

THE MACROECONOMIC EFFECTS OF LARGE-SCALE ASSET PURCHASE PROGRAMMES*

Han Chen, Vasco Cúrdia and Andrea Ferrero

We simulate the Federal Reserve second Large-Scale Asset Purchase programme in a DSGE model with bond market segmentation estimated on US data. GDP growth increases by less than a third of a percentage point and inflation barely changes relative to the absence of intervention. The key reasons behind our findings are small estimates for both the elasticity of the risk premium to the quantity of long-term debt and the degree of financial market segmentation. Without the commitment to keep the nominal interest rate at its lower bound for an extended period, the effects of asset purchase programmes would be even smaller.

In response to the recent financial crisis, the major central banks around the world have engaged in multiple rounds of large-scale asset purchase (LSAP) programmes, typically financed with the creation of excess reserves. The Federal Reserve purchased a total of \$1.75 trillion in agency debt, mortgage-backed securities and Treasuries starting in early 2009, followed by a second Treasury-only programme in the fall of 2010.¹ Also in early 2009, the Bank of England set up an asset purchase facility whose size is, as of July 2012, £275 billion, approximately equivalent to \$445 billion at the current exchange rate. The European Central Bank initiated purchases of up to €60 billion (almost \$80 billion) in Euro area covered bonds in mid-2009. In October 2010, the Bank of Japan unveiled a ¥5 trillion (approximately \$62 billion) asset purchase programme, which had been progressively increased to ¥70 trillion by July 2012.

The objective of the various LSAP programmes, often referred to as ‘Quantitative Easing’ (or QE), is to support aggregate economic activity in periods when the traditional instrument of monetary policy (the short-term nominal interest rate) is not available due to the zero bound constraint. The general idea is that asset purchases operate directly on different segments of the yield curve, reducing rates at different maturities while the short-term rate is at zero.

* Corresponding author: Vasco Cúrdia, Federal Reserve Bank of New York, 33 Liberty Street, 3rd Floor, NY 10045, New York. Email: vasco.curdia@ny.frb.org.

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¹ In September 2011, the Fed announced an additional \$400 billion programme, this time as a pure swap between short-term and long-term assets not involving the creation of additional reserves.

Several papers find evidence that LSAP programmes have indeed been effective in reducing long-term rates. For example, Gagnon *et al.* (2011) estimate that the first round of asset purchases by the Federal Reserve lowered the 10-year Treasury yield by 58 basis points (bp).²

Yet, agreement on the effectiveness of LSAP programmes in supporting the macroeconomy is far from universal. From a theoretical perspective, LSAP programmes were criticised before their implementation, based on some version of the irrelevance result in Wallace (1981). Quantitative easing of this type is also completely ineffective in the baseline New Keynesian model of Eggertsson and Woodford (2003). In this framework, injecting reserves in exchange for longer term securities is a neutral operation. To the extent that market participants take full advantage of arbitrage opportunities, LSAP programmes should have no effect on real economic outcomes. Cúrdia and Woodford (2011) extend this result to a New Keynesian model with credit frictions. If households perceive the assets purchased (such as short-term government bonds) as equivalent to reserves, again LSAP programmes have no effect on the macroeconomy.³ *Ex post*, the criticism has continued due to the difficulty of identifying empirically the effects of asset purchases from other macroeconomic forces (e.g. Cochrane, 2011).

In this article, we estimate the effects of LSAP on macroeconomic variables in a dynamic stochastic general equilibrium (DSGE) model with segmented asset markets. General equilibrium effects are at the heart of Wallace's irrelevance theorem. By going beyond the effects of asset purchases on interest rates, we can evaluate the extent of the criticisms against this type of programmes. At the same time, we want to give LSAP programmes a chance. We introduce limits to arbitrage and market segmentation in a simple form that encompasses frictionless financial markets. Therefore, our strategy is to identify the degree of segmentation – and ultimately the effectiveness of asset

Table 1
Estimated Impact of LSAPs on the 10-Year Treasury Yield in the Literature

Papers	Total impact	Impact per \$100 Bil
Hamilton and Wu (2010)	–13 bp	–3 bp
Doh (2010)	–39 bp	–4 bp
D'Amico and King (2010)	–45 bp	–15 bp
Bomfim and Meyer (2010)	–60 bp	–3 bp
Gagnon <i>et al.</i> (2011)	–58 bp to –91 bp	–3 bp to –5 bp
Neely (2011)	–107 bp	–6 bp
Krishnamurthy and Vissing-Jorgensen (2011)	–33 bp (LSAP II)	–5 bp
D'Amico <i>et al.</i> (2011)	–55 bp (LSAP II)	–9 bp
Swanson (2011)	–15 bp (Twist)	

² A selected sample of other estimates include 13 bp in Hamilton and Wu (2010), 39 bp in Doh (2010), 45 bp in D'Amico and King (2010) and 107 bp in Neely (2010). Krishnamurthy and Vissing-Jorgensen (2011) find that LSAP II reduced the 10-year yield by about 16 bp. See more details in Table 1.

³ Asset purchase programmes may be an effective tool to boost the economy if the government buys securities that are not equivalent to reserves, either because not all households can invest in those assets or because financial frictions impair investment. Recent research along these lines, such as Cúrdia and Woodford (2011), Del Negro *et al.* (2011) and Gertler and Karadi (2011), has mostly focused on private credit markets. Here, instead, we study frictions that rationalise a role for government purchases of long-term bonds.

purchases on macroeconomic activity – directly from the data, without assuming *a priori* that LSAP programmes are bound to fail.

To implement this approach, we augment a standard DSGE model with nominal and real rigidities, along the lines of Christiano *et al.* (2005) and Smets and Wouters (2007), with segmented bond markets. In particular, we follow Andrés *et al.* (2004, henceforth ALSN) and assume that investors have heterogeneous preferences for assets of different maturities (a ‘preferred habitat’ motive, similar to Vayanos and Vila, 2009). We do not model the details of why assets of different maturities are imperfect substitutes. Rather, we postulate that this type of market segmentation exists and estimate the importance of this friction for the transmission mechanism of monetary policy.

The form of asset market segmentation that we use in this article implies that the long-term interest rate matters for aggregate demand distinctly from the expectation of short-term rates. In this world, even if the short-term rate is constrained by the zero lower bound (ZLB) for a long period of time, monetary policy can still be effective by directly influencing current long-term rates. In addition, we assume that the risk premium that arises in the model as a consequence of transaction costs is a positive function of the supply of long-term Treasury securities. This assumption captures, in reduced form, the notion that asset purchase programmes are most effective in flattening the yield curve by reducing the risk premium (Gagnon *et al.*, 2011).

We estimate the model on US data with standard Bayesian methods for the post-war sample, including the recent years. Our main experiment is a counterfactual evaluation of what would have happened to output, inflation and the other macroeconomic variables in the absence of LSAP programmes.⁴

First, we calibrate the size of the asset purchase programme to match a \$600 billion reduction of long-term debt in the hands of the private sector, as announced in the US at the time of LSAP II. At the same time, we consider that the central bank announces the commitment to hold the interest rate at the ZLB for four quarters.⁵ The posterior median effect on GDP growth is an increase of 0.13% (annualised), while the posterior median inflation increase is 3 bp (annualised). The corresponding effect on the *level* of GDP is estimated to be very long lasting – 6 years after the start of the programme the level of GDP is still 0.07% above the path that would have prevailed in the absence of the LSAP programme.

Counterfactual simulations suggest that the commitment to hold the short-term nominal interest rate at the ZLB increases the response of real activity and inflation roughly by factors of three and two, respectively, and introduces upward skewness in the uncertainty surrounding the median estimates. Furthermore, the boost from the commitment to the ZLB is increasingly larger with the length of such a commitment. Overall, in our model, the effects of LSAP II are slightly smaller – and considerably more uncertain – than a 25-bp cut in the short-term rate.

These results suggest that the effects of LSAP programmes on macroeconomic variables, such as GDP and inflation, are likely to be modest. In the online Appendix, we consider several robustness exercises and find that the effects on GDP growth are

⁴ These simulations present us with the key challenge of incorporating the zero bound of nominal interest rates. We deal with this problem using the techniques developed in Cúrdia and Woodford (2011).

⁵ This assumption is consistent with the ‘extended period’ language in the FOMC statements at the time of LSAP II and the market expectations as implied in surveys of private forecasters.

unlikely to exceed a third of a percentage point. The inflationary consequences of asset purchase programmes are consistently very small throughout all scenarios considered. As a comparison, using the FRB/US model, Chung *et al.* (2011) find that LSAP II induced a reduction in the risk premium of only 20 bp but increased the level of GDP by about 0.6% and the inflation rate by 0.1%. Baumeister and Benati (2010), using a vector autoregression (VAR) with time-varying coefficients, consider a change in the term premium of 60 bp and estimate a median increase in GDP growth of 3% and on inflation of 1%. Our results are therefore more moderate than in the existing literature, especially compared with the VAR methodology. Importantly, our results only touch upon the positive dimension of LSAP programmes. Harrison (2010) evaluates the macroeconomic consequences of the optimal amount of asset purchases in a version of this model without capital. His findings are consistent with ours in the sense that asset purchases can improve aggregate welfare but their quantitative relevance appears to be limited.

