The intermediate-goods-producing firm

Intermediate-goods-producing firm j hires K_{jt} ***units of capital and*** N_{jt} ***units of labour to produce output,*** Y_{jt}***, according to the following constant-returns to- scale technology:***

$$Y_{jt} \leq A_t N_{jt}^{1-\alpha} K_{jt}^{\alpha} \qquad \textbf{(12)}$$

where A_t ***represents the technology shock, assumed to be common to all firms and to evolve exogenously over time. The technology shock,*** A_t***, is assumed to follow the autoregressive process***

$$\log A_t = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}$$

where $\rho_A \in (-1, 1)$ ***is an autoregression coefficient and*** ε_{At} ***is a serially uncorrelated shock that is normally distributed with zero mean and standard deviation*** σ_A***.***

It is well known that money is super neutral in a monopolistic competition framework unless some sort of nominal friction is added to the model (e.g., Rotemberg 1982). Here, nominal rigidity is introduced by the presence of price-adjustment costs. It is assumed that the intermediate-goods-producing firm faces a quadratic cost of adjusting its nominal price given by the following function:

$$AC_{jt} = \frac{\phi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - 1\right)^2 Y_t$$

where $\phi_p \geq 0$ ***is the price-adjustment cost parameter. These real costs are measured in terms of the final good. Rotemberg (1982) interprets this quadratic adjustment cost specification as capturing the negative effects of price changes on consumer-firm relationships, which increase in magnitude with the size of the price change and with the overall scale of economic activity, as summarized by total output of the finished good. The price markup is constant under complete price flexibility (*** $\phi_p = 0$ ***), but it is endogenous when prices are rigid.***

This cost are measured in terms of the final good, and they directly affect labour demand. With price-adjustment costs, the intermediate firm's optimization problem is dynamic; the intermediate firm j chooses contingency plans for N_{jt}***,*** K_{jt}***,*** Y_{jt}***, and*** P_{jt} ***for all*** $t \geq 0$ ***that maximize its expectation of the discounted sum of its profit flows conditional on the information available at time zero:***

$$\max_{\{K_{jt},N_{jt},P_{jt}\}} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{\Pi_{jt}}{P_t} \middle| S_{jt} \right]$$

where the instantaneous profit function is given by

$$\Pi_{jt} = P_{jt} Y_{jt} - W_t N_{jt} - R_t K_{jt} - P_t AC_{jt}$$

subject to constraints ($Y_{jt} \equiv \left(\frac{P_{jt}}{P_t}\right)^{-\theta} \in_{jt} Y_t \Rightarrow P_{jt} = P_t Y_{jt}^{-\frac{1}{\theta}} (\in_{jt} Y_t)^{\frac{1}{\theta}}$ ***) and (12), to which the Lagrangian multiplier*** $\xi_t \geq 0$ ***is associated. The firm's discount factor is given by the stochastic process (*** $\beta^t \Lambda_t$ ***), where*** Λ_t ***denotes the marginal utility of real income. In equilibrium, this factor represents a pricing kernel for contingent claims. The first-order conditions with respect to*** K_{jt} ***,*** N_{jt} ***,*** P_{jt} ***, and*** ξ_t ***are given***

$$\ell = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[\frac{P_{jt}}{P_t} Y_{jt} - \frac{W_t}{P_t} N_{jt} - \frac{R_t}{P_t} K_{jt} - AC_{jt} \right] + \xi_t \left[P_t \left(A_t N_{jt}^{1-\alpha} K_{jt}^{\alpha} \right)^{-\frac{1}{\theta}} (\in_{jt} Y_t)^{\frac{1}{\theta}} - P_{jt} \right] \middle| S_{jt} \right\}$$

$$\ell = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[\left(\frac{P_{jt}}{P_t} \right)^{1-\theta} \in_{jt} Y_t - \frac{W_t}{P_t} N_{jt} - \frac{R_t}{P_t} K_{jt} - AC_{jt} \right] + \xi_t \left[P_t \left(A_t N_{jt}^{1-\alpha} K_{jt}^{\alpha} \right)^{-\frac{1}{\theta}} (\in_{jt} Y_t)^{\frac{1}{\theta}} - P_{jt} \right] \middle| S_{jt} \right\}$$

