Introducing Liquidity Difference of Bonds into a Simple DSGE Framework

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[A Preliminary Memo]

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1 The Story

To be added...

2 Baseline Model

The per-period utility function of consumers are given by

$$U(c_t, h_t, \frac{M_t}{P_t}) = \ln c_t + \phi_h \ln(1 - h_t) + \frac{(M_t/P_t)^{1-\phi_m}}{1 - \phi_m} \tag{1}$$

The budget constraint is given by

$$c_t + k_{t+1} + \tfrac{B_{S,t}}{P_t R_{S,t}}(1 + \xi_{S,t}) + \tfrac{B_{L,t}}{P_t R_{L,t}}(1 + \xi_{L,t}) + \tfrac{B_t}{P_t R_t} + \tfrac{M_t}{P_t} \tag{2}$$

$$= \quad \tfrac{B_{S,t-1}}{P_t R_t} + \tfrac{B_{L,t-1}}{P_t R_t} + \tfrac{B_{t-1}}{P_t} + \tfrac{M_{t-1}}{P_t} + w_t h_t + (r_t + 1 - \delta) k_t \tag{3}$$

where $\xi_{i,t}$ is the bond adjustment cost measuring the liquidity differences of bond S and bond L, defined as

$$\xi_{i,t} = \frac{\theta_i}{2} (\frac{B_{i,t}}{B_{i,t-1}} - \pi)^2 y_t \tag{4}$$

Solving consumers' first order conditions, we get [1]

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[1]Lowercase letters are for real variables.

$$m_t^{-\phi_m}\pi_t = \lambda_t\pi_t - \beta\lambda_{t+1} \tag{5}$$
$$\lambda_t = \beta\lambda_{t+1}(r_{t+1} + 1 - \delta) \tag{6}$$
$$\lambda_t = \tfrac{1}{c_t} \tag{7}$$
$$\lambda_t\pi_{t+1} = \beta\lambda_{t+1}R_t \tag{8}$$
$$\lambda_t\pi_{t+1}(\frac{1+\xi_{S,t}}{R_{S,t}} + \frac{\theta_S}{R_{S,t}}Q_{S,t}) = \beta\lambda_{t+1}(\tfrac{1}{R_{t+1}} - \tfrac{\theta_S}{R_{S,t}}\tfrac{b_{S,t+1}}{b_{S,t}}Q_{S,t+1}\pi_{t+1}) \tag{9}$$
$$\lambda_t\pi_{t+1}(\frac{1+\xi_{L,t}}{R_{L,t}} + \frac{\theta_S}{R_{L,t}}Q_{S,t}) = \beta\lambda_{t+1}(\tfrac{1}{R_{t+1}} - \tfrac{\theta_L}{R_{L,t}}\tfrac{b_{L,t+1}}{b_{L,t}}Q_{S,t+1}\pi_{t+1}) \tag{10}$$

where $Q_{i,t} = \frac{b_{i,t}}{b_{i,t-1}}\pi_t(\frac{b_{i,t}}{b_{i,t-1}}\pi_t - \pi)$. Firms' FOCs link factor price and factors' marginal product

$$w_t = (1-\alpha)\frac{y_t}{h_t} \tag{11}$$
$$r_t = \alpha\frac{y_t}{k_t} \tag{12}$$

The production function is standard Cobb-Douglas

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} \tag{13}$$

And market clearing condition closes the model

$$c_t + k_{t+1} - (1-\delta)k_t = y_t - \frac{b_S}{R_{S,t}}\xi_{S,t} - \frac{b_L}{R_{L,t}}\xi_{L,t} \tag{14}$$

Monetary policy is introduced via a Taylor Rule and a OMO operation of bonds

$$\frac{R_t}{R} = (\frac{\pi_t}{\pi})^{\rho_\pi}(\frac{y_t}{y})^{\rho_y}e^{\epsilon_R} \tag{15}$$
$$\frac{m_t}{m_{t-1}} = (\frac{b_{S,t}}{b_{S,t-1}})^{-\eta_S}e^{\epsilon_S} \tag{16}$$
$$\frac{m_t}{m_{t-1}} = (\frac{b_{L,t}}{b_{L,t-1}})^{-\eta_L}e^{\epsilon_L} \tag{17}$$

(5) - (17) characterize the dynamic system of the model.