Our results do not depend on whether asset purchases are financed via reserves or sales of short-term debt, to the extent that these two assets are close to perfect substitutes. Therefore, according to our model, the effects of the Federal Reserve's last round of asset purchases (also known as 'Operation Twist Again') should be in line with the estimates from LSAP II after controlling for the scale factor and for any differences in the duration of the commitment to the zero interest rate.

The rest of this article proceeds as follows. The next Section presents the model. Section 2 discusses the data, the estimation of the model, some basic analysis of parameter estimates and an evaluation of how the model explains the level and slope of the term structure of interest rates. We discuss the LSAP simulation in Section 3. Finally, Section 4 concludes. The companion online Technical Appendix presents additional details on the model equations and steady state, data, implementation of the ZLB commitment, additional robustness exercises and some diagnostics on shock and variance analysis with respect to the components of the yield curve.

1. Model

Two types of households, *unrestricted* (denoted by u) and *restricted* (denoted by r), populate the economy and supply differentiated labour inputs. Competitive labour agencies combine these inputs into a homogeneous composite. Competitive capital producers transform the consumption good into capital. Monopolistic competitive firms hire the labour composite and rent capital to produce intermediate goods. Competitive final goods producing firms package intermediate goods into a homogeneous consumption good. Finally, the government sets monetary and fiscal policy.

1.1. Households

The key modification relative to a standard medium-scale DSGE model (Christiano *et al.*, 2005; Smets and Wouters, 2007) is the introduction of segmentation and transaction costs in bond markets, as in ALSN.

A continuum of measure one of households populates the economy. Household $j = u, r$ enjoys consumption C_t^j (relative to productivity Z_t , as in An and Schorfheide (2007))

and dislikes hours worked L_t^j .⁶ Households supply differentiated labour inputs indexed by i but perfectly share consumption risk within each group. The life-time utility function for a generic households j is

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta_j^s b_{t+s}^j \left\{ \frac{1}{1 - \sigma_j} \left(\frac{C_{t+s}^j}{Z_{t+s}} - h \frac{C_{t+s-1}^j}{Z_{t+s-1}} \right)^{1-\sigma_j} - \frac{\phi_{t+s}^j [L_{t+s}^j(i)]^{1+\nu}}{1 + \nu} \right\}, \tag{1}$$

where $\beta_j \in (0,1)$ is the individual discount factor, b_t^j is a preference shock, $\sigma_j > 0$ is the coefficient of relative risk aversion, $h \in (0,1)$ is the habit parameter, $\nu \geq 0$ is the inverse elasticity of labour supply and ϕ_t^j is a labour supply shock.⁷ Both the preference and labour supply shocks follow stationary AR(1) processes in logs.

Two types of bonds exist. Short-term bonds B_t are one-period securities purchased at time t that pay a nominal return R_t at time $t + 1$. Following Woodford (2001), long-term bonds are perpetuities that cost $P_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t + s + 1$, for $\kappa \in (0,1]$.⁸ We abstract from money and consider the limit of a cashless economy as in Woodford (1998).

The fraction ω_u of unrestricted households trade in both short-term and long-term government bonds. Unrestricted households, however, pay a transaction cost ζ_t per-unit of long-term bond purchased. This transaction cost is paid to a financial intermediary as a fee for its service. The financial intermediary distributes its profits, whose per-capita nominal value is \mathcal{P}_t^i , as dividends to all shareholders (regardless of type). The remaining fraction of the population $\omega_r = 1 - \omega_u$ consists of restricted households who only trade in long-term bonds but pay no transaction costs.⁹

The flow budget constraint differs depending on whether the household belongs to the unrestricted or restricted group. For an unrestricted household that can trade both short and long-term bonds, we have

$$P_t C_t^u + B_t^u + (1 + \zeta_t) P_{L,t} B_t^{L,u} \leq R_{t-1} B_{t-1}^u + \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,u} + W_t^u(i) L_t^u(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^i - T_t^u. \tag{2}$$

For a restricted household that can only trade in long-term securities but does not pay transaction costs, we have

$$P_t C_t^r + P_{L,t} B_t^{L,r} \leq \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,r} + W_t^r(i) L_t^r(i) + \mathcal{P}_t + \mathcal{P}_t^{cp} + \mathcal{P}_t^i - T_t^r. \tag{3}$$

⁶ We express utility as a function of de-trended consumption to ensure the existence of a balanced growth path with constant relative risk aversion preferences. Imposing log-utility of consumption may be an excessively restrictive assumption in our model which is mainly concerned about asset pricing.

⁷ We allow for heterogeneity in preference shocks, discount factors and coefficient of relative risk aversions because these factors affect the household's consumption-saving decisions and financial market segmentation directly influences these optimality conditions. As such, this heterogeneity can potentially influence the simulation results in a substantial way.

⁸ If $\kappa = 1$, this security is a consol.

⁹ We discuss in more detail the implications of transaction costs and bond market segmentation in Sections 1.6.1 and 1.6.2

In (2) and (3), P_t is the price of the final consumption good, $W_t^j(i)$ is the wage set by a household of type $j = \{u, r\}$ who supplies labour of type i , \mathcal{P}_t and \mathcal{P}_t^{cp} are profits from ownership of intermediate goods producers and capital producers, respectively, and T_t^j are lump-sum taxes.¹⁰

One advantage of assuming that the entire stock of long-term government bonds consists of perpetuities is that the price in period t of a bond issued s periods ago $P_{L-s,t}$ is a function of the coupon and the current price, $P_{L,t}$. In the online Technical Appendix, we show how we can write the budget constraints for the two types of households recursively as a function of the price of the bond in period t and the yield to maturity of the bond, $R_{L,t}$.

Household j consumption-saving decisions are the result of the maximisation of (1) subject to (2) if $j = u$ or (3) if $j = r$. See the online Technical Appendix for details and Section 1.6 for some discussion.

1.1.1. *Labour agencies and wage-setting decision*

Perfectly competitive labour agencies combine differentiated labour inputs into a homogeneous labour composite L_t according to the technology

$$L_t = \left[\int_0^1 L_t(i)^{\frac{1}{1+\lambda_w}} \mathbf{d}i \right]^{1+\lambda_w},$$

where $\lambda_w \geq 0$ is the steady-state wage markup.

Profit maximisation gives the demand for the i th labour input

$$L_t(i) = \left[\frac{W_t(i)}{W_t} \right]^{-(1+\lambda_w)/\lambda_w} L_t. \tag{4}$$

From the zero-profit condition for labour agencies, we obtain an expression for the aggregate wage index W_t as a function of the wage set by the i th household

$$W_t = \left[\int_0^1 W_t(i)^{-\frac{1}{\lambda_w}} \mathbf{d}i \right]^{-\lambda_w}.$$

Households are monopolistic suppliers of differentiated labour inputs $L_t(i)$ and set wages on a staggered basis (Calvo, 1983) taking the demand for their input as given. In each period, the probability of resetting the wage is $1 - \zeta_w$, while with the complementary probability the wage is automatically increased by the steady-state rates of inflation (Π) and of productivity growth (e^γ),

$$W_{t+s}^j(i) = (\Pi e^\gamma)^s \tilde{W}_t^j(i), \tag{5}$$

for $s > 0$, where $\tilde{W}_t^j(i)$ is the wage chosen at time t in the event of an adjustment. A household of type j that can reset the wage at time t chooses $\tilde{W}_t^j(i)$ to maximise

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta_j \zeta_w)^s \left\{ \Xi_{t+s}^{j,p} (\Pi e^\gamma)^s \tilde{W}_t^j(i) L_{t+s}^j(i) - \frac{\varphi_{t+s}^j [L_{t+s}^j(i)]^{1+\nu}}{1+\nu} \right\},$$

¹⁰ Each household receives the same dividend from intermediate goods and capital producers and pays the same amount of lump-sum taxes.

where $\Xi_t^{j,p}$ is the marginal utility of consumption in nominal terms, subject to (4) and (5). The online Technical Appendix presents the first-order condition for this problem.

1.2. Capital Producers

Competitive capital producers make investment decisions, choose the utilisation rate and rent capital to intermediate good producing firms. By choosing the utilisation rate u_t , capital producers end up renting in each period t an amount of ‘effective’ capital equal to

$$K_t = u_t \bar{K}_{t-1}.$$

Capital producers accumulate capital according to

$$\bar{K}_t = (1 - \delta)\bar{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t, \tag{6}$$

where $\delta \in (0,1)$ is the depreciation rate, μ_t is an investment-specific technology shock that follows a stationary AR(1) process in logs and $S(\cdot)$ is the cost of adjusting investment (with $S'(\cdot) \geq 0$ and $S''(\cdot) > 0$).¹¹

Capital producers discount future profits at the marginal utility of the average shareholder

$$\Xi_{t+s}^p \equiv \omega_u \beta_u^s \Xi_{t+s}^{u,p} + \omega_r \beta_r^s \Xi_{t+s}^{r,p}.$$

This variable is the appropriate discount factor of future dividends because ownership of capital producing firms is equally distributed among all households.¹² Capital producers maximise the expected discounted stream of dividends to their shareholders

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Xi_{t+s}^p [R_{t+s}^k u_{t+s} \bar{K}_{t+s-1} - P_{t+s} a(u_{t+s}) \bar{K}_{t+s-1} - P_{t+s} I_{t+s}],$$

subject to the law of motion of capital (6), where R_t^k is the return per unit of effective capital. Note that we assume that utilisation subtracts real resources measured in terms of the consumption good, $a(u_t) \bar{K}_{t-1}$.¹³

1.3. Final Goods Producers

Perfectly competitive final goods producers combine differentiated intermediate goods $Y_t(f)$, supplied by a continuum of firms f of measure 1, into a homogeneous good Y_t according to the technology

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\lambda_f}} df \right]^{1+\lambda_f},$$

where $\lambda_f \geq 0$ is the steady-state price markup. The resulting demand for the f th intermediate good is

¹¹ Furthermore, we assume that $S(\vartheta') = S'(\vartheta') = 0$.