Differentiation

$$N_{jt} : E_t \left\{ -\Lambda_t \frac{W_t}{P_t} + \frac{(1-\alpha)}{\theta} \xi_t \left(A_t N_{jt}^{-\alpha} K_{jt}^{\alpha} \right) \left(A_t N_{jt}^{1-\alpha} K_{jt}^{\alpha} \right)^{-\frac{1}{\theta}-1} P_t (\in_{jt} Y_t)^{\frac{1}{\theta}}{}_t \middle| S_{jt} = 0 \right\}$$

$$\Lambda_t \frac{W_t}{P_t} N_{jt} = \frac{(1-\alpha)}{\theta} \xi_t Y_{jt}^{-\frac{1}{\theta}} P_t E_t (\in_{jt} Y_t)^{\frac{1}{\theta}}{}_t \middle| S_{jt}$$

Making Logarithm

$$\log \Lambda_t + \log W_t - \log P_t + \log N_{jt} = \log \frac{(1-\alpha)}{\theta} + \log \xi_t - \frac{1}{\theta} \log Y_{jt} + \log P_t + \log E_t \left\{ (\in_{jt} Y_t)^{\frac{1}{\theta}}{}_t \middle| S_{jt} \right\}$$

$$E_t \left\{ (\in_{jt} Y_t)^{\frac{1}{\theta}}{}_t \middle| S_{jt} \right\} = E_t \left\{ \exp(\frac{1}{\theta} \varepsilon_{jt} + \frac{1}{\theta} z_t) \middle| s_{jt} \right\} = \exp \left\{ E_t \left[(\frac{1}{\theta} \varepsilon_{jt} + \frac{1}{\theta} z_t) \middle| s_{jt} \right] + \frac{1}{2} \mathrm{var} \left[(\frac{1}{\theta} \varepsilon_{jt} + \frac{1}{\theta} z_t) \middle| s_{jt} \right] \right\}$$

$$E_t\left[(\frac{1}{\theta}\varepsilon_{jt}+\frac{1}{\theta}z_t)\middle|s_{jt}\right]=\frac{\operatorname{cov}(\frac{1}{\theta}\varepsilon_{jt}+\frac{1}{\theta}z_t,s_{jt})}{\operatorname{var}(s_{jt})}s_{jt}=\frac{\frac{\lambda}{\theta}\sigma_\varepsilon^2+\frac{1-\lambda}{\theta}\sigma_z^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2+\sigma_v^2}(\lambda\varepsilon_{jt}+(1-\lambda)z_t+v_{jt})$$

$$\Omega_s=\operatorname{var}\left[(\frac{1}{\theta}\varepsilon_{jt}+\frac{1}{\theta}z_t)\middle|s_{jt}\right]$$

Since $\frac{1}{\theta}\varepsilon_{jt}$ and $\frac{1}{\theta}z_t$ are Gaussian, the conditional variance Ω_s will not depend on the observed s_{jt} and will be given by

$$\Omega_s=\operatorname{var}(\frac{1}{\theta}\varepsilon_{jt}+\frac{1}{\theta}z_t)-\frac{\left[\operatorname{cov}(\frac{1}{\theta}\varepsilon_{jt}+\frac{1}{\theta}z_t,s_{jt})\right]^2}{\operatorname{var}(s_{jt})}$$

The steady state

$$\log\bar{\Lambda}+\log\bar{W}-\log\bar{P}+\log\bar{N}_j=\log\frac{(1-\alpha)}{\theta}+\log\bar{\xi}-\frac{1}{\theta}\log\bar{Y}_j+\log\bar{P}+\frac{\frac{\lambda}{\theta}\sigma_\varepsilon^2+\frac{1-\lambda}{\theta}\sigma_z^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2+\sigma_v^2}(\lambda\bar{\varepsilon}_j+(1-\lambda)\bar{z}+0)+\frac{1}{2}\Omega_s$$

Linearization

$$\hat{\Lambda}_t+\hat{W}_t-2\hat{P}_t+\hat{N}_{jt}=\hat{\xi}_t-\frac{1}{\theta}\hat{Y}_{jt}+\frac{\frac{\lambda}{\theta}\sigma_\varepsilon^2+\frac{1-\lambda}{\theta}\sigma_z^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2+\sigma_v^2}(\lambda\hat{\varepsilon}_{jt}+(1-\lambda)\hat{z}_t+\hat{v}_{jt})$$

