(******** **Linear system of rational expectations equations + expectational errors. Note that** $\mathrm{Ex_t}$ **means** $\mathrm{E_t x_{t+1}}$. ********)

$\mathrm{Omegah} = (\lambda_{star} - \beta\,(1+\lambda_a)\,\gamma_h)\,(\lambda_{star} - \gamma_h);$

$\mathrm{Omegae} = (\lambda_{star} - \beta\,\gamma_e)\,(\lambda_{star} - \gamma_e);$

$\Delta\mathrm{log}\lambda\mathrm{start} = (1-(1-\phi)\,\alpha)^{-1}\left(\lambda z_t + \nu z_t - \nu z_{t-1} + (1-\phi)\,\alpha\left(\lambda q_t + \nu q_t - \nu q_{t-1}\right)\right);$ (* $\Delta\log\,\lambda_{*,t}$ *)

$\Delta\mathrm{log}\lambda\mathrm{kt} = (1-(1-\phi)\,\alpha)^{-1}\left(\lambda z_t + \nu z_t - \nu z_{t-1} + \lambda q_t + \nu q_t - \nu q_{t-1}\right);$ (* $\Delta\log\,\lambda_{k,t}$ *)

$\mathrm{E}\Delta\mathrm{log}\lambda\mathrm{start} = (1-(1-\phi)\,\alpha)^{-1}\left(\rho_{\lambda z}\,\lambda z_t + (\rho_{\nu z}-1)\,\nu z_t + (1-\phi)\,\alpha\left(\rho_{\lambda q}\,\lambda q_t + \left(\rho_{\nu q}-1\right)\nu q_t\right)\right);$

(* $\mathrm{E_t}\Delta\log\,\lambda_{*,t+1}$ *)

$\mathrm{E}\Delta\mathrm{log}\lambda\mathrm{kt} = (1-(1-\phi)\,\alpha)^{-1}\left(\rho_{\lambda z}\,\lambda z_t + (\rho_{\nu z}-1)\,\nu z_t + \rho_{\lambda q}\,\lambda q_t + \left(\rho_{\nu q}-1\right)\nu q_t\right);$ (* $\mathrm{E_t}\Delta\log\,\lambda_{k,t+1}$ *)

REequations = {

(* ---------- **Household equations** -------------- *)

(* **1. Marginal utility of consumption.** *)

$\mathrm{Omegah}\,\mu h_t == -\left(\lambda_{star}^2 + \beta\,(1+\lambda_a)\,\gamma_h^2\right)\mathrm{ch}_t + \lambda_{star}\,\gamma_h\,(\mathrm{ch}_{t-1} - \Delta\mathrm{log}\lambda\mathrm{start}) +$
$\beta\,(1+\lambda_a)\,\gamma_h\,\lambda_{star}\,(\mathrm{Ech}_t + \mathrm{E}\Delta\mathrm{log}\lambda\mathrm{start}) - \beta\,\gamma_h\,\lambda_a\,(\lambda_{star} - \gamma_h)\,\rho_{\lambda a}\,\lambda a_t,$

(* **2. Residential housing decision** *)

$\mathrm{ql}_t + \mu h_t == \dfrac{\lambda_a}{1+\lambda_a}\,\rho_{\lambda a}\,\lambda a_t + \beta\,(1+\lambda_a)\left(\mathrm{Eql}_t + \mathrm{E}\mu h_t\right) + (1-\beta\,(1+\lambda_a))\,(\rho_\varphi\,\varphi_t - \mathrm{Lh}_t),$

(* **3. Labor supply** *)

$w_t + \mu h_t == \psi_t + \eta\,n_t,$

(* **4. Saving decision** *)

$\mu h_t - R_t == \dfrac{\lambda_a}{1+\lambda_a}\,\rho_{\lambda a}\,\lambda a_t + \mathrm{E}\mu h_t - \mathrm{E}\Delta\mathrm{log}\lambda\mathrm{start},$

(* ---------- **Entrepreneur equations** -------------- *)

(* **5. Marginal utility of consumption.** *)

$\mathrm{Omegae}\,\mu e_t ==$
$-\left(\lambda_{star}^2 + \beta\,\gamma_e^2\right)\mathrm{ce}_t + \lambda_{star}\,\gamma_e\,(\mathrm{ce}_{t-1} - \Delta\mathrm{log}\lambda\mathrm{start}) + \beta\,\gamma_e\,\lambda_{star}\,(\mathrm{Ece}_t + \mathrm{E}\Delta\mathrm{log}\lambda\mathrm{start}),$