¹² The same consideration applies below to intermediate goods producers.

¹³ As in Christiano *et al.* (2011), we choose an implicit functional form for $a(u_t)$ such that $u = 1$ in steady state and $a(1) = 0$.

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t. \tag{7}$$

From the zero-profit condition for intermediate goods producers, we obtain an expression for the aggregate price index P_t as a function of the price set by the f th intermediate good producer

$$P_t = \left[\int_0^1 P_t(f)^{-\frac{1}{\lambda_f}} df \right]^{-\lambda_f}.$$

1.4. *Intermediate Goods Producers*

A continuum of measure one of monopolistic competitive firms combines rented capital and hired labour to produce intermediate goods according to a standard Cobb–Douglas technology

$$Y_t(f) = K_t(f)^\alpha [Z_t L_t(f)]^{1-\alpha}, \tag{8}$$

where Z_t is a labour-augmenting technology process which evolves according to

$$\log\left(\frac{Z_t}{Z_{t-1}}\right) = (1 - \rho_z)\gamma + \rho_z \log\left(\frac{Z_{t-1}}{Z_{t-2}}\right) + \epsilon_{z,t}.$$

Cost minimisation yields an expression for the marginal cost which only depends on aggregate variables

$$MC(f)_t = MC_t = \frac{(R_t^k)^\alpha W_t^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha} Z_t^{1-\alpha}}. \tag{9}$$

Intermediate goods producers set prices on a staggered basis (Calvo, 1983). In each period, a firm can readjust prices with probability $1 - \zeta_p$ independently of previous adjustments. We depart from the basic formulation of staggered price setting in assuming the firms that cannot adjust in the current period index their price to the steady-state inflation rate Π . The problem for a firm that can adjust at time t is to choose the price $\tilde{P}_t(f)$ that maximises

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Xi_{t+s}^p [\tilde{P}_t(f) \Pi^s - \lambda_{f,t+s} MC_{t+s}] Y_{t+s}(f),$$

subject to (7) conditional on no further adjustments after t , where $\lambda_{f,t}$ is a goods markup shock that follows a stationary AR(1) process in logs.

1.5. *Government Policies*

The central bank follows a conventional feedback interest rate rule similar to Taylor (1993), amended to include interest rate smoothing (Clarida *et al.*, 2000) and using the growth rate of output instead of the output gap (Justiniano *et al.*, 2011)

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_m} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}}\right)^{\phi_y}\right]^{1-\rho_m} e^{\epsilon_{m,t}},$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the inflation rate, $\rho_m \in (0,1)$, $\phi_\pi > 1$, $\phi_y \geq 0$ and $\epsilon_{m,t}$ is an i.i.d. innovation.¹⁴

The presence of long-term bonds modifies the standard government budget constraint

$$B_t + P_{L,t}B_t^L = R_{t-1,t}B_{t-1} + (1 + \kappa P_{L,t})B_{t-1}^L + P_tG_t - T_t. \tag{10}$$

The left-hand side of (10) is the market value, in nominal terms, of the total amount of bonds (short-term and long-term) issued by the government at time t . The right-hand side is the total deficit at time t , that is, the cost of servicing bonds maturing in that period plus spending G_t net of taxes.

We assume that the government controls the supply of long-term bonds following a simple autoregressive rule for their de-trended market value in real terms

$$\frac{P_{L,t}B_t^L}{P_tZ_t} = \left(\frac{P_{L,t-1}B_{t-1}^L}{P_{t-1}Z_{t-1}}\right)^{\rho_B} e^{\epsilon_{B,t}}, \tag{11}$$

where $\rho_B \in (0,1)$ and $\epsilon_{B,t}$ is an i.i.d. exogenous shock. We interpret LSAP programmes as shocks to the composition of outstanding government liabilities compared with the historical behaviour of these series.

Finally, the government adjusts the real primary fiscal surplus in response to the lagged real value of long-term debt, as in Davig and Leeper (2006) and Eusepi and Preston (2011),

$$\frac{T_t}{P_tZ_t} - \frac{G_t}{Z_t} = \Phi \left(\frac{P_{L,t-1}B_{t-1}^L}{P_{t-1}Z_{t-1}}\right)^{\phi_T} e^{\epsilon_{T,t}}, \tag{12}$$

where $\phi_T > 0$ and $\epsilon_{T,t}$ follows a stationary AR(1) process. All fiscal variables in rule (12) are cyclically adjusted (i.e. expressed relative to the level of productivity) and the constant Φ is such that in steady state the fiscal rule is just an identity. Note that the presence of asset market segmentation breaks Ricardian equivalence in this model. Therefore, fiscal financing decisions have real consequences on the allocation. Given a strong enough feedback (a high enough value of the coefficient ϕ_T), rule (12) ensures that the primary surplus adjusts to satisfy the government intertemporal budget constraint.

1.6. Equilibrium and Solution Strategy

In equilibrium, households and firms maximise their objectives subject to their constraints and all markets clear. In particular, the resource constraint is

¹⁴ The presence of output growth, instead of the output gap, in the interest rate rule avoids the complication of solving for and estimating the system of equations that characterise the flexible price equilibrium of the model. In practice, GDP growth relative to trend is often cited as one of the main indicators of real activity for the conduct of monetary policy.

$$Y_t = \omega_u C_t^u + \omega_r C_t^r + I_t + G_t + a(u_t) \bar{K}_{t-1}.$$

We solve the model by taking a first-order log-linear approximation around a steady state in which quantities are normalised by the level of productivity Z_t and relative prices are expressed as function of P_t . The online Appendix shows the full set of non-linear normalised equilibrium relations, characterises the steady-state solution and presents the full set of log-linearised equations that constitute the basis for the estimation.

These conditions are standard in modern DSGE models (Christiano *et al.*, 2005; Smets and Wouters, 2007) with the exception of the households' consumption-saving decisions. Here, we focus on these Euler equations to sharpen the intuition about the effects of segmentation in the bond market. This discussion should also clarify the channels through which asset purchase programmes can support macroeconomic outcomes.

Since only unrestricted households trade in short-term bonds, the pricing equation for these securities is

$$1 = \beta_u \mathbb{E}_t \left(e^{-\gamma-z_{t+1}} \frac{\Xi_{t+1}^u}{\Xi_t^u} \frac{R_t}{\Pi_{t+1}} \right), \tag{13}$$

where Ξ_t^u is the marginal utility of de-trended consumption in real terms for an unrestricted household and $e^{-\gamma-z_{t+1}}$ is the correction factor due to productivity growth.

Both unrestricted and restricted households trade long-term bonds. For unrestricted households, the pricing equation of these securities is

$$(1 + \zeta_t) = \beta_u \mathbb{E}_t \left(e^{-\gamma-z_{t+1}} \frac{\Xi_{t+1}^u}{\Xi_t^u} \frac{P_{L,t+1}}{P_{L,t}} \frac{R_{L,t+1}}{\Pi_{t+1}} \right). \tag{14}$$

For constrained households, the pricing condition is

$$1 = \beta_r \mathbb{E}_t \left(e^{-\gamma-z_{t+1}} \frac{\Xi_{t+1}^r}{\Xi_t^r} \frac{P_{L,t+1}}{P_{L,t}} \frac{R_{L,t+1}}{\Pi_{t+1}} \right). \tag{15}$$

Restricted households have a different marginal utility of consumption and do not pay the transaction cost.

1.6.1. Transaction costs and the risk premium

The presence of transaction costs for unrestricted households in the market for long-term bonds gives rise to a risk premium. Using (14), we define $R_{L,t}^{EH}$ as the counterfactual yield to maturity on a long-term bond at time t in the absence of transaction costs, given the same path for the marginal utility of consumption of unrestricted households. No arbitrage implies that this fictitious bond should have the same risk-adjusted return as the long-term security actually traded. We measure the risk premium as the difference between these two yields to maturity, up to a first-order approximation

$$\hat{R}_{L,t} - \hat{R}_{L,t}^{EH} = \frac{1}{D_L} \sum_{s=0}^{\infty} \left(\frac{D_L - 1}{D_L} \right)^s \mathbb{E}_t \zeta_{t+s}, \tag{16}$$

where D_L is the steady-state duration of the two securities.¹⁵ Expression (16) shows that the risk premium in this economy equals the present discounted value of current and expected future transaction costs.

In ALSN, the risk premium has two components: one endogenous and one exogenous. The endogenous component arises because households face a portfolio adjustment cost, function of the relative quantity of money relative to long-term assets. The idea is that long-term bonds entail a loss of liquidity that households hedge by increasing the amount of money in their portfolio. The transaction costs in the market for long-term bonds are treated as purely exogenous.

