

1 A Cash In Advance Model

1.1 NL

$$\mathcal{U}_{c,t} = \left(\frac{1}{c_t - \varsigma c_{t-1}} \right) \quad (1)$$

$$\mathcal{U}_{n,t} = \psi_n n_t^\eta \quad (2)$$

$$\mathcal{U}_{c,t} = \beta \mathbb{E}_t \left\{ \frac{\mathcal{U}_{c,t+1}}{\pi_{t+1}} \right\} R_t^D \quad (3)$$

$$w_t = \frac{\mathcal{U}_{n,t}}{\mathcal{U}_{c,t}} \quad (4)$$

$$c_t = \frac{m_{t-1}}{\pi_t} + n_t w_t \quad (5)$$

$$c_t = n_t w_t + \Pi_t - \left[\tau_t + \left(m_t - \frac{m_{t-1}}{\pi_t} \right) \right] \quad (6)$$

$$g_t = \tau_t + \left(m_t - \frac{m_{t-1}}{\pi_t} \right) \quad (7)$$

or

$$c_t = y_t - g_t \quad (8)$$

Remove m_t from the system (9)

Remove τ_t from the system (10)

$$y_t = \frac{A_t n_t}{V_t^p} \quad (11)$$

$$mc_t = \frac{w_t}{A_t} \quad (12)$$

$$\Pi_t = y_t - w_t n_t \quad (13)$$

$$1 = (1 - \omega_p) (p_t^*)^{1-\lambda_p} + \omega_p \left(\frac{\pi_{t-1}^{\iota_p} \pi^{1-\iota_p}}{\pi_t} \right)^{1-\lambda_p} \quad (14)$$

$$Q3_t = \mathcal{U}_{c,t} y_t m c_t + \beta \omega_p \mathbb{E}_t \left\{ Q3_{t+1} \left(\frac{\pi_t^{\iota_p} \pi^{1-\iota_p}}{\pi_{t+1}} \right)^{-\lambda_p} \right\} \quad (15)$$

$$Q4_t = \mathcal{U}_{c,t} y_t + \beta \omega_p \mathbb{E}_t \left\{ Q4_{t+1} \left(\frac{\pi_t^{\iota_p} \pi^{1-\iota_p}}{\pi_{t+1}} \right)^{1-\lambda_p} \right\} \quad (16)$$

$$p_t^* = \left(\frac{\lambda_p}{\lambda_p - 1} \right) \frac{Q3_t}{Q4_t} \quad (17)$$

$$R_t^D = (R_{t-1}^D)^{\rho^R} \left(\left(\frac{y_t}{y_{t-1}} \right)^{\phi_y} \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} R^D \right)^{(1-\rho^R)} \exp(\epsilon_t^R) \quad (18)$$

$$\ln A_t = \rho^A \ln A_{t-1} + \epsilon_t^R$$

$$\ln g_t = \rho^G \ln g_{t-1} + \epsilon_t^G$$

1.2 LL

$$\hat{\mathcal{U}}_{c,t} = \frac{-(\hat{c}_t - \varsigma \hat{c}_{t-1})}{(1-\varsigma)} \quad (19)$$

$$\hat{\mathcal{U}}_{n,t} = \eta \hat{n}_t \quad (20)$$

$$\hat{\mathcal{U}}_{c,t} = \mathbb{E}_t \left\{ \hat{\mathcal{U}}_{c,t+1} + \hat{R}_t^D - \hat{\pi}_{t+1} \right\} \quad (21)$$

$$\hat{w}_t = \hat{\mathcal{U}}_{n,t} - \hat{\mathcal{U}}_{c,t} \quad (22)$$

$$\hat{c}_t = \frac{1}{c} \left(nw (\hat{n}_t + \hat{w}_t) + \frac{m}{\pi} (\hat{m}_{t-1} - \hat{\pi}_t) \right) \quad (23)$$

$$\hat{c}_t = \frac{1}{c} \left[nw (\hat{n}_t + \hat{w}_t) + \Pi \hat{\Pi}_t - \tau \hat{\tau}_t + \frac{m}{\pi} (\hat{m}_{t-1} - \hat{\pi}_t) - m \hat{m}_t \right] \quad (24)$$

$$\hat{g}_t = \left(\frac{1}{g} \right) \left[\tau \hat{\tau}_t + m \hat{m}_t - \frac{m}{\pi} (\hat{m}_{t-1} - \hat{\pi}_t) \right] \quad (25)$$

or

$$\hat{c}_t = \left(\frac{1}{c} \right) (y \hat{y}_t - g \hat{g}_t) \quad (26)$$

Remove \hat{m}_t from the system

Remove $\hat{\tau}_t$ from the system

$$\quad (28)$$

$$\hat{y}_t = \hat{A}_t + \hat{n}_t - \hat{V}_t^p \quad (29)$$

$$\hat{m}_t = \hat{w}_t - \hat{A}_t \quad (30)$$

$$\hat{\Pi}_t = \left(\frac{1}{\Pi} \right) (y \hat{y}_t - w n (\hat{w}_t + \hat{n}_t)) \quad (31)$$

