. IRFs Debt/Output, Current Account, Net Exports

The last row in Figure 5 plots the IRFs in the M2 version of the model. In this row, we plot the IRFs after a one-standard-deviation level shock that is accompanied by a $\kappa-$standard deviation shock to volatility. The pattern of the IRFs is qualitatively the same as in the first row. The lesson from this third row is that our results are robust to the correlation between innovations.

We conclude by pointing out two features of our model. First, our results come in a model without working capital, a mechanism often added to improve the performance of international macro models. As shown in the online Appendix, working capital makes our findings even stronger. Second, we do not have any of the real-option effects of risk emphasized by the literature, for example, when we have irreversibilities (Bloom 2009). Introducing those effects explicitly is difficult with our perturbation approach because of the nondifferentiability of threshold decision rules created by real-option environments. However, real-option effects would increase the impact of shocks to volatility on investment. Therefore, our results are likely to be a lower bound to the implications of time-varying risk. Bloom, Jaimovich, and Floetotto (2008) explore the real-option effects of volatility shocks in a model calibrated for the US economy, but a more thorough investigation of the interaction between our higher-order terms and real-option effects remains an open question.

Ecuador.—Next, we turn to Ecuador, whose IRFs are plotted in Figure 7. The IRFs are similar to those in the Argentinian case. There is a decline in economic activity with responses qualitatively similar to, although somewhat smaller than, those for Argentina. After a shock to volatility, consumption drops 0.44 percent upon impact, investment 0.66 percent, and debt 0.08 percent. Investment falls for five quarters

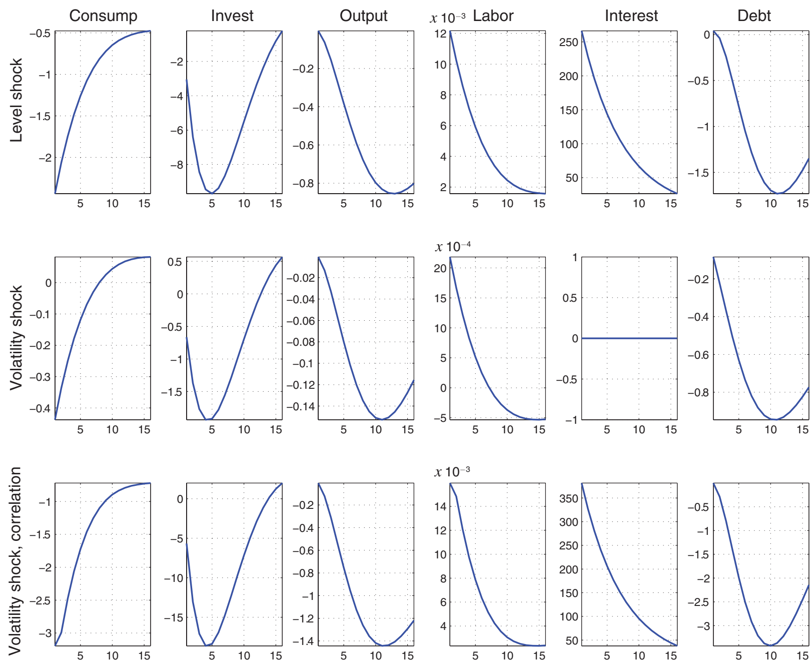

FIGURE 7. IRFs, ECUADOR

and output for around three years, when debt also reaches its lowest value, 0.95 percent below its original level. It is perhaps surprising, given Ecuador's large debt-to-output ratio (net exports are 3.9 percent of output), that the results, even if still large, are smaller than for Argentina. The key for this finding is that Ecuador enjoys a smaller standard deviation in the innovation to volatility shocks, η_r.

It is interesting, however, to look at the third row of IRFs, when the innovations are correlated. While a shock to the interest rate raises it by 266 annual basis points, a correlated shock raises it by 382 basis points. This is due to the high estimated correlation of 0.89. After a one-standard-deviation shock to the interest rate and a 0.89-standard-deviation shock to its volatility, output takes a dive, falling 1.44 percent after three years. When we evaluate this last row in conjunction with the results of our econometric exercise, we can venture the hypothesis that Ecuador's debacle in the late 1990s started with a sharp volatility shock in 1998, 2.5 standard deviations in size.