3 Steady State

First calibrate π, b_S and b_L, then the steady state of the model is given by

$$\frac{k}{y} = \frac{\alpha\beta}{1-\beta(1-\delta)} \tag{18}$$

$$\frac{c}{y} = 1 - \delta\frac{k}{y} \tag{19}$$

$$h = \frac{1}{1 + \frac{\phi_h}{1-\alpha}\frac{c}{y}} \tag{20}$$

$$y = h(\frac{k}{y})^{(}\frac{\alpha}{1-\alpha}) \tag{21}$$

$$R = \frac{\pi}{\beta} \tag{22}$$

$$m = (\frac{\pi-\beta}{\pi c})^{-\frac{1}{\phi_m}} \tag{23}$$

$$R_S = R_L = \frac{\pi R}{\beta} \tag{24}$$

Log-linearizing around this steady state, we could obtain the state space representation of the model.

4 Two Sector Model

In this section we extend the baseline model to a two sector setting. Assume there are two types of consumers (investors) $\{A, B\}$, with different combinations of bond adjustment cost.

Consumers' FOCs become

$$m_t^{A^{-\phi_m}}\pi_t = \lambda_t^A\pi_t - \beta\lambda_{t+1}^A \tag{25}$$

$$\lambda_t^A = \beta\lambda_{t+1}^A(r_{t+1} + 1 - \delta) \tag{26}$$

$$\lambda_t^A = \frac{1}{c_t^A} \tag{27}$$

$$\phi_h\frac{1}{1-h_t^A} = w_t\lambda_t^A \tag{28}$$

$$\lambda_t^A\pi_{t+1} = \beta\lambda_{t+1}^A R_t \tag{29}$$

$$\lambda_t^A\pi_{t+1}(\frac{1+\xi_{S,t}^A}{R_{S,t}} + \frac{\theta_S^A}{R_{S,t}}Q_{S,t}^A) = \beta\lambda_{t+1}^A(\frac{1}{R_{t+1}} - \frac{\theta_S^A}{R_{S,t}}\frac{b_{S,t+1}^A}{b_{S,t}^A}Q_{S,t+1}^A\pi_{t+1}) \tag{30}$$

$$\lambda_t^A\pi_{t+1}(\frac{1+\xi_{L,t}^A}{R_{L,t}} + \frac{\theta_S^A}{R_{L,t}}Q_{S,t}^A) = \beta\lambda_{t+1}^A(\frac{1}{R_{t+1}} - \frac{\theta_L^A}{R_{L,t}}\frac{b_{L,t+1}^A}{b_{L,t}^A}Q_{S,t+1}^A\pi_{t+1}) \tag{31}$$

And the same for type B. In total consumers' FOCs give 14 equations. (11) - (17) are exactly the same, and give 7 equations. Finally we have 6 aggreagtion equations.

$$\tau c_t^A + (1-\tau)c_t^B = c_t \tag{32}$$

$$\tau k_t^A + (1-\tau)k_t^B = k_t \tag{33}$$

$$\tau m_t^A + (1-\tau)m_t^B = m_t \tag{34}$$

$$\tau h_t^A + (1-\tau)h_t^B = h_t \tag{35}$$

$$\tau b_{S,t}^A + (1-\tau)b_{S,t}^B = b_{S,t} \tag{36}$$

$$\tau b_{L,t}^A + (1-\tau)b_{L,t}^B = b_{L,t} \tag{37}$$

where τ_t is the fraction of type A investors. Also notice that at steady states, there is no bond adjustment so 2 types of investors are indentical. So the steady state values are exactly the same as the baseline model.