The clearing of market for good j

$$\hat{\Lambda}_t+\hat{W}_t-2\hat{P}_t+\hat{N}_t=\hat{\xi}_t-\frac{1}{\theta}\hat{Y}_t+\frac{\frac{\lambda}{\theta}\sigma_\varepsilon^2+\frac{1-\lambda}{\theta}\sigma_z^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2+\sigma_v^2}(\lambda\hat{\varepsilon}_t+(1-\lambda)\hat{z}_t+\hat{v}_t)$$

K_{jt} :

$$\hat{\Lambda}_t + \hat{W}_t - 2\hat{P}_t + \hat{K}_t = \hat{\xi}_t - \frac{1}{\theta}\hat{Y}_t + \frac{\frac{\lambda}{\theta}\sigma_\varepsilon^2 + \frac{1-\lambda}{\theta}\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2 + \sigma_v^2}(\lambda\hat{\varepsilon}_t + (1-\lambda)\hat{z}_t + \hat{v}_t)$$

Price setting

P_{jt} :

$$E_t\left\{\Lambda_t(1-\theta)\frac{1}{P_t}\left(\frac{P_{jt}}{P_t}\right)^{-\theta} \in_{jt} Y_t - \Lambda_t\Phi_p\left(\frac{P_{jt}}{P_{jt-1}}-1\right)\frac{Y_t}{P_{jt-1}} + \xi_t - \beta\Phi_p\left[-\frac{P_{jt+1}}{P_{jt}^{\,2}}\left(\frac{P_{jt+1}}{P_{jt}}-1\right)\Lambda_{t+1}Y_{t+1}\right]\Big| S_{jt}\right\} = 0$$

$$\Lambda_t(1-\theta)\frac{1}{P_t}\left(\frac{P_{jt}}{P_t}\right)^{-\theta} E_t\left\{\in_{jt} Y_t \big| S_{jt}\right\} - \Lambda_t\Phi_p\left(\frac{P_{jt}}{P_{jt-1}}-1\right)\frac{Y_t}{P_{jt-1}} + \xi_t - \beta\Phi_p\left[-\frac{P_{jt+1}}{P_{jt}^{\,2}}\left(\frac{P_{jt+1}}{P_{jt}}-1\right)\Lambda_{t+1}Y_{t+1}\right] = 0$$

$$E_t\left\{\in_{jt} Y_t \big| S_{jt}\right\} = E_t\left\{\exp(\varepsilon_{jt} + z_t)\big| s_{jt}\right\} = \exp\left\{\left[E_t(\varepsilon_{jt} + z_t)\big| s_{jt}\right] + \frac{1}{2}\text{var}\left[(\varepsilon_{jt} + z_t)\big| s_{jt}\right]\right\}$$

$$E_t\left[(\varepsilon_{jt} + z_t)\big| s_{jt}\right] = \frac{\text{cov}(\varepsilon_{jt} + z_t, s_{jt})}{\text{var}(s_{jt})} s_{jt} = \frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2 + \sigma_v^2}(\lambda\varepsilon_{jt} + (1-\lambda)z_t + v_{jt})$$

$$\Omega_s = \text{var}\left[(\varepsilon_{jt} + z_t)\big| s_{jt}\right] = \text{var}(\varepsilon_{jt} + z_t) - \frac{\left[\text{cov}(\varepsilon_{jt} + z_t, s_{jt})\right]^2}{\text{var}(s_{jt})}$$

$$\Lambda_t(1-\theta)\frac{1}{P_t}\left(\frac{P_{jt}}{P_t}\right)^{-\theta} \exp\left\{\left[E_t(\varepsilon_{jt} + z_t)\big| s_{jt}\right] + \frac{1}{2}\Omega_s\right\} - \Lambda_t\Phi_p\left(\frac{P_{jt}}{P_{jt-1}}-1\right)\frac{Y_t}{P_{jt-1}} + \xi_t - \beta\Phi_p\left[-\frac{P_{jt+1}}{P_{jt}^{\,2}}\left(\frac{P_{jt+1}}{P_{jt}}-1\right)\Lambda_{t+1}Y_{t+1}\right] = 0$$