Venezuela.—Our next IRFs are those of Venezuela in Figure 8. Although the qualitative shape of the IRFs is similar to the two previous cases, now the response to a volatility shock is milder. The similar net export-to-output ratios in Ecuador and Venezuela could have made us suspect that these countries should experience equivalent contractions following a volatility shock. Yet a look at Figures 7 and 8 reveals that consumption drops nearly nine times as much in Ecuador as in Venezuela; large indebtedness alone does not generate large recessions. Furthermore, the size of the volatility shock, η_r, is essentially the same for the two countries. What matters in this case is the difference in the average standard deviation of the level shock, σ_r (the

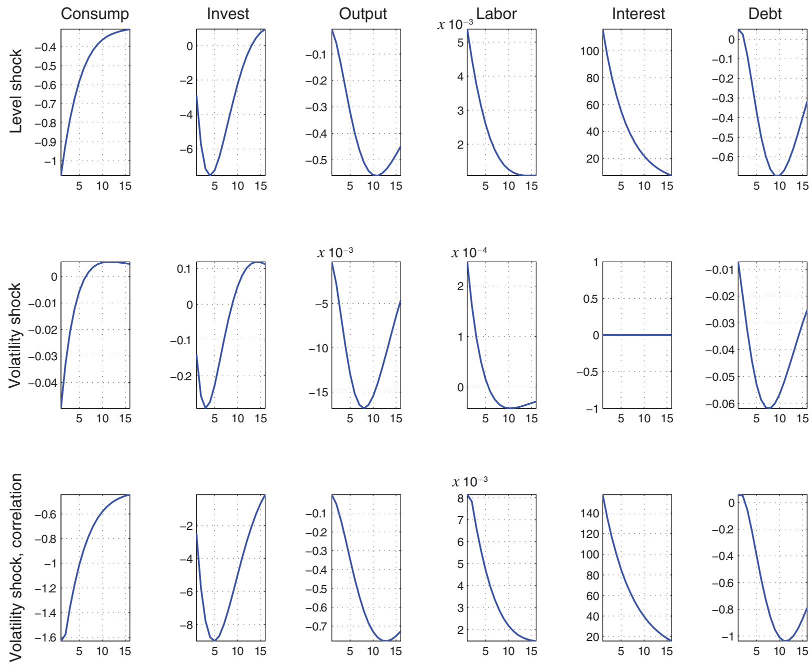

Figure 8. IRFs, Venezuela

posterior median of σ_r for Venezuela is -6.88 while for Ecuador it is -6.06). A higher σ_r increases the mean volatility of the economy and, with it, the size of the IRFs.

To better compare the IRFs across countries, we propose the following experiment. At time t, the economy is hit by a one-standard-deviation volatility shock, which is followed by a shock to the interest rate level, u_r, at time $t + 1$. An Ecuadorian household facing this scenario understands that annualized interest rates will increase tomorrow by as much as 4 percentage points. The same sequence of events means that Venezuelans will see an increase in annualized interest rates of 1.7 percentage points. Clearly, Ecuador faces a rather stringent situation, which explains the larger recession in this country.

Brazil.—Figure 9 presents Brazil's responses to level and volatility shocks. The main result for Brazil's case is, once more, the similarity of the IRFs to previous findings, although now output's response is muted, even more so than in the case of Venezuela. The stronger response to volatility shocks in Venezuela than in Brazil is accounted for by Venezuela's larger shocks and debt-to-output ratio. This remark further illustrates how the mechanism through which volatility affects real variables is the increased exposure to consumption risk implied by D_t when volatility rises.

