first order approximation to the unconditional means of endogenous variables with their non stochastic steady state values. This neglects important effects of uncertainty on the average level of households' welfare. A first order approximation to the policy functions would give an incorrect second order approximation of the welfare function [18].

To overcome this limitation and obtain a second-order accurate approximation, we adopt a perturbation technique introduced by Fleming (1971) and applied to various types of economic models by Judd and coauthors[19] and recently generalized by Schmitt-Grohe and Uribe (2002)[20] (SU henceforth). Second order approximations are quite convenient to implement since, even capturing the effects of uncertainty, do not suffer from the "curse of dimensionality"[21].In fact, following SU, given the first-order terms of the Taylor expansions of the functions expressing the model's solution, the second-orther terms can be identified by solving a linear system of equations whose terms are the first order terms and the derivatives up to the second order of the equilibrium conditions evaluated at the non-stochastic steady state.

5.2 Welfare Measure and Optimal Rules

How should monetary policy be conducted in a world economy with credit frictions at the household level? In order to answer this question, we rely on utility-based welfare calculations, assuming that the benevolent monetary authority maximize the utility of the households subject to the model's equilibrium conditions. Formally, the optimal policy maximize the household's life-time utility:

[18] See Woodford (2002) and Kim et al. (200?) for a discussion of situations in which second-order accurate welfare evaluations can be obtained using first-order approximations to the policy functions.

[19] See Judd and Guu (1993,1997) for applications to deterministic and stochastic, continuous and discrete-time growth models in one state variable, Gaspar and Judd (1997) for multidimentional stochastic models in continuous time approximated up to the fourth-order, Judd (1998) presents the general method , Jin and Judd (2001) extended these methods to more general rational expectations models .

[20] The derive a second-order approximation to the policy function of a general class of dynamic, discrete-time, rational expectations models. They show that in a second-order expansion of the policy functions, the coefficients on the linear and quadratic terms in the state vector are independent of the volatility of the exogenous shocks. Thus, only the constant term is affected by uncertainty.

[21] Models with large numbers of state variables can be solved without much computational effort.

$$V_t \equiv E_t \left[\sum_{i=1}^{2} \eta_i \sum_{j=0}^{\infty} \beta_i^j U(c_{i,t+j}, h_{i,t+j}, L_{i,t+j}) \right]$$

where η_i are the weights on households' utilities. Where $U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_h \ln h_{it} - \nu_L \frac{L_{i,j}^{1+\varphi_L}}{1+\varphi_L}$.

We measure welfare as the conditional expectation at time zero ($t = 0$), time in which all state variables of the economy equal their steady state values. Since different policy regimes, even not affecting the non-stochastic steady state, are associated with different stochastic steady states, in order to not neglect the welfare effects during the transition from one to another steady state, we use a conditional welfare criterion. Thus, we evaluate welfare conditional on the initial state being the non stochastic steady state[22].

We evaluate the optimal setting of monetary policy in the constrained class of simple interest rate rules.

$$R_t = \Theta(X)$$

Where X represent easily observable macroeconomic indicators tested as possible arguments of the rule:

$$X = \left[R_{t-1}, \frac{\pi_t}{\pi_{ss}}, \frac{y_t}{y_{ss}}, \frac{q_t}{q_{ss}}, \frac{b_{2t}}{b_{2ss}} \right]$$

As implementability condition is required policies to deliver local uniqueness of the rational expectations equilibrium. Following SU we require that the associate equilibrium be locally unique. The configuration of parameters satisfying the requirements and yielding the highest welfare gives the optimal implementable rule. In characterizing optimal policy we search over a grid considering different ranges of the parameters. Then, we compute the total welfare associated to the different parametrizations of the rule.

[22] An alternative to condition on a particular initial state could be to condition on a distribution of values for the initial state. Anyway, when there is a time-inconsistency problem, the optimality of the rule may depens on the initial conditions. A way to overcome this problem could be to find the rule that would prevail under commitment from a "timeless perspective" see Giannoni and Woodford (2002).