We retain the distinction between endogenous and exogenous component of the risk premium while abstracting from the portfolio adjustment cost component. Instead, we directly assume that transaction costs are a function of the ratio of the market value of long-term debt to short-term debt in the hands of the public, plus an error

$$\zeta_t \equiv \zeta \left(\frac{P_{L,t} B_{z,t}^L}{B_{z,t}}, \epsilon_{\zeta,t} \right), \quad (17)$$

where $B_{z,t}^L \equiv B_t^L / (P_t Z_t)$, and $B_{z,t} \equiv B_t / (P_t Z_t)$. We do not take a stand on the explicit functional form of $\zeta(\cdot)$. We only require the function and its first derivative to be positive when evaluated in steady state (i.e. $\zeta(P_L B_z^L / B_z, 0) > 0$ and $\zeta'(P_L B_z^L / B_z, 0) > 0$). The first assumption ensures the presence of a positive steady-state risk premium, as in the data. The second assumption guarantees that the yield on long-term bonds drops following a reduction in their outstanding amount. This element gives LSAP programmes a chance to work through the mechanism identified in the reduced form estimates (Gagnon *et al.*, 2011).

Up to a log-linear approximation, our parsimonious formulation of transaction costs is observationally equivalent to the two frictions in ALSN. The idea that transaction costs depend directly on the aggregate stock of bonds captures the same intuition (i.e. a liquidity cost) of the adjustment cost function in the original formulation.¹⁶

1.6.2. *Limits to arbitrage*

The assumption of market segmentation captures, in reduced form, the observation that in reality some fraction of the population mostly saves through pension funds and other types of long-term institutional investors. These financial intermediaries are specialists in certain segments of the market and their transaction costs are likely to be small. Conversely, households who invest in long-term bonds mostly for diversification motives may face higher transaction costs.¹⁷ The parameter ω_u measures this segmentation and is one of the key objects of interest in our estimation results.

¹⁵ The details of the derivation are in the online Appendix. In defining the yield to maturity of a bond in the absence of transaction costs, we adjust the parameter κ to guarantee that the fictitious security has the same steady-state duration as the actual long-term bond.

¹⁶ An alternative approach to study the effects of LSAP programmes on risk and term premia would be to use higher order approximation methods and estimate the model with the particle filter (van Binsbergen *et al.*, 2010).

¹⁷ See ALSN for a more detailed discussion of this interpretation.

The key implication of bond market segmentation is that not all agents in the model can take full advantage of arbitrage opportunities. Unrestricted households can arbitrage away, up to some transaction cost, differences in risk-adjusted expected returns between short- and long-term bonds (13 and 14) but restricted households do not have this possibility. Equation (15) fully characterises the savings behaviour of restricted households.

This friction provides a rationale for asset purchase programmes to influence macroeconomic outcomes, thus breaking the irrelevance result in Wallace (1981). In particular, in our model, a programme targeted to purchases of long-term securities reduces the risk premium (17), changing their expected return. Without segmentation, this programme would affect the yield to maturity of the long-term bond (14) but would have no effects on the real allocation. Because unrestricted households can invest in both securities, their portfolios would adjust until the two expected returns are equated again, implying a different yield to maturity on the long-term bond. In equilibrium, expected returns, inclusive of transaction costs, would be unchanged, hence avoiding any change of the stochastic discount factor. Thus, no real variable in this economy is affected.

Conversely, with segmented bond markets, LSAP programmes do affect the real economy. The change in long-term yields induces a change in the expected return of the restricted households, which are not subject to the transaction costs. Because the expected return is different from the restricted households' perspective, their stochastic discount factor has to adjust. This change alters their intertemporal profile of consumption (15) and indirectly influences both the pricing decisions of intermediate producing firms and the investment decisions of capital producers. Ultimately, general equilibrium forces imply that consumption for both types of agents, investment and production respond as well.¹⁸ The simulations in Section 3 illustrate the magnitude of the LSAP stimulus on aggregate demand and inflation.

2. Empirical Analysis

We estimate the model with Bayesian methods, as surveyed for example by An and Schorfheide (2007). Bayesian estimation combines prior information on the parameters with the likelihood function of the model to form the posterior distribution. We construct the likelihood using the Kalman filter based on the state space representation of the rational expectations solution of the model.¹⁹ In the remainder of this Section, we first describe the data used and then present parameter prior and posterior distributions.

2.1. Data

We use quarterly data for the US from the third quarter of 1987 (1987q3) to the third quarter of 2009 (2009q3) for the following seven series: real GDP per capita, hours

¹⁸ In practice, another effect of asset purchase programmes could be the incentive for households to shift their portfolios toward riskier assets, such as equity and corporate bonds. In the model, this mechanism is absent as the equity shares are non-tradable.

¹⁹ We impose a zero posterior density for parameter values that imply indeterminacy, which is equivalent to a truncation of the joint prior distribution.

worked, real wages, core personal consumption expenditures deflator, nominal effective Federal Funds rate, the 10-year Treasury constant maturity yield, and the ratio between long-term and short-term US Treasury debt.²⁰ All data are extracted from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve Bank of St. Louis. In the online Appendix we describe more precisely how the data are constructed and how they map to the state variables in the model.

2.2. *Prior Choice*

Tables 2 and 3 (columns 2–5) summarise the prior distributions of each parameter. We use a Gamma distribution for the parameters that economic theory suggests should be positive to constrain their support on the interval $[0, \infty]$. For those parameters that span only the unit interval, we use the Beta distribution. For the standard deviation of shock innovations, we use the Inverse-Gamma distribution.

The steady-state value for inflation is centred at 2%, in line with the mandate-consistent level of inflation commonly assigned to the Federal Open Market Committee (FOMC). The steady-state growth rate is centred at 2.5% (annualised). The discount factor has a prior that implies a real interest rate of about 2% (annualised). The steady-state spread between the 10-year treasury yield and the federal funds rate has a prior centred at 0.75% (annualised), similar to the average in the data.

Table 2
Parameter Prior and Posterior Distribution: Structural Parameters

	Prior				Posterior				
	Dist	5%	Median	95%	Mode	Mean	5%	Median	95%
400γ	G	1.7382	2.4667	3.3752	1.9922	1.9867	1.5413	1.9805	2.4515
400π	G	1.2545	1.9585	2.8871	2.2404	2.1477	1.5075	2.1376	2.8237
$400(\beta_u^{-1} - 1)$	G	0.6272	0.9792	1.4436	0.4943	0.4890	0.3282	0.4828	0.6706
400ζ	G	0.3913	0.7224	1.2029	0.4761	0.5127	0.2735	0.4968	0.8082
B^{LMV}/B	G	0.6953	0.9867	1.3501	0.8222	0.8502	0.7164	0.8533	0.9712
S''	G	2.5090	3.9170	5.7743	4.4277	4.8371	3.3764	4.7815	6.5330
a''	G	0.0683	0.1836	0.3877	0.2093	0.2322	0.0994	0.2159	0.4189
h	B	0.4302	0.6029	0.7597	0.8370	0.7898	0.6421	0.8020	0.9007
σ_u	G	0.6832	1.8360	3.8768	3.0151	3.4958	1.9891	3.3548	5.4795
σ_r	G	0.6832	1.8360	3.8768	1.5635	2.2370	0.8692	2.0825	4.1268
$100\zeta'$	G	0.3067	1.2846	3.4294	0.2420	0.3763	0.0862	0.3274	0.8257
ω_u	B	0.3214	0.7334	0.9646	0.9832	0.9322	0.8237	0.9468	0.9934
Ξ^u/Ξ^r	G	0.3416	0.9180	1.9384	0.7917	1.1403	0.4537	1.0730	2.0683
C^u/C^r	G	0.3416	0.9180	1.9384	0.7641	1.0533	0.3921	0.9760	1.9747
χ_{wu}	B	0.2486	0.6143	0.9024	0.5170	0.5566	0.2825	0.5611	0.8187
v	G	1.2545	1.9585	2.8871	1.6814	1.9658	1.2518	1.9295	2.7996
ζ_w	B	0.3351	0.5000	0.6649	0.6860	0.7309	0.6292	0.7342	0.8205
ζ_p	B	0.3351	0.5000	0.6649	0.9260	0.9287	0.9116	0.9288	0.9452
ϕ_T	G	0.7825	1.4448	2.4058	1.2660	1.3147	0.6890	1.2543	2.1216
ρ_r	B	0.5242	0.7068	0.8525	0.8547	0.8556	0.8182	0.8565	0.8903
ϕ_π	G	1.0164	1.7026	2.6453	1.5644	1.6090	1.3706	1.5979	1.8866
ϕ_y	G	0.1366	0.3672	0.7754	0.2975	0.3295	0.2487	0.3251	0.4247

²⁰ We use an extended sample, starting in 1959q3, to initialise the Kalman filter, but the likelihood function itself is evaluated only for the period starting in 1987q3, conditional on the previous sample.