C. *Variance Decomposition*

An additional exercise is to measure the contribution of each of the three shocks in our model to aggregate fluctuations. The task is complicated because, with a

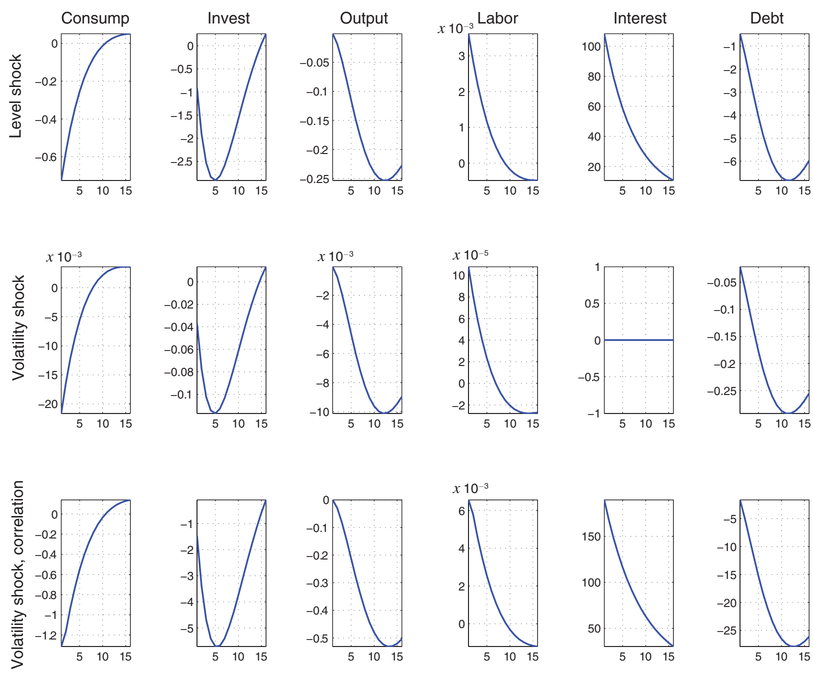

Figure 9. IRFs, Brazil

third-order approximation to the policy function and its associated nonlinear terms, we cannot neatly divide total variance among the three shocks as we would do in the linear case.

A possibility is to set the realizations of one or two of the shocks to zero and measure the volatility of the economy with the remaining shocks. The agents in the model still think that the shocks are distributed by the law of motion that we specified: it just happens that their realizations are zero in the simulation. We explore six possible combinations: (i) the benchmark case with all three shocks; (ii) when we have a shock only to productivity; (iii) when we have a shock to productivity and to the interest rate (with volatility fixed at its unconditional value); (iv) when we have a shock only to the interest rate; (v) when we have shocks to interest rate and to volatility; and (vi) when we have shocks only to volatility.

Table 8 reports the results for Argentina (the results for the other three countries can be found in the online Appendix). When we allow only productivity to change over time, output has fluctuations that are around 105 percent of the observed ones. Remember that, in the absence of good data on the Solow residual, we are calibrating productivity shocks to match output volatility, and hence this 105 percent is not sensu stricto a measurement of the impact of productivity innovations. A more informative finding is that, counterfactually, the standard deviation of consumption falls below the standard deviation of output. This result is of interest because one of the most salient characteristics of the business cycle of emerging economies is that consumption is more volatile than output. In a model with such a strong desire for consumption smoothing as this one, it is difficult to get around this result when only productivity shocks are considered.

TABLE 8—VARIANCE DECOMPOSITION: ARGENTINA

	All three shocks	Only prod.	Prod. and rate	Rate	Rate and volatility	Only vol.
σ_y	5.30	5.01	5.09	0.68	1.23	0.16
σ_c	8.12	2.72	5.04	4.36	7.76	0.77
σ_i	20.6	5.10	13.3	12.2	20.1	3.09
σ_{nx}	2.53	1.77	4.52	6.64	4.15	4.16

When we add a real interest rate shock, the volatility of output does not increase much. The reason is that, since both shocks are independent, their effects often cancel each other (for instance, a positive technological shock happens at the same time as a rise in the real interest rate). In comparison, the simultaneous presence of both shocks substantially raises the volatility of consumption, which now becomes almost as volatile as output. While the household wants to smooth out productivity shocks, it prefers to pay back the debt and adjust consumption as a response to a positive shock to the real interest rate. For a similar reason, investment becomes more volatile. These two mechanisms are seen more clearly in the case with only interest rate shocks. While output variability drops to only 0.68, the standard deviation of consumption is still 4.36 and the standard deviation of investment 12.20.

