Dynare of GHK2000

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The detrended equilibrium equations:

$$\frac{\theta}{c_t} = \lambda_t$$

$$\frac{1-\theta}{1-l_t} = \lambda_t\, w_t\, (1-\tau_l)$$

$$q_t(1-\tau_k)r_{e,t} = bh_t^{\omega-1}$$

$$1 = \frac{\lambda_{t+1}}{\lambda_t}\frac{\beta}{g}\left[(1-\tau_k)\, h_{t+1}q_t r_{e,t+1} + \frac{\left(1-\frac{b}{\omega}h_{t+1}^{\omega}\right)}{\frac{q_{t+1}}{q_t}}\right]$$

$$1 = \frac{\lambda_{t+1}}{\lambda_t}\frac{\beta}{g}\left[(1-\tau_k)r_{s,t+1} + (1-\delta_s)\right]$$

$$\alpha_e z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e-1}\left(k_{s,t}\right)^{\alpha_s} g^{\alpha_e+\alpha_s-1}\, l_t^{1-\alpha_s-\alpha_e} = r_{e,t}$$

$$\alpha_s z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e}\left(k_{s,t}\right)^{\alpha_s-1} g^{\alpha_e+\alpha_s-1}\, l_t^{1-\alpha_s-\alpha_e} = r_{s,t}$$

$$(1-\alpha_s-\alpha_e)z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e} k_{s,t}^{\alpha_s}\, g^{\alpha_e+\alpha_s-1}\, l_t^{-\alpha_s-\alpha_e} = w_t$$

$$y_t = z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e} k_{s,t}^{\alpha_s} g^{\alpha_e+\alpha_s-1}\, l_t^{1-\alpha_e-\alpha_s}$$

$$y_t = c_t + i_{e,t} + i_{s,t}$$

$$k_{e,t+1}\gamma_q g = \left(1-\frac{b}{\omega}\, h_t^{\omega}\right)k_{e,t}\gamma_q + i_{e,t}q_t$$

$$k_{s,t+1}g = (1-\delta_s)k_{s,t} + i_{s,t}\;\; where\ \delta_s \in (0,1)$$

$$z_{t+1} = \gamma_{z,t+1}e^{\zeta_{t+1}}$$

$$q_{t+1} = \gamma_q^{t+1}e^{\eta_{t+1}}$$

$$where\ \eta_t = \rho\eta_{t-1} + \xi_t\;\; with\ \rho \in (0,1)\ and\ \xi_t \sim N(0,\sigma).$$

The static variables only enter FOCs at time t are: $y\ c\ w\ l\ q\ z\ i_e\ i_s\ r_e\ r_s$

The purely predetermined variables only enter FOCs at time t and t-1,t are: $\lambda\ k_e\ k_s h$

Since r_e, r_s are stationary in the balanced growth model, λ and w are out of our interest , we can replace r_e, r_s, λ and w to get the following equations:

$$\frac{1-\theta}{1-l_t} = \frac{\theta}{c_t}\;(1-\tau_l)(1-\alpha_s-\alpha_e)z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e} k_{s,t}^{\alpha_s}\, g^{\alpha_e+\alpha_s-1}\, l_t^{-\alpha_s-\alpha_e}$$

$$q_t(1-\tau_k)\alpha_e z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e-1}\left(k_{s,t}\right)^{\alpha_s} g^{\alpha_e+\alpha_s-1}\, l_t^{1-\alpha_s-\alpha_e} = bh_t^{\omega-1}$$

$$c_{t+1} = \frac{\beta}{g}c_t\left[(1-\tau_k)\, h_{t+1}q_t\alpha_e z_{t+1}\left(h_{t+1}k_{e,t+1}\gamma_q\right)^{\alpha_e-1}\left(k_{s,t+1}\right)^{\alpha_s} g^{\alpha_e+\alpha_s-1}\, l_{t+1}^{1-\alpha_s-\alpha_e} + \frac{\left(1-\frac{b}{\omega}h_{t+1}^{\omega}\right)}{\frac{q_{t+1}}{q_t}}\right]$$

$$\mathrm{c}_{t+1} = \frac{\beta}{g}\mathrm{c}_t\left[(1-\tau_k)\alpha_s z_{t+1}\left(h_{t+1}k_{e,t+1}\gamma_q\right)^{\alpha_e}\left(k_{s,t+1}\right)^{\alpha_s-1} g^{\alpha_e+\alpha_s-1}\, l_{t+1}^{1-\alpha_s-\alpha_e} + (1-\delta_s)\right]$$

