In Trigari (2009), the value of an open vacancy V_t is expressed in terms of current consumption (see her equation (13)). She writes:

$$V_t = -\frac{\kappa}{\lambda_t} + E_t\beta_{t,t+1}\left[q_t(1-\rho_{t,t+1})\int_0^{\underline{a}_{t+1}} J_{t+1}(a_{t+1})\frac{dF(a_{t+1})}{F(\underline{a}_{t+1})} + (1-q_t)V_{t+1}\right] \tag{1}$$

with κ the utility cost of having an open vacancy,λ_t the marginal utility of consumption, ρ_t the job separation rate, q_t the probability to fill a job and J_t the value of a job for a firm expressed in terms of current consumption of final goods. Such an equation simply says that an open vacancy yields a negative return in period t. In period $t+1$ the vacancy is filled and not destroyed with probability $q_t(1-\rho_{t+1})$ such that the firm obtains J_{t+1}. By contrast, with probability $(1-q_t)$ the vacancy remains unfilled.

In this first equation, **I have some troubles with the first term of the right hand side** $-\frac{\kappa}{\lambda_t}$. Indeed, the utility cost of posting a vacancy is divided by the marginal utility of consumption. Such a writing seems very uncommon because other papers in the field do not have such an expression (in general, κ is not divided by λ_t).

Then, with this writing of V_t Trigari obtains the following vacancy posting condition (equation (16)-(17)) in Trigari (2009, JMCB)):

$$\frac{\kappa}{\lambda_t q_t} = E_t\beta_{t,t+1}(1-\rho_{t+1})\int_0^{\underline{a}_{t+1}} J_{t+1}(a_{t+1})\frac{dF(a_{t+1})}{F(\underline{a}_{t+1})} \tag{2}$$

Again the marginal utility of consumption λ_t appears in the denominator of the left hand side of this equation[1]. This corresponds to my main problem with the model[2].

The firms' problem:

Having say that, my goal is now to retrieve the vacancy posting condition of Trigari (2009) from the firm problem. Intermediate good producers maximize their discounted profit by choosing the optimal level of employment n_t and the optimal number of vacancies v_t:

$$E_t\sum_{t=0}^{\infty}\beta^t\frac{\lambda_t}{\lambda_0}\left(x_t f(h_t)n_t - \kappa v_t - w_t(a_t)h_t n_t\right) \tag{3}$$

Subject to the perceived law of motion of employment:

$$n_t = (1-\rho_{t-1})n_{t-1} + q_{t-1}v_{t-1} \tag{4}$$

[1]The vacancy posting condition can be rewritten as $\frac{\kappa}{\lambda_t q_t} = E_t\beta_{t,t+1}(1-\rho_{t+1})\left(x_{t+1}h_{t+1} - w_{t+1}h_{t+1} + \frac{\kappa}{\lambda_{t+1}q_{t+1}}\right)$. Description of undefined variables will follow.

[2]Observe also that the expression $\frac{\kappa}{\lambda_t}$ appears in the wage equation and in the expression of the threshold determining endogenous job separation.

with x_t the relative price of intermediate goods, w_t the hourly wage rate which depends on the idiosyncratic realization of the preference shock (which governs endogenous separation in the model). n_t is employment, v_t vacancies and q_t the probability to fill a vacancy from firms' viewpoint.

At this stage, my main question is: should we divide κ by λ_t directly in equation (3)? Or such an expression would be the result of the derivation of the firms' problem? Without a direct division of κ by λ_t, I am unable to retrieve Trigari's expression... Furthermore, what is the implication of this writing for the model dynamics? Indeed, there is no reason to have a constant marginal utility of consumption after an aggregate shock.