$$\hat{\pi}_t = \left(\frac{\iota_p}{1 + \iota_p \beta} \right) \hat{\pi}_{t-1} + \left(\frac{\beta}{1 + \iota_p \beta} \right) \mathbb{E}_t \{ \hat{\pi}_{t+1} \} + \frac{(1 - \beta \omega_p)(1 - \omega_p)}{\omega_p (1 + \iota_p \beta)} (-\hat{X}_t) \quad (32)$$

$$\hat{R}_t^D = \rho^R \hat{R}_{t-1}^D + (1 - \rho^R) \left(\phi_y (\hat{y}_t - \hat{y}_{t-1}) + \phi_\pi \hat{\pi}_t \right) + \epsilon_t^R \quad (33)$$

1.3 Steady State

$$\mathcal{U}_c = \frac{1}{c(1 - \varsigma)} \quad (34)$$

$$\mathcal{U}_n = \psi_n n^\eta \quad (35)$$

$$\mathcal{U}_c = \beta \left\{ \frac{\mathcal{U}_c}{\pi} \right\} R^D \implies R^D = \frac{\pi}{\beta} \quad (36)$$

$$w = \frac{\mathcal{U}_n}{\mathcal{U}_c} \implies w = \psi_n n^\eta c (1 - \varsigma) \quad (37)$$

$$c = \frac{m}{\pi} + nw \quad (38)$$

$$c = nw + \Pi - \left[\tau + \left(m - \frac{m}{\pi} \right) \right] \quad (39)$$

$$g = \tau + \left(m - \frac{m}{\pi} \right) \quad (40)$$

which can be rearranged as

$$c = y - g$$

$$y = \frac{An}{V^p} \quad (41)$$

$$mc = \frac{w}{A} \implies w = A mc \quad (42)$$

$$\Pi = y - wn \quad (43)$$

$$1 = (1 - \omega_p)(p^*)^{1-\lambda_p} + \omega_p \left(\frac{\pi^{\iota_p} \pi^{1-\iota_p}}{\pi} \right)^{1-\lambda_p} \implies p^* = 1 \quad (44)$$

$$Q3 = \mathcal{U}_c y mc + \beta \omega_p \left\{ Q3 \left(\frac{\pi^{\iota_p} \pi^{1-\iota_p}}{\pi} \right)^{-\lambda_p} \right\} \implies Q3 = \frac{\mathcal{U}_c y mc}{1 - \beta \omega_p} \quad (45)$$

$$Q4 = \mathcal{U}_c y + \beta \omega_p \left\{ Q4 \left(\frac{\pi^{\iota_p} \pi^{1-\iota_p}}{\pi} \right)^{1-\lambda_p} \right\} \implies Q4 = \frac{\mathcal{U}_c y}{1 - \beta \omega_p} \quad (46)$$

$$p^* = \left(\frac{\lambda_p}{\lambda_p - 1} \right) \frac{Q3}{Q4} \implies 1 = \left(\frac{\lambda_p}{\lambda_p - 1} \right) mc \implies mc = \frac{1}{\left(\frac{\lambda_p}{\lambda_p - 1} \right)} \quad (47)$$

1.3.1 Solving analytically for SS

$$\pi = 1 \quad (48)$$

$$R^D = \frac{\pi}{\beta} \quad (49)$$

$$mc = \frac{1}{\left(\frac{\lambda_p}{\lambda_p - 1} \right)} \quad (50)$$

$$w = A mc \quad (51)$$

$$\begin{aligned}
y &= \frac{An}{V^p} \\
\boxed{\frac{n}{y}} &= \frac{V^p}{A}
\end{aligned} \tag{52}$$

$$\begin{aligned}
c &= y - g \\
\frac{c}{y} &= 1 - \frac{g}{y} \\
\boxed{\frac{c}{y}} &= 1 - g_y \quad ; g_y \text{ is a parameter}
\end{aligned} \tag{53}$$

$$\begin{aligned}
w &= \psi_n n^\eta c (1 - \varsigma) \\
n^\eta c &= \frac{w}{\psi_n (1 - \varsigma)} \\
y^{1+\eta} \left(\frac{n}{y}\right)^\eta \left(\frac{c}{y}\right) &= \frac{w}{\psi_n (1 - \varsigma)} \\
y^{1+\eta} &= \frac{\left(\frac{w}{\psi_n (1 - \varsigma)}\right)}{\left(\boxed{\frac{n}{y}}\right)^\eta \left(\boxed{\frac{c}{y}}\right)} \\
y &= \left(\frac{\left(\frac{w}{\psi_n (1 - \varsigma)}\right)}{\left(\boxed{\frac{n}{y}}\right)^\eta \left(\boxed{\frac{c}{y}}\right)} \right)^{\left(\frac{1}{1+\eta}\right)}
\end{aligned} \tag{54}$$

$$c = \boxed{\frac{c}{y}} y \tag{55}$$

$$n = \boxed{\frac{n}{y}} y \tag{56}$$

$$g = g_y y \tag{57}$$

$$\Pi = y - wn \tag{58}$$