Table 3
Parameter Prior and Posterior Distribution: Shock Process Parameters

	Prior				Posterior				
	Dist	5%	Median	95%	Mode	Mean	5%	Median	95%
ρ_z	B	0.0976	0.3857	0.7514	0.1286	0.1486	0.0448	0.1430	0.2706
ρ_μ	B	0.5701	0.7595	0.8971	0.8293	0.8477	0.7906	0.8489	0.8998
ρ_b	B	0.5701	0.7595	0.8971	0.9603	0.9597	0.9388	0.9614	0.9764
ρ_ϕ	B	0.5701	0.7595	0.8971	0.5200	0.5063	0.3767	0.5074	0.6345
ρ_B	B	0.6146	0.8135	0.9389	0.9773	0.9652	0.9396	0.9659	0.9880
ρ_ζ	B	0.6146	0.8135	0.9389	0.9614	0.9402	0.9012	0.9426	0.9700
ρ_g	B	0.5701	0.7595	0.8971	0.7581	0.7397	0.5542	0.7491	0.8943
σ_z	IG1	0.1663	0.3433	1.2367	0.7426	0.7624	0.6742	0.7588	0.8634
σ_{λ_j}	IG1	0.1663	0.3433	1.2367	1.6306	1.9235	1.1103	1.7930	3.1600
σ_μ	IG1	0.1663	0.3433	1.2367	2.8430	3.0102	2.2984	2.9725	3.8662
σ_b	IG1	0.1663	0.3433	1.2367	3.0703	3.7799	2.2957	3.5860	5.8901
σ_ϕ	IG1	0.1663	0.3433	1.2367	0.4518	0.9048	0.3100	0.7628	1.9982
σ_B	IG1	0.1663	0.3433	1.2367	0.3686	0.3656	0.2930	0.3578	0.4660
σ_T	IG1	0.1663	0.3433	1.2367	0.2348	0.5521	0.1670	0.3457	1.3682
σ_m	IG1	0.0819	0.1700	0.6217	0.1130	0.1167	0.1019	0.1161	0.1335
σ_ζ	IG1	0.0819	0.1700	0.6217	0.2137	0.2667	0.1900	0.2592	0.3694
σ_g	IG1	0.1663	0.3433	1.2367	0.2439	0.3894	0.1688	0.3429	0.7619

We follow Del Negro and Schorfheide (2008) for the priors of standard parameters. The investment adjustment cost convexity parameter S'' has prior mean of 4 and standard deviation of 1. The utilisation cost elasticity parameter a'' has prior mean 0.2 and standard deviation 0.1, implying that in response to a 1% increase in the return to capital, utilisation rates rise by about 0.17%. We calibrate the share of capital in production α to 0.33 and the capital depreciation rate δ to 2.5% per quarter.

The habit formation coefficient for both types of agents has prior mean of 0.6 and standard deviation 0.1, also fairly common in the literature. The parameter controlling the labour supply elasticity ν has a prior centred at 2. Similarly to Smets and Wouters (2007), we estimate the intertemporal elasticity of substitution of consumption for households, except that in our model we have two types of agents. The prior on σ_u and σ_r is relatively flat (centred at 2 with standard deviation of 1) and equal for both types, so that the data can be informative about their value.

The fraction of unrestricted agents ω_u is the crucial parameter to identify the degree of bond market segmentation in the model. At the mean, our prior implies that 70% of the households are unrestricted. As we show below, this degree of segmentation, conditional on the rest of the priors, is consistent with substantial effects of LSAP in the model. A standard deviation of 0.2, however, makes the distribution flat enough that the 90% prior interval is (0.32, 0.96) and encompasses very large to minimal effects. The other key parameter is the elasticity of the risk premium to changes in the market value of long debt ζ' . The prior for this parameter has a mean 1.5/100 and a standard deviation big enough to match the range of estimates shown in Table 1 (see discussion above). The cash-flow parameter that controls the duration of long-term bonds (given the yield to maturity) is calibrated to imply a duration of 30 quarters, similar to the average duration in the secondary market for 10-year US Treasury bills. We consider

short-term debt to include both government bonds with less than 1 year to maturity as well as central bank liabilities in the form of reserves, vault cash and deposits. In the US, the average for this quantity since 1974 is about 16% of annual GDP. For long-term bonds, we consider all government bonds with maturity greater than 1 year, which in the US is also about 16% of annual GDP since 1974.

Table 2 contains three non-standard parameters (Ξ^u/Ξ^r , C^u/C^r and χ_{wu}) which refer to steady-state ratios hard to pin down directly from the data. We decided not to calibrate these ratios to avoid biasing the estimation and the simulations in either direction. The posterior distribution for these three parameters turns out to deviate negligibly from our prior. Furthermore, the uncertainty in these ratios translates into uncertainty in the dynamics of the model and in the effects of asset purchases on macroeconomic variables.

The priors for the wage and price rigidity parameters ζ_w and ζ_p are centred at 0.5 with a standard deviation of 0.1, as in Smets and Wouters (2007). The fiscal rule parameter, ϕ_T , is centred at 1.5 and its posterior does not differ too much from the prior. For the monetary policy rule, we consider fairly standard parameter priors. The interest rate smoothing parameter ρ_r is centred at 0.7. The response to output growth ϕ_y is centred at 0.4. The prior mean for the response to inflation ϕ_π , centred at 1.75, is slightly higher than the usual value of 1.5 in Taylor (1993). The 90% prior interval, however, is completely above one, consistent with the Taylor principle.

The shocks follow AR(1) processes, with autocorrelation coefficient ρ_i centred at 0.75, except for the autocorrelations of productivity shocks (equal to 0.4 so that the growth rate shock is not too persistent) and of the risk premium and debt shocks (equal to 0.8). The prior mean of the innovations have standard deviations σ_i centred at 0.5, except for the innovation to the monetary policy shock and the risk premium shock whose standard deviation is smaller because these variables refer to quarterly changes in interest rates.

2.3. Parameter Posterior Distribution

To obtain the posterior distribution, we first obtain the posterior mode.²¹ We then use a normal approximation around the mode to form a jump distribution to generate a sample of parameter vector draws representative of the posterior based on the Metropolis random walk Markov Chain Monte Carlo (MCMC) simulation method.²² The last 5 columns of Tables 2 and 3 report the posterior distribution of each parameter.

The first result that emerges from Tables 2 and 3 is that the measure of market segmentation is very small – the posterior 90% interval for ω_u is (0.824, 0.993) with a median of 0.947 and a mode of 0.983. Given our prior, the data strongly push against a model with a significant degree of market segmentation. *Ceteris paribus*, we should

²¹ We extract the mode by maximising the posterior density function, which can be a very challenging task with a high-dimensional problem like the one in this article. To reduce the chances of extracting a local maximum, we perform 50 maximisations of the mode starting at different guesses of the parameter vector. For each solution, we further test at least 10 times whether a new maximisation with a guess parameter vector in a small neighbourhood of that solution can achieve a higher level of the posterior density function.

²² After obtaining four separate chains of 100,000 draws, we compute the covariance matrix (with a 25% burn-in) and generate four new chains of 100,000 draws. We repeat this step two more times with 200,000 and 500,000 draws respectively. At this stage, we use these last four chains to extract the parameter posterior distribution properties and to simulate the effects of asset purchases.

expect small macroeconomic effects of asset purchases. To check the stability of the estimate of market segmentation (and, in general, of other parameters), we re-estimated the model with alternative samples. While in our baseline estimation the sample ends in 2009q3 (just before the first US LSAP programme), we considered three alternative endings: 2007q2 (before the recent financial turbulence), 2008q3 (before the federal funds rate reached the ZLB) and 2011q2 (the most recent available data). The parameter estimates always remain very comparable.²³

The other key parameter is the elasticity of the risk premium to asset purchases ζ' . If this elasticity were zero, asset purchases would affect neither the risk premium nor the real economy. The posterior distribution turns out to be concentrated at low levels, although different from zero, with a median of 0.327/100 and a 90% interval of (0.086, 0.826), suggesting a fairly small impact of the quantity of debt on the risk premium and the 10-year yield. This finding collocates our estimate of the elasticity of the risk premium to the quantity of debt at the lower end of the spectrum in the literature.

The sensitivity of consumption to the interest rate is estimated to be 3.4 for the unrestricted type and 2.1 for the restricted type at the posterior median. These numbers suggest a specification of utility far enough from the usual log-utility assumption but also significant heterogeneity in the sensitivity to the interest rate for the two types. Finally, the posterior moments for the nominal rigidity parameters and policy rule coefficients are consistent with several contributions in the DSGE literature (Del Negro and Schorfheide, 2008). Importantly, price rigidities are estimated to be quite high relative to the micro-evidence. These parameters may significantly influence the simulations. Therefore, in the robustness analysis, we repeat our baseline experiment with ζ_p set at the prior mean.

2.4. Interest Rate Diagnostics

This subsection briefly discusses a number of interest rate diagnostics (variance, variance decomposition, historical shock decomposition). More details, including Tables and plots, are available in the online Appendix.

As we use data on both short- and long-term interest rates, our model can match the exact path of these variables through Kalman filtering and smoothing. Nonetheless, we can also compute the model-based unconditional moments for each variable. In particular, we focus on the variance of interest rates that the model is able to produce given the posterior distributions of the parameters.²⁴ Our DSGE model captures more than half of the variance of the federal funds rate (FFR) in the data (0.44 *versus* 0.81) and about one quarter of the 10-year yield (0.12 *versus* 0.47).²⁵

²³ One caveat is that most of our sample corresponds to a period of relative macroeconomic and financial stability in the US. Because the recent crisis may have exacerbated financial frictions, we subject our main experiment to a robustness check where we allow for an (exogenous) increase in the degree of segmentation.

²⁴ To compute the model-based unconditional variance of a certain variable, we draw a vector of parameters from the joint posterior distribution, compute the unconditional variance of the variable of interest, repeat the procedure 1,000 times and then take the median.

²⁵ The performance for long-term rates is not much worse than three-factors affine models of the term structure of interest rates. For example, the R^2 of the regression for the 10-year yield in Balduzzi *et al.* (1996) is 31%.