Thus, the endogenous variables reduced to 10, and equilibrium equations reduced to 10.

(1) $\mathrm{c_t} = \frac{\theta}{1-\theta}(1-l_t)(1-\mathrm{\tau_l})(1-\alpha_s-\alpha_e)z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e}k_{s,t}^{\alpha_s}\, g^{\alpha_e+\alpha_s-1}\, l_t^{-\alpha_s-\alpha_e}$

(2) $q_t(1-\tau_k)\alpha_e z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e-1}\left(k_{s,t}\right)^{\alpha_s} g^{\alpha_e+\alpha_s-1}\; l_t^{1-\alpha_s-\alpha_e} = b h_t^{\omega-1}$

(3) $\mathrm{c}_{t+1} = \frac{\beta}{g}\mathrm{c}_t\left[(1-\tau_k)\, h_{t+1}q_t\alpha_e z_{t+1}\left(h_{t+1}k_{e,t+1}\gamma_q\right)^{\alpha_e-1}\left(k_{s,t+1}\right)^{\alpha_s} g^{\alpha_e+\alpha_s-1}\; l_{t+1}^{1-\alpha_s-\alpha_e} + \frac{\left(1-\frac{b}{\omega}h_{t+1}^{\omega}\right)}{\frac{q_{t+1}}{q_t}}\right]$

(4) $\mathrm{c}_{t+1} = \frac{\beta}{g}\mathrm{c}_t\left[(1-\tau_k)\alpha_s z_{t+1}\left(h_{t+1}k_{e,t+1}\gamma_q\right)^{\alpha_e}\left(k_{s,t+1}\right)^{\alpha_s-1} g^{\alpha_e+\alpha_s-1}\, l_{t+1}^{1-\alpha_s-\alpha_e} + (1-\delta_s)\right]$

(5) $y_t = z_t\left(h_t k_{e,t}\gamma_q\right)^{\alpha_e}k_{s,t}^{\alpha_s}g^{\alpha_e+\alpha_s-1}\; l_t^{1-\alpha_e-\alpha_s}$

(6) $y_t = c_t + i_{e,t} + i_{s,t}$

(7) $k_{e,t+1}\gamma_q g = \left(1-\frac{b}{\omega}\, h_t^{\omega}\right)k_{e,t}\gamma_q + i_{e,t}q_t$

(8) $k_{s,t+1}g = (1-\delta_s)k_{s,t} + i_{s,t}\;\; where\; \delta_s \in (0,1)$

$$z_{t+1} = \gamma_{z,t+1}e^{\zeta_{t+1}} \Rightarrow \log(z_t) = \log(\gamma_z) + \zeta_t \Rightarrow \log(\gamma_z) = \log(z_t) - \zeta_t$$

$$\log(z_{t+1}) = \log(\gamma_z) + \zeta_{t+1} = \log(z_t) + \zeta_{t+1} - \zeta_t$$