The fourth case is when we have rate and volatility shocks. The standard deviation of output rises to 1.23, 25 percent of the observed volatility, consumption goes to 7.76, and investment to 20.1. The final case is when we have only volatility shocks. In this situation, the standard deviation of output is low, 0.16 (after all, volatility per se appears only in the third-order term of the policy function). For output, the interaction effect of the rate and volatility shocks is noticeable: jointly they generate a standard deviation of 1.23, while separately they induce standard deviations of 0.68 and 0.16. The difference is accounted for by the cross-terms of interest rate and volatility shocks that appear in the policy function of the agents. Volatility alone, however, makes a relatively important contribution to the fluctuations of consumption (the standard deviation is 0.77 with volatility shocks alone) and investment (standard deviation of 3.09).

D. *Cross-Country Correlations*

One last interesting consequence of volatility shocks is that they might help to circumvent a feature of international macro models driven by shocks to spreads. If these shocks are common across countries, they might imply cross-correlations of output that are counterfactually high.

Volatility shocks reduce this cross-correlation through two effects. First, as documented in Section II, each country in our sample experiences quite a distinct process for volatility. Second, countries endogenously respond in an asymmetric way to these volatility shocks because of their different average levels of debt. To assess these mechanisms, we compute the cross-correlations of output for Argentina, Ecuador, Brazil, and Venezuela in two cases: (i) when countries have (a) the same innovations to the technology shocks; (b) the spread shocks observed in the data; and (c) there are no volatility shocks (that is, volatility is always at its mean level);

and (ii) when we keep (a) and (b) from case (i), but now we feed the model with the empirical volatility shocks for the country spreads that we estimated with our particle filter smoother. We fix the technology shocks to be equal across countries merely for convenience. While doing so increases the cross-correlation of output (in the data, technology shocks as defined by our model have a correlation less than one), at the same time it allows us to factor out the possible indirect effects of those shocks that may make the interpretation of results harder.

Our main finding is that volatility shocks significantly lower the cross-country correlation of output. In case (i), the simulated average cross-country correlation of output is 0.80, higher than in our sample, where it is only 0.58. In comparison, in case (ii), the simulated average cross-country correlation is 0.70. Thus, the introduction of volatility shocks cuts nearly in half the distance between the cross-correlations of output in the model and in the data. For instance, for the pair Argentina-Ecuador, the cross-correlation of output falls from 0.62 to 0.49 or for Brazil-Ecuador from 0.71 to 0.59. It falls less for the pair Brazil-Venezuela precisely because we know that, for these countries, volatility shocks are less important.

IV. Causes and Policy Implications of Volatility

One weakness of our model is that we do not offer a theory of why real interest rate volatility evolves over time. Instead, we model it as an exogenously given process. Clearly, our position misses important aspects of the data. For instance, the elevated level of volatility in advanced economies from 2008 to 2010 is directly linked to the weakening of underlying economic conditions caused by an unusual realignment of asset prices.

We have presented a strong case, however, that volatility shocks may be a significant mechanism behind the business cycle, at least for some countries. This justifies our venturing a few conjectures on the origins and policy implications of volatility. The themes below are elaborated in further detail in Fernández-Villaverde and Rubio-Ramírez (2010).