(* **6. Commercial housing decision** *)

$\mathrm{ql}_t + (1-(1-\mathrm{Iq}_l)\,\theta\,\lambda_{star}\,\mu_{boe})\,\mu e_t == \mathrm{Iq}_l\,\theta\,\lambda_{star}\,\mu_{boe}\left(\theta_t + \mu b_t\right) + (1-\theta\,\lambda_{star}\,\mu_{boe})\,\mathrm{E}\mu e_t +$
$(1-\beta-\beta\,\theta\,\lambda_a)\left(\mathrm{Ey}_t - \mathrm{Le}_t\right) + (\beta + \mathrm{Iq}_l\,\theta\,\lambda_{star}\,\mu_{boe})\,\mathrm{Eql}_t + \mathrm{Iq}_l\,\theta\,\lambda_{star}\,\mu_{boe}\,\mathrm{E}\Delta\mathrm{log}\lambda\mathrm{start},$

(* **7. Labor demand** *)

$w_t == y_t - n_t,$

(* **8. Investment decision** *)

$\mathrm{qk}_t == \lambda_K^2\,\Omega\,(i_t - i_{t-1} + \Delta\mathrm{log}\lambda\mathrm{kt}) - \beta\,\lambda_K^2\,\Omega\,(\mathrm{Ei}_t - i_t + \mathrm{E}\Delta\mathrm{log}\lambda\mathrm{kt}),$

(* **9. Capital decision** *)

$\mathrm{qk}_t + \left(1-(1-\mathrm{Iq}_k)\,\theta\,\theta K\,\lambda_q^{-1}\,\mu_{boe}\right)\mu e_t ==$
$\left(1-\theta\,\theta K\,\lambda_q^{-1}\,\mu_{boe}\right)\mathrm{E}\mu e_t + \alpha\,\beta\,(1-\phi)\,y_k\left(\mathrm{Ey}_t - k_t\right) + \beta\,(1-\delta)\,\lambda_K^{-1}\left(\mathrm{Eqk}_t - \mathrm{E}\Delta\mathrm{log}\lambda\mathrm{kt}\right) +$
$\mathrm{Iq}_k\,\theta\,\theta K\,\lambda_q^{-1}\,\mu_{boe}\left(\mu b_t + \mathrm{Eqk}_t + \left(1-\rho_{\nu q}\right)\nu q_t - \rho_{\lambda q}\,\lambda q_t + \theta_t\right),$

(* **10. Demand for loans or borrowing demand** *)

$R_t == \mu e_t - \dfrac{\lambda_a}{1+\lambda_a}\,\mu b_t - \dfrac{1}{1+\lambda_a}\left(\mathrm{E}\mu e_t - \mathrm{E}\Delta\mathrm{log}\lambda\mathrm{start}\right),$

(* **11. Production function** *)

$y_t + (1-\phi)\,\alpha\,\Delta\mathrm{log}\lambda\mathrm{kt} == \alpha\,\phi\,\mathrm{Le}_{t-1} + \alpha\,(1-\phi)\,k_{t-1} + (1-\alpha)\,n_t,$

(* **12. Law of motion for capital accumulation** *)

$k_t == (1-\delta)\,\lambda_K^{-1}\,k_{t-1} + i_k\,i_t - (1-\delta)\,\lambda_K^{-1}\,\Delta\mathrm{log}\lambda\mathrm{kt},$

(* **13. Budget (flow of funds) constraint** *)

$\alpha\,y_t + b_y\,R^{-1}\,(b_t - R_t) ==$
$\mathrm{ce}_y\,\mathrm{ce}_t + i_y\,i_t + \mathrm{qlLeOY}\,(\mathrm{Le}_t - \mathrm{Le}_{t-1}) + b_y\,\lambda_{star}^{-1}\,(b_{t-1} - \Delta\mathrm{log}\lambda\mathrm{start}),$

