

Consider a p -order VAR for the $1 \times m$ vector of observed variables y_t

$$y_t = \sum_{k=1}^p y_{t-k} A_k + u_t$$

where

$$u_t \sim \mathcal{N}(0, \Sigma_u).$$

Let z_t be the $mp \times 1$ vector

$$Y = ZA + \mathcal{U}$$

Dummy observation prior for the VAR can be constructed using the VAR likelihood function $T = \lambda T$ artificial data simulated from the DSGE model (Y^*, Z^*) , combined with diffuse priors.

The prior is given by

$$p_0(A, \Sigma | Y^*, Z^*) \propto |\Sigma|^{-\frac{\lambda T + m + 1}{2}} \exp \left\{ -\frac{1}{2} \text{tr} \left[\Sigma^{-1} \left(Y^{*'} Y^* - A' Z^{*'} Y^* - Y^{*'} Z^* A + A' Z^{*'} Z^* A \right) \right] \right\}$$

implying that Σ follows an inverted Wishart distribution and A conditional on Σ is gaussian.

Del Negro Schorfheide use the DSGE theoretical autocovariances matrices for a given $n \times 1$ vector of model parameters θ , denoted $\Gamma_{YY}(\theta), \Gamma_{ZY}(\theta), \Gamma_{YZ}(\theta), \Gamma_{ZZ}(\theta)$ instead of artificial sample moments. In addition, the p -th order VAR approximation of the DSGE provides the first moment of the prior distributions through the population least-square regression

$$A^*(\theta) = \Gamma_{ZZ}(\theta)^{-1} \Gamma_{ZY}(\theta) \tag{1.6.1}$$

$$\Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YZ}(\theta) \Gamma_{ZZ}(\theta)^{-1} \Gamma_{ZY}(\theta) \tag{1.6.2}$$

Conditional on the deep parameters of the DSGE model θ and the weight λ , the priors for the VAR parameters are given by

$$\text{vec}(A) | \Sigma, \theta, \lambda \sim \mathcal{N} \left(\text{vec}(A^*(\theta)), \Sigma \otimes [\lambda T \Gamma_{ZZ}(\theta)]^{-1} \right) \tag{1.6.3}$$

$$\Sigma | \theta, \lambda \sim \mathcal{IW}(\lambda T \Sigma^*(\theta), \lambda T - mp - m) \tag{1.6.4}$$

where $\Gamma_{ZZ}(\theta)$ is assumed to be non-singular and $\lambda \geq \frac{mp+m}{T}$ for the priors to be proper.

Problem Dynare does not explicitly state the DSGE-based prior for the BVAR. It should, however, be possible to construct it according to (1.6.1), (1.6.2), (1.6.3) and (1.6.4).