Following the literature, we can interpret a shock to the volatility of spreads from three perspectives. First, higher volatility may reflect more risk surrounding the world financial markets. Times generally understood as uncertain, such as the Asian crisis, the Long Term Capital Management fiasco, or the Great Recession of 2007–2009, are associated with heightened volatility. A second interpretation builds on the idea that volatility is related to the volume of information (Stephen Ross 1989; and Torben Andersen 1996). During turbulent times, news arrives more frequently (or perhaps keener attention is devoted to it), inducing large volumes of trade in foreign debt and rising volatility in interest rates. Furthermore, since the markets for debt issued by emerging economies are much smaller than those for developed economies, this could explain, in part, why the arrival of information brings larger swings in the former than in the latter. A third possible source of volatility is political instability. The quintessential example, without a doubt, is the political and subsequent economic turmoil in Mexico after the assassination of the presidential candidate Luis Donaldo Colosio in 1994. On that occasion, the sudden changes in the volatility of Mexico's country spread were triggered, to a considerable extent, by a home-brewed event that had few links, by itself, to fundamentals of the economy such as productivity or physical capital. Other recent instances of politically induced volatility shocks include the 2002 Venezuelan coup d'état attempt and the 2008 debt renegotiation announcements in Ecuador. Again, the higher political instability of emerging economies may account for the higher volatility of spreads.

These conjectures on the origins of volatility lead us to guess that models that explicitly deal with the arrival and processing of information by financial markets and their interactions with the macroeconomy and the political-economic equilibrium are natural starting points for the task of endogenizing volatility.

The final step is to think about two main policy implications. The first, and most obvious, is that if volatility shocks affect aggregate fluctuations in a quantitatively noticeable fashion, it could be relevant for policymakers to consider volatility when implementing fiscal and monetary policy. In the same way that, to make a simple analogy, the classical Ramsey approach suggests that debt should be used as an absorber of technology shocks (Varadarajan V. Chari, Larry J. Christiano, and Patrick J. Kehoe 1994), an extended Ramsey optimal policy would prescribe how debt, and fiscal policy in general, need to respond to volatility shocks. With respect to monetary policy, an optimal interest rate rule followed by the central bank could also depend on the level of volatility in addition to the traditional dependence on the levels of inflation and the output gap. In fact, recently, Geert Bekaert, Marie Hoerova, and Marco Lo Duca (2010) have documented that, in the United States, the Federal Reserve reacts to increased stock market volatility by easing monetary policy.

A second policy consideration is that countries subject to volatility shocks could require a more sophisticated management of their debt. In particular, countries may have to evaluate both the future paths of level and volatility of the interest rates when they decide their debt maturity structure. This is key in an environment with noncontingent public debt, arguably a fair description of reality.[14] The majority of the existing results in the literature that demonstrate the near optimality of noncontingent debt and how to manage it rely on the existence of a set of maturities richer than the set of events (Angeletos 2002, theorem 1). Unfortunately, once we factor in level and volatility shocks, emerging economies may face too many different states to assume that the standard maturities traded (especially for small emerging economies) are rich enough. Thus, volatility highlights the importance of characterizing optimal government debt management strategies in a world with limited maturities traded, a field currently not fully explored from a theoretical perspective.

V. Summary and Directions for Future Research

Our empirical evidence shows that time-varying volatility is a key feature of the real interest rate faced by emerging economies. This changing volatility has a quantitatively important effect on the dynamics of an otherwise standard small open economy business cycle model, even when the real interest rate remains constant. The mechanism behind the real effects of volatility is that households with precautionary behavior will change their holding of foreign debt as a response to changes in volatility to reduce future fluctuations of marginal utility.

[14] With a complete set of Arrow securities, countries could hedge any volatility risk, making the whole problem rather uninteresting.

Our investigation opens the door to a set of interesting questions. First, and most obviously, why does volatility change over time? Is it related to some states of the economy? How does it interact with other phenomena, such as debt default, debt renegotiation, or financial market integration? Can we filter from the data changes in default probabilities from changes in the level and volatility of the market price for risk?

Second, we would like to evaluate the possibilities of having time-varying volatilities in other aspects of the economy. As an example among many others, Aguiar and Gopinath (2007) have argued that one contributing factor behind business cycle fluctuations in emerging economies might be recurrent changes in the productivity growth trend. It could be profitable to explore the consequences of introducing stochastic volatility in these changes.

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