In a symmetric equilibrium, all intermediate-goods-producing firms are identical. They make the same decisions, so that $P_{jt} = P_t, Y_{jt} = Y_t, \varepsilon_{jt} = \varepsilon_t, v_{jt} = v_t$

$$\Lambda_t(1-\theta)\frac{1}{P_t}\exp\left\{\left[\frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2 + \sigma_v^2}(\lambda\varepsilon_t + (1-\lambda)z_t + v_t)\right] + \frac{1}{2}\Omega_s\right\} - \Lambda_t\Phi_p\left(\frac{P_t}{P_{t-1}}-1\right)\frac{Y_t}{P_{t-1}} + \xi_t - \beta\Phi_p\left[-\frac{P_{t+1}}{P_t^2}\left(\frac{P_{t+1}}{P_t}-1\right)\Lambda_{t+1}Y_{t+1}\right] = 0$$

$$(1-\theta)\frac{\Lambda_t}{P_t}\exp\left\{\left[A(\lambda\log\in_t+(1-\lambda)\log Z_t+v_t)\right]+\frac{1}{2}\Omega_s\right\}-\Lambda_t\Phi_p\left(\frac{P_t}{P_{t-1}}-1\right)\frac{Y_t}{P_{t-1}}+\xi_t-\beta\Phi_p\left[-\frac{P_{t+1}}{P_t^2}\left(\frac{P_{t+1}}{P_t}-1\right)\Lambda_{t+1}Y_{t+1}\right]=0$$

$$A=\frac{\lambda\sigma_\varepsilon^2+(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2+\sigma_v^2}$$

$$(1-\theta)\frac{\Lambda_t}{P_t}\in_t^{A\lambda}Z_t^{A(1-\lambda)}\exp\left\{Av_t+\frac{1}{2}\Omega_s\right\}-\Phi_p\frac{\Lambda_tP_tY_t}{P_{t-1}^2}+\Phi_p\frac{\Lambda_tY_t}{P_{t-1}}+\xi_t+\beta\Phi_p\frac{P_{t+1}^2\Lambda_{t+1}Y_{t+1}}{P_t^3}-\beta\Phi_p\frac{P_{t+1}\Lambda_{t+1}Y_{t+1}}{P_t^2}=0$$

$$\Phi_p\frac{\Lambda_tP_tY_t}{P_{t-1}^2}=\Phi_p\frac{\bar{\Lambda}e^{\hat{\Lambda}_t}\bar{P}e^{\hat{P}_t}\bar{Y}e^{\hat{Y}_t}}{\bar{P}^2e^{2\hat{P}_{t-1}}}=\Phi_p\frac{\bar{\Lambda}\bar{Y}}{\bar{P}}\frac{e^{\hat{\Lambda}_t}e^{\hat{P}_t}e^{\hat{Y}_t}}{e^{2\hat{P}_{t-1}}}=\Phi_p\frac{\bar{\Lambda}\bar{Y}}{\bar{P}}(1+\hat{\Lambda}_t)(1+\hat{P}_t)(1+\hat{Y}_t)(1-2\hat{P}_{t-1})$$

$$=\Phi_p\frac{\bar{\Lambda}\bar{Y}}{\bar{P}}\left[1+\hat{\Lambda}_t+\hat{P}_t+\overset{\simeq 0}{\hat{\Lambda}_t\hat{P}_t}\right]\left[1+\hat{Y}_t-2\hat{P}_{t-1}-\overbrace{2\hat{Y}_t\hat{P}_{t-1}}^{\simeq 0}\right]=$$

$$=\Phi_p\frac{\bar{\Lambda}\bar{Y}}{\bar{P}}\left\{1+\hat{Y}_t-2\hat{P}_{t-1}+\hat{\Lambda}_t+\overset{\simeq 0}{\hat{\Lambda}_t\hat{Y}_t}-2\overset{\simeq 0}{\hat{\Lambda}_t\hat{P}_{t-1}}+\hat{P}_t+\overset{\simeq 0}{\hat{P}_t\hat{Y}_t}-2\overset{\simeq 0}{\hat{P}_t\hat{P}_{t-1}}\right\}$$

$$=\Phi_p\frac{\bar{\Lambda}\bar{Y}}{\bar{P}}\left(1+\hat{\Lambda}_t+\hat{Y}_t+\hat{P}_t-2\hat{P}_{t-1}\right)$$