While our DSGE model, like most, fails to completely explain the term structure, we nevertheless provide a theory of how changes in long-term rates affect the real economy.

Different shocks explain the variance of short and long-term interest rates. For the FFR, the shock to the marginal efficiency of investment is the single most important driver of the variance at business cycle frequencies (periodic components with cycles between 6 and 32 quarters), consistent with the findings in Justiniano *et al.* (2010). In the shorter run, shocks to the monetary policy rule play a non-trivial role while at longer horizons preference shocks become relevant. The volatility of long-term rates is mostly accounted for (roughly 60%) by shocks to the risk premium, with preference and marginal utility of investment shocks splitting the remaining 40% of the variance more or less equally.

The historical shock decomposition is related to the variance decomposition but is conditional on the actual path of the data. The product of the historical shock decomposition is the marginal contribution of each shock to the path of each variable in the model. As with the variance, we report results for the median across 1,000 draws. Shocks to the risk premium pushed down the FFR since 1994, between 2 and 3 percentage points. Since 2007, monetary policy shocks have been exerting the opposite pressure. Our model hence suggests that economic conditions, not discretionary policy decisions, account for the low policy rate during the recent financial crisis.²⁶ The shock to the marginal efficiency of investment – the key factor in the variance decomposition – captures the cyclical movements in the FFR fairly well, with two notable exceptions: the early 1990s, when μ_t exercises downward pressures while the FFR is actually going up, and the end of the sample, when μ_t shock is pushing the FFR up. The same risk premium shocks that put downward pressure on the FFR since 1994 exert the opposite force on long-term rates, although other shocks (in particular, preference and productivity) partly offset this dynamics, especially at the end of the sample.

3. Simulating LSAP II

Our baseline experiment corresponds to a simulation of the US LSAP II programme, announced with the FOMC statement of 3 November 2010. The central bank buys long-term bonds (in exchange for short-term bonds) over the course of four quarters, holds its balance sheet constant for the following 2 years and progressively shrinks its holdings of long-term securities over the final 2 years of the simulation. We calibrate the size of the asset purchase programme to match a \$600 billion reduction of long-term debt in the hands of the private sector. Figure 1 illustrates the path of the market value of long-term bonds in the hands of the private sector (in deviations from trend) following the central bank purchases.²⁷

We also impose that the FFR stays at the ZLB for the first four quarters after the beginning of the asset purchase programme (the ‘extended period’ language), con-

²⁶ Because the FFR eventually hits the ZLB in the Fall of 2008, one way to recast this result is that the interest rate rule in the model calls for negative nominal interest rates.

²⁷ To be precise, we perform the simulation by feeding a series of shocks to the rule controlling the level of long-term bonds in the hands of the public that is announced to all agents in the economy. As such, the private sector is aware of the whole path when forming expectations about the future.

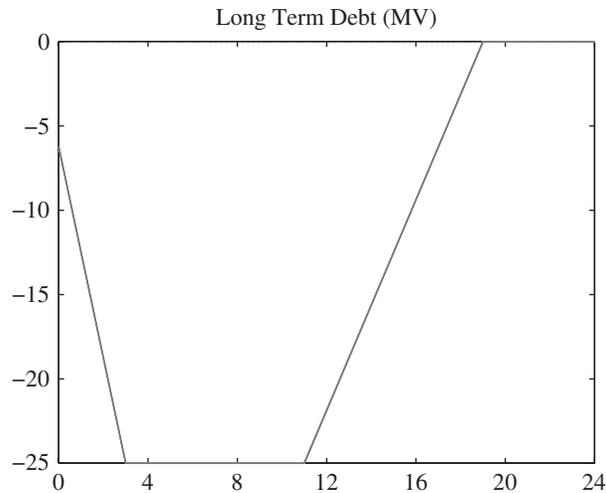


Fig. 1. *Simulated Path of the Market Value of Long-Term Debt*

sistent with the survey evidence from Blue Chip.²⁸ In the online Appendix, we explain the exact details of how we implement this commitment to the ZLB.

We begin by showing our main simulation of LSAP II at the prior distribution and then repeat the same experiment at the posterior. The following two subsections discuss the role of the commitment to the ZLB and how LSAP compares to interest rate policy shocks. In the online Appendix we present several robustness results.

3.1. *Simulation at the Prior Distribution*

This subsection illustrates how the choice of the priors constrains the macroeconomic effects of asset purchase programmes via Monte Carlo simulations. Specifically, we obtain 1,000 random draws for the parameter vector using the prior distribution. We then use each of these draws to solve the model and extract the path of the state variables in response to the LSAP experiment described above. Finally, we compute moments and percentiles of this sample of responses for the variables of interest.²⁹

Figure 2 shows the response of output growth, output level, inflation, FFR, 10-year yield and risk premium to the simulated LSAP II experiment at the prior distribution, all in annualised percentage rates. The level of output corresponds to percentage deviations from trend, as opposed to a rate of change, and thus is not annualised. These plots represent the marginal contribution of LSAP II, i.e. the deviations of each variable relative to the path that would have prevailed absent the policy intervention. The continuous line is the prior median response, while the grey-shaded area corre-

²⁸ Blue Chip has been asking the survey participants about the expected duration of the ZLB since the end of 2008. Until the recent (FOMC statement of 9 August 2011) change in the Federal Open Market Committee language that introduced a specific date for the expected liftoff, market participants had always maintained the expectation that the FFR would remain at the ZLB for the four/five quarters after the question was asked.

²⁹ The results are not sensitive to increasing the number of draws.

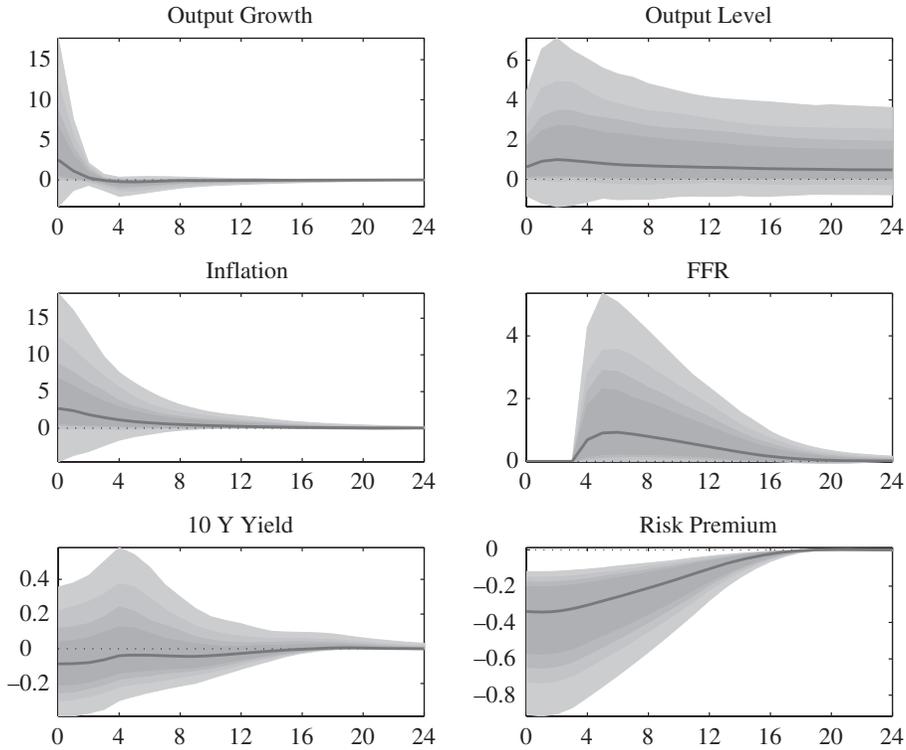


Fig. 2. Responses to Calibrated LSAP II Experiment at the Prior Distribution

sponds to the 50th, 60th, 70th and 80th prior probability intervals, from darker to lighter shading respectively.

The prior for the elasticity of the risk premium to the quantity of long-term debt implies a median response at the peak of about 30 bp, consistent with the estimates in Gagnon *et al.* (2011). The uncertainty bands cover pretty much the whole range of estimates in the empirical literature discussed in the introduction and summarised in Table 1. As a consequence of the change in the risk premium, output growth, the output level and inflation are higher than in the absence of asset purchases. By construction, the asset purchase programme achieves the desired effect in the model. The key question is how big these effects are. Our prior is fairly generous, encompassing very large effects, but also relative agnostic, as to extract as much information as possible from the data without imposing too many *ex ante* restrictions. To be more precise, the median prior response of output growth is 2.7% and the median response of inflation is 1.08%, roughly in line with the results in Baumeister and Benati (2010). Using a VAR model with time-varying coefficients, these authors find that a 60-bp reduction in long-term rates increases GDP growth by 3% and the inflation rate of the GDP deflator by 1% at the posterior median.

Our prior may be seen as too generous to the extent that we allow the effects of LSAP to be potentially quite extreme (e.g. the 80th percentile is above 15% for GDP growth and inflation). The literature, however, does not rule out these extreme outcomes. For

example, Kapetanios *et al.* (2011) present VAR evidence for the effects of similar policies on GDP growth in the UK that can be as high as 5% at the mean, depending on the estimation method. Our choice of fairly uninformative priors gives the model a chance to generate such large effects.

