

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t U(c_t) \\ \text{s.t. } & c_t + I_t = A_t k_t^\alpha \end{aligned} \quad (1)$$

We assume a CRRA utility function :

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \quad (2)$$

The investment function takes the following form :

$$\begin{cases} I_t = k_{t+1} - (1-\delta)k_t + FC \\ I = \delta k \end{cases} \quad (3)$$

The Lagrangian associated with the following program is :

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \lambda_t (A_{t+1} k_t^\alpha - c_t - k_{t+1} + (1-\delta)k_t - FC) \right\}$$

FOC :

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \iff c_t^{-\gamma} = \lambda_t \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \iff \lambda_t = \beta \lambda_{t+1} (\alpha A k_{t+1}^{\alpha-1} + 1 - \delta) \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \iff A_t k_t^\alpha = c_t + k_{t+1} - (1-\delta)k_t + FC \quad (6)$$

In the steady state :

$$\begin{cases} c^{-\gamma} = \lambda \\ 1 = \beta (\alpha A k^{\alpha-1} + 1 - \delta) \\ A k^\alpha = c + \delta k \\ I = \delta k \end{cases} \iff \begin{cases} \lambda = (A k^\alpha - \delta k)^{-\gamma} \\ k = \left(\frac{1 - \beta - \beta \delta}{A \alpha \beta} \right)^{\frac{1}{\alpha-1}} \\ c = A k^\alpha - \delta k \\ I = \delta k \end{cases}$$