(* **14. Borrowing constraint** *)

```
    b_t == Iq_l θ_t + Iq_l θ λ_star qlLeOB ( Eql_t + Le_t + EΔlogλstart ) +
      Iq_k (1 - θ λ_star qlLeOB) ( Eqk_t + k_t + (1 - ρ_vq) νq_t - ρ_λq λq_t ),

    (* ---------- Aggregation equations -------------- *)
    (* 15. Resource constraint *)
    y_t == ch_y ch_t + ce_y ce_t + i_y i_t,
    (* 16. Housing resource constraint *)
    0 == L_h Lh_t + L_e Le_t,

    (* ---------- Exogeneous processes -------------- *)
    (* We use gφ instead of φ,
    because φ is an expression and will be autormatically subtituted in. *)
    λa_t == ρ_λa λa_{t-1} + ελa_t, (* 17. Household patience shock *)
    λz_t == ρ_λz λz_{t-1} + ελz_t, (* 18. Neutral tech trend growth shock *)
    νz_t == ρ_νz νz_{t-1} + ενz_t, (* 19. Neutral tech level shock *)
    λq_t == ρ_λq λq_{t-1} + ελq_t, (* 20. Biased tech trend growth shock *)
    νq_t == ρ_νq νq_{t-1} + ενq_t, (* 21. Biased tech level shock *)
    gφ_t == ρφ gφ_{t-1} + εφ_t, (* 22. Household housing intratemporal (MRS) shock *)
    gψ_t == ρψ gψ_{t-1} + εψ_t, (* 23. Household labor intratemporal (MRS) shock *)
    gθ_t == ρθ gθ_{t-1} + εθ_t, (* 24. (Borrowing) collateral shock *)

    (* ---------- Expectatonal errors -------------- *)
    (* Note that η has been already used in REequations. *)
    ch_t == Ech_{t-1} + ηch_t, (* 25. Household consumption expectation. *)
    ce_t == Ece_{t-1} + ηce_t, (* 26. Entrepreneur consumption expectation. *)
    μh_t == Eμh_{t-1} + ημh_t, (* 27. Household marginal utility expectation. *)
    μe_t == Eμe_{t-1} + ημe_t, (* 28. Entrepreneur marginal utility expectation. *)
    ql_t == Eql_{t-1} + ηql_t, (* 29. Housing price expectation. *)
    y_t == Ey_{t-1} + ηy_t, (* 30. Output expectation. *)
    i_t == Ei_{t-1} + ηi_t, (* 31. Investment expectation. *)
    qk_t == Eqk_{t-1} + ηqk_t (* 32. Tobin's q expectation. *)
   };
(* Variables *)
x0 = {ql_t, ch_t, ce_t, i_t, n_t, R_t, w_t, y_t, Lh_t, Le_t, k_t, b_t, μh_t, μe_t, qk_t, μb_t, λa_t,
   λz_t, νz_t, λq_t, νq_t, gφ_t, gψ_t, gθ_t, Ech_t, Ece_t, Eμh_t, Eμe_t, Eql_t, Ey_t, Ei_t, Eqk_t};
Length[REequations]
Length[x0]
(* Save forever the following forever for getting rid of hats. *)
(*
REequationsWithNoOverHat=REequations/.OverHat[z_]:>z;
Length[REequationsWithNoOverHat];
x0hat = {c^_t,i^_t,q^_t,ℓ^_t,k^_t,qk^_t,uc^_t,z^_t,a^_t,ξ^_t,Ec^_t,Euc^_t,Ei^_t,Eqk^_t};
x0 = x0hat/.OverHat[z_]:>z;
*)

(*
(* Important trick here:*)
ε_t = {ε_{z t},ε_{q t},ε_{a t},ε_{ξ t}};
ε = ε_t/.Subscript[z_,Times[q_,t]]:>Subscript[ToString[z]<>ToString[q],t];

(* The following is yet not working.  See
```

```
 ComplicatedSubscripts2StringFromMark.nb in /FromMarkFisher. *)
ηt = {ηct,ηuct,ηit,ηyt,ηqkt};
η = ηt/.Subscript[z_,q__t]:>Subscript[ToString[z]<>ToString[q],t];
*)

(* Shocks *)
ε0 = {ελat, ελzt, ενzt, ελqt, ενqt, εφt, εψt, εθt};
(* Expectational terms *)
η0 = {ηcht, ηcet, ημht, ημet, ηqlt, ηyt, ηit, ηqkt};
(* because η has been already used in REequations. *)
```