In response to higher output and inflation, the central bank eventually increases the interest rate in accordance with the policy rule but only after the end of the commitment to the ZLB. The evolution of the 10-year yield reflects the combined effect of the responses of the risk premium and the expected future short-term interest rate (expectations hypothesis). The former puts negative pressure on the long yield, whereas the latter exerts the opposite pressure. The outcome depends on how effective asset purchases ultimately are in boosting the economy. If LSAP programmes have a significant effect on output and inflation, the policy rule dictates a strong response of the federal funds rate which can potentially dominate over the negative impact on the risk premium and lead to an equilibrium increase in the 10-year yield.

3.2. *Simulation at the Posterior Distribution*

In the previous subsection, we concluded that, according to this model and our choice of the priors for the parameters, LSAP programmes can boost output and inflation while the effect on the 10-year yield is somewhat ambiguous, depending on the interplay between the risk premium and the expectation hypothesis. In this Section, we combine the prior with the data for the past 20 years or so to form a posterior distribution of the parameters. We then use the posterior to revisit our simulations of the effects of LSAP II. Figure 3 shows the same variables and simulation as Figure 2, but now using parameter draws from the posterior distribution.

The policy intervention reduces the risk premium by 11 bp on impact at the posterior median, reflecting the small elasticity of this variable to the quantity of debt discussed earlier. Combined with a small estimated degree of segmentation, not surprisingly the effects of LSAP II on aggregate activity are modest. On impact, GDP growth increases by 0.13% at the posterior median. The uncertainty is skewed on the upside to about 0.6%, partly due to the ZLB. After three quarters, the effect on output growth is less than a half of its peak (which occurs on impact) and completely vanishes after eight quarters. The effects on the level of output are modest too. The peak in this case occurs after 6 quarters at about 0.1% (posterior median) but now the effects persist longer – after 24 quarters, the output level is still more than 0.05% higher than without asset purchases. The reason for the high level of persistence of the level of real economic activity is that the asset purchase programme induces small but long-lasting movements in real interest rates.³⁰ This modest but persistent effect on GDP level is likely to be important from a welfare perspective – even more so if we consider that the 90th probability interval allows for an increase in the level of GDP as high as 0.5%. The effect on inflation is very small, 3 annualised bp at the median, and skewed upward, but even the 95th percentile is only about 15 bp.

³⁰ The persistence of both inflation and the nominal interest rate after exiting the ZLB is quite evident from Figure 3.

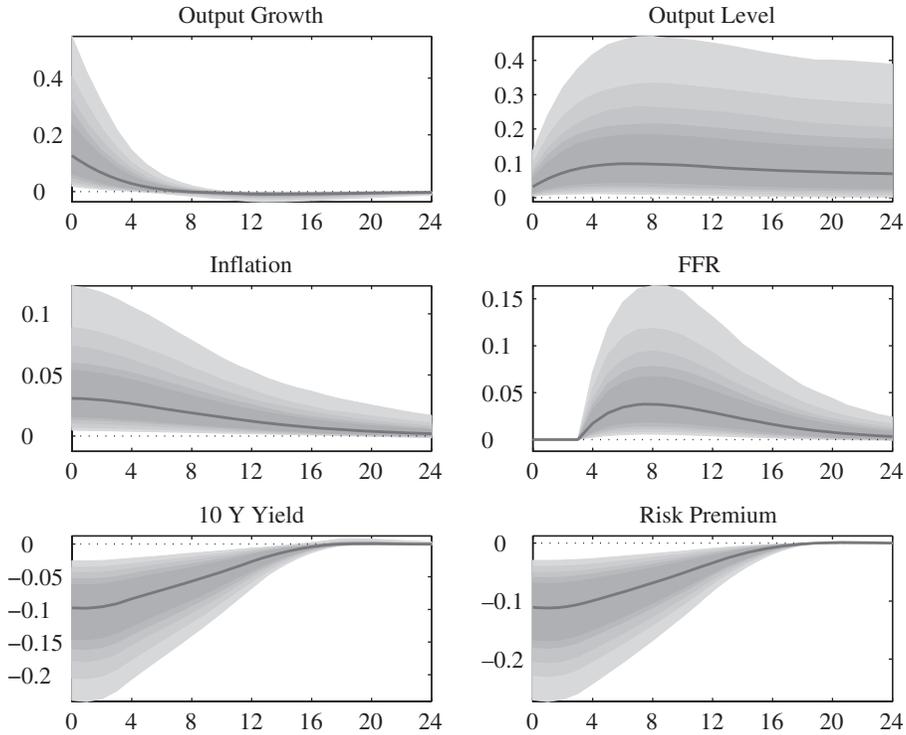


Fig. 3. Responses to Calibrated LSAP II Experiment at the Posterior Distribution

In spite of the small magnitudes, the positive boost of asset purchases to GDP growth and inflation puts upward pressure on the interest rate. After the four quarters at the ZLB which correspond to the commitment period, the FFR becomes positive but the median increase is only 4 bp. Because asset purchases introduce little stimulus, the central bank does not raise interest rates by much upon exiting the ZLB. The upward skewness in the FFR reflects the skewness in the effects on GDP growth and inflation. Later, we disentangle the effects of the non-linearity introduced by the ZLB from the pure estimation uncertainty.

The drop in the 10-year yield almost coincides with the reduction in the risk premium. Because the FFR only increases few basis points upon exiting the ZLB, the expectation hypothesis component of long-term rates plays a minor role.

Importantly, the duration of the ZLB commitment interacts with the LSAP programme. Recently, the Federal Reserve has extended its commitment to keep the nominal interest rate at zero for a longer period.³¹ The marginal effect of increasing the ZLB commitment by one extra quarter almost doubles the effects on GDP growth and GDP level while the effect on inflation is 50% bigger.³² These results show very clearly that in this model the ZLB commitment is very powerful in stimulating the

³¹ At the time we are writing this article, the commitment is ‘at least through late 2014’.

³² More details are available in the online Appendix.

economy, due to the strongly forward looking behaviour of the agents in the economy, and its effects increase non-linearly with the number of quarters of the commitment.

To summarise, the effects of LSAP II on GDP and inflation are modest, especially compared to the simulation at the prior, although the effects on the output level are quite persistent. The main reason for this result is that the two crucial parameters that control the effects of asset purchases on real activity – the degree of segmentation and the elasticity of the risk premium to the quantity of debt in the hands of the private sector – are estimated to be small. Yet, the posterior distributions for both these parameters are skewed so that we cannot completely discard the possibility of larger effects. Together with the ZLB, the long tails of the posterior estimates for ω_u and ζ' contribute to the upward skewness of the response of GDP growth and inflation in the baseline simulation.

Our results are at the lower end of the spectrum in the existing literature. Beside the estimates in Baumeister and Benati (2010) mentioned earlier, Chung *et al.* (2011), using the FRB/US model, assume that LSAP II induces a reduction in the risk premium of only 20 bp but increases the level of GDP by about 0.6% and inflation rate by 0.1%.

The results in this subsection are subject to the caveat, discussed for example in D'Amico *et al.* (2011), that the use of aggregate data on debt may bias the results, weakening the effects of asset purchases on yields. One possible rationalisation of this bias is that our treatment of the sample as homogeneous may have overlooked a structural change in the underlying structure of financial markets caused by the recent crisis. Alternatively, active debt management policy by the Treasury, in an attempt to minimise the financing costs of debt issuance, may make the elasticity of the risk premium to the quantity of debt hard to estimate. Under this hypothesis, the Treasury internalises the asset market friction to minimise the interest rate cost of marginal funding so that, *ex post*, the data display very little relationship between yield spreads and relative supply of assets at different maturities.³³

In the working paper version of this study (Chen *et al.*, 2011), we estimate the model without using observations on the quantity of debt. In that case, our prior on the elasticity of the risk premium to the quantity of debt encompasses most of the estimates in the empirical literature. Because the elasticity is not well-identified in this case, the posterior median coincides with the prior, which corresponds to a cumulative effect of –30 bp on the risk premium in response to LSAP II (Gagnon *et al.*, 2011). Under this specification, LSAP II increases GDP growth by 0.4% at the posterior median, while the impact on inflation is very similar to the case in which the quantity of debt is used in the estimation.³⁴

3.3. *The Role of the ZLB*

In this subsection, we show that the commitment of the central bank to keep the short-term nominal interest rate at the ZLB for an ‘extended period’ amplifies the effects of LSAP II. According to our simulations, asset purchases boost GDP growth and increase

³³ We thank the referee for suggesting this interpretation.

³⁴ Interestingly, the estimated degree of segmentation does not change appreciably between the two cases. Hence, the different effect on GDP growth is to attribute entirely to the elasticity of the risk premium to the quantity of debt.