To simply the AR (1) of z, denote $\mathrm{Z} = \zeta_{t+1} - \zeta_t$, then

(9) $\log(z_{t+1}) = \log(z_t) + \mathrm{Z}$

$$q_{t+1} = \gamma_q^{t+1}e^{\eta_{t+1}} \Rightarrow \log(\mathrm{q_t}) = \log(\gamma_q) + \eta_t \Rightarrow \log(\mathrm{q_t}) = \log(\gamma_q) + \rho\eta_{t-1} + \xi_t$$

$$\log(\mathrm{q_{t+1}}) - \rho\eta_t + \xi_{t+1} = \log(\gamma_q) \Rightarrow \log(\mathrm{q_t}) = \log(\mathrm{q_{t+1}}) - \rho(\eta_t - \eta_{t-1}) + \xi_t + \xi_{t+1}$$

$$\Rightarrow \log(\mathrm{q_{t+1}}) = \log(\mathrm{q_t}) + \rho(\eta_t - \eta_{t-1}) - (\xi_t + \xi_{t+1})$$

$$\eta_t - \eta_{t-1} = \frac{\xi_t - \xi_{t-1}}{1-\rho}$$

$$\Rightarrow \log(\mathrm{q_{t+1}}) = \log(\mathrm{q_t}) + \frac{\rho(\xi_t - \xi_{t-1})}{1-\rho} - (\xi_t + \xi_{t+1}) = \frac{\rho(\xi_t - \xi_{t-1})}{1-\rho} - \frac{1-\rho}{1-\rho}\xi_t - \xi_{t+1}$$

To simply the AR (1) of q, denote $\Xi = \xi_t + \xi_{t+1}$, then

(10) $\log(\mathrm{q_{t+1}}) = \log(\mathrm{q_t}) + \frac{\rho}{1-\rho}\Xi -$

7 Steady state equations, converse to the ratios we have:

$$l^* = \frac{\theta}{1-\theta}(1-\mathrm{\tau_l})(1-\alpha_s-\alpha_e)(1-l^*)\frac{y^*}{\mathrm{c}^*}$$

$$\Rightarrow \frac{\mathrm{c}^*}{y^*} = \frac{\theta}{1-\theta}\frac{1}{l^*}(1-\mathrm{\tau_l})(1-\alpha_s-\alpha_e)(1-l^*)$$

$$g = \beta\,[(1-\tau_k)\alpha_s y^*(k_s^*)^{-1} + (1-\delta_s)]$$

$$\Rightarrow \frac{k_s^*}{y^*} = \frac{(1-\tau_k)\alpha_s}{\frac{g}{\beta} - (1-\delta_s)}$$

$$\gamma_q = \frac{\beta}{g}\left[(1-\tau_k)\, q^* \alpha_e y^* (k_e^*)^{-1} + 1 - \frac{b}{\omega} h^{*\omega}\right]$$

$$\Rightarrow q^* = \frac{k_e^*}{y^*} \frac{\gamma_{q*}\frac{g}{\beta} - \left(1 - \frac{b}{\omega} h^{*\omega}\right)}{\alpha_e(1-\tau_k)}$$

$$q^* = \gamma_{q*} e^{\eta_*}$$

$$\eta_* = \rho\eta_* + \xi_* \Rightarrow \eta_* = \frac{\xi_*}{1-\rho} \Rightarrow \; q^* = \; \gamma_{q*} ex\,\mathrm{p}\left(\frac{\xi_*}{1-\rho}\right)$$

$$(1-\tau_k)\alpha_e y^* q^* \left(k_e^* \gamma_q\right)^{-1} = b h^{*\omega} \Rightarrow q^* = \frac{b h^{*\omega} \gamma_q}{(1-\tau_k)\alpha_e} \frac{k_e^*}{y^*} \Rightarrow \frac{k_e^*}{y^*} = \frac{q^*(1-\tau_k)\alpha_e}{b h^{*\omega} \gamma_q}$$

$$\frac{i_e^*}{y^*} = \frac{k_e^*}{y^*} \frac{\gamma_q}{q^*}\left[\gamma_q g - \left(1 - \frac{b}{\omega}\; h^{*\omega}\right)\right]$$

$$\frac{i_s^*}{y^*} = 1 - \frac{c^*}{y^*} - \frac{i_e^*}{y^*}$$

$$\frac{k_s^*}{y^*} = \frac{i_s^*}{y^*} \frac{1}{g - (1-\delta_s)}$$