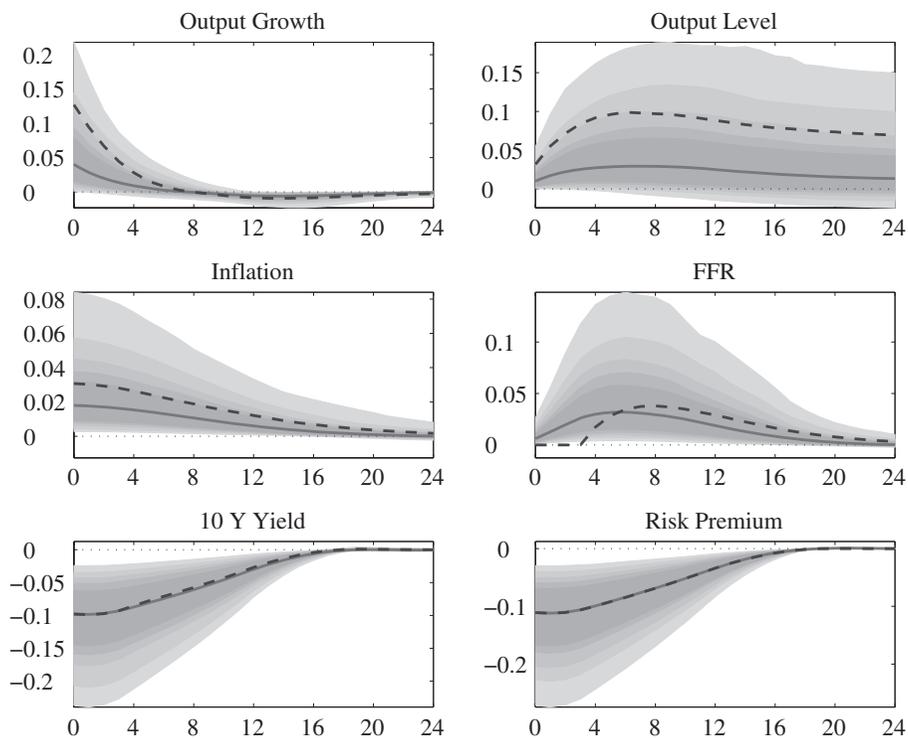


Fig. 4. Responses to LSAP II Experiment with (dashed line) and without (continuous line) ZLB Commitment

inflation, thus leading the central bank to increase the FFR. This endogenous interest rate response mitigates the macroeconomic stimulus of asset purchase programmes through the conventional monetary policy channel. A commitment to keep the short-term nominal interest rate at the ZLB for an ‘extended’ period of time prevents the endogenous response of the monetary authority and magnifies the contribution of asset purchases on macroeconomic outcomes. Here, we quantify the magnitude of such a mitigation effect.

Figure 4 shows the responses in the case in which we do not impose the commitment to the ZLB. For reference, the dashed line corresponds to the baseline simulation with the commitment to the ZLB imposed. Quantitatively, the ZLB commitment more than triples the effects of asset purchases on GDP growth. Without this commitment, output growth increases by 0.04%, compared to 0.13% in the baseline experiment. Inflation increases by less than 2 bp (0.018%), compared to 3 bp in the baseline case.

Interestingly, while the profile for the FFR differs from the baseline, the 10-year yield is almost identical. The cumulative effect of the increase in short rates on long rates via the expectation-hypothesis component is the same. Without ZLB, the increase in nominal rates occurs earlier but is smoother.

Importantly, the responses to LSAP II remain skewed upward, regardless of whether the ZLB is imposed or not. This observation suggests that the role of the skewness in

the posterior distribution of the degree of segmentation and of the semi-elasticity of the risk premium to the quantity of debt plays a central role in explaining the upside uncertainty of the response of macroeconomic variables.

3.4. *Comparison with a Standard Monetary Policy Shock*

One of the motivations for central banks to engage in asset purchases is to support output and inflation at times in which the ZLB constrains conventional interest rate setting. To give a sense of the relative effectiveness of these two policies, this Section compares the effects of asset purchase programmes discussed so far with a standard monetary policy shock, that is, an unexpected reduction of the short-term nominal interest rate.

Figure 5 shows the response of the key macroeconomic variables to an unexpected reduction of 25 bp in the short-term interest rate. The median effect on GDP growth is somewhat stronger than in the baseline simulation previously discussed while the median effect on inflation is very much comparable.³⁵ Furthermore, the effects of the interest rate shock on the output level are not only stronger but also more persistent than those of LSAP. The implied decrease in long-term rates, however, is much smaller, only 1 bp.³⁶ Moreover, the long-term rate quickly turns positive. This result is not surprising given that the risk premium does not change. Therefore, the expectation hypothesis component completely pins down the long-term interest rate in this case.

Another significant difference is the smaller uncertainty about the effects in the case of an interest rate shock. The absence of the ZLB constraint may in part explain why the posterior bands are more symmetric. Yet, as discussed in the previous subsection, even in the absence of a commitment to the ZLB, uncertainty remains skewed upward, mostly due to the skewness of the posterior estimates of the degree segmentation and of the semi-elasticity of the term premium to the quantity of debt. Because asset market frictions play a smaller role in case of a shock to the short-term interest rate, the uncertainty in the response of GDP growth and inflation becomes smaller and much more symmetric.

Overall, in this model, the effects of LSAP II on output and inflation are slightly smaller than those of a surprise reduction of the FFR by 25 bp, and much more uncertain. This conclusion stands in contrast with Furher and Moore (1995), who find that output is four times more sensitive to long-term than short-term rates. According to this metric, the 11-bp reduction in the risk premium triggered by LSAP II should be equivalent to a reduction of the FFR of about 44 bp. Our results are thus much less generous to changes in the risk premium, confirming our previous finding that the model simulations yield weaker effects of LSAP II on output and inflation than what the VAR literature suggests.

³⁵ The median effects of the monetary policy shock on output and inflation in our model are slightly smaller than in standard estimated DSGE models, such as Smets and Wouters (2007). The key parameters that determine this result are the higher estimated degree of price rigidity and the lower sensitivity of demand to the interest rate (higher coefficient of risk aversion).

³⁶ Our estimates thus imply a much smaller sensitivity of long-term rates to shocks to the short-term rate. As a point of comparison, Gurkaynak *et al.* (2005) estimate that the typical response of long rates to a cut of 100 bp in the FFR is 15 bp.

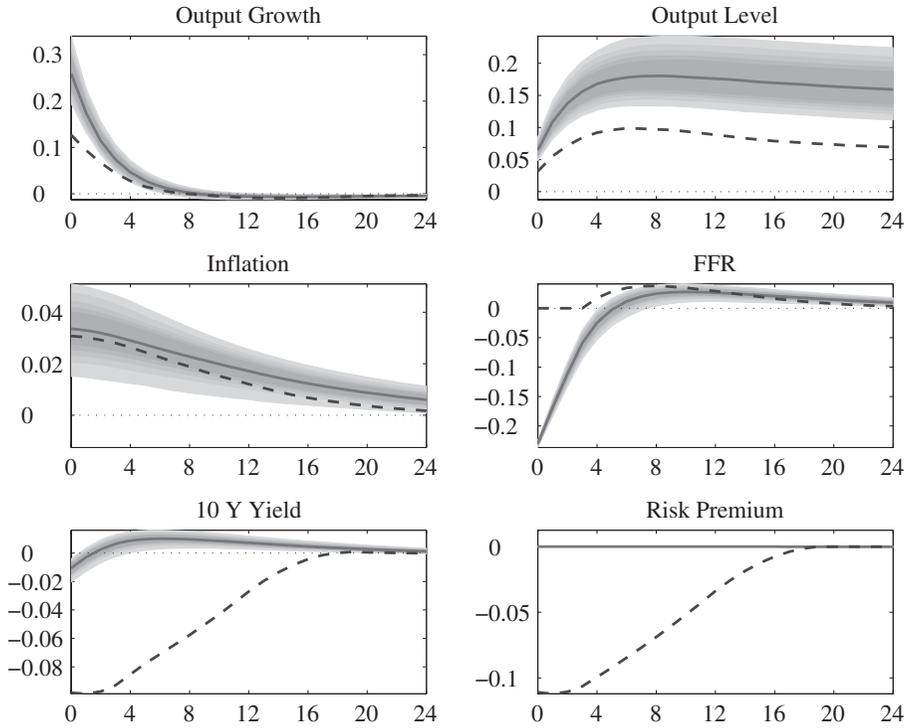


Fig. 5. Responses to an Annualised 25 bp Innovation to the FFR (continuous line), Compared with the Baseline LSAP II Experiment (dashed line)

4. Conclusions

Using an estimated medium-scale DSGE model, we find that the effects of recent asset purchase programmes on macroeconomic variables, such as GDP growth and inflation, are likely to be modest, although with a lasting impact on the level of GDP. Asset purchase programmes are in principle effective at stimulating the economy because of limits to arbitrage and market segmentation between short-term and long-term government bonds. The data, however, provide little support for these frictions to be pervasive.

In the online Appendix, we consider several robustness exercises and find that the effects on GDP growth are not very likely to exceed a third of a percentage point. The inflationary consequences of asset purchase programmes are consistently small. Combining LSAP programmes with a commitment to keep interest rates low for some period of time allows these programmes to be more effective in boosting GDP growth and inflation.

Our results do not depend on whether asset purchases are financed via reserves or sales of short-term debt, to the extent that money and short-term bonds are close to perfect substitutes. Therefore, according to our model, the effects of the Federal Reserve’s last round of asset purchases (also known as ‘Operation Twist Again’) should be in line with the estimates from LSAP II after controlling for the scale factor.

Links to the working paper version of this article and the online Appendix are available at the authors' webpages.

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Additional Supporting information may be found in the online version of this article:

Appendix A. Model.

Appendix B. Normalised Equations.

Appendix C. Model Steady State.

Appendix D. Log-linear Approximation.

Appendix E. Data.

Appendix F. Implementing the Commitment to the Zero Lower Bound.

Appendix G. Robustness.

Appendix H. Diagnostics.

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