SOE TNT Models

November 18, 2018

1 SOE Model with Capital

1.1 Households

Households maximize the present value of their lifetime utility given by,

$$\sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

where β is the intertemporal discount factor, L represents total hours worked and C is consumption, which we assumed to be a CES aggregation of tradable, C^T, and non-tradable, C^N, consumption

$$C_t = \left[\varphi^{1/\epsilon}\left(C_t^N\right)^{1-\frac{1}{\epsilon}} + (1+\varphi)^{1/\epsilon}\left(C_t^T\right)^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where ϵ is the elasticity of substitution and $0 < \varphi < 1$ is a parameter governing the share of each type of goods in consumption expenditures. Tradable consumption is in turn a Cobb-Douglas aggregation of exportable, C^X, and importable, C^M, goods:

$$C^T = \left(\frac{C^X}{\chi}\right)^{\chi}\left(\frac{C^M}{1-\chi}\right)^{1-\chi} \tag{2}$$

with χ determining the share of exportable in good in tradable total expenditure. Demands for each type of goods are given by

$$C_t^N = \varphi\left(\frac{P_t}{P_t^N}\right)^{\epsilon} C_t \tag{3}$$

$$C_t^T = (1-\varphi)\left(\frac{P_t}{P_t^T}\right)^{\epsilon} C_t \tag{4}$$

$$C_t^X = \chi\left(\frac{P_t^T}{P_t^X}\right) C_t^T \tag{5}$$

$$C_t^M = (1-\chi)\, P_t^T C_t^T \tag{6}$$

Household members could work in either the exportable sector or the non-tradable sector so that hours worked are given by,

$$L_t = L_t^X + L_t^N \tag{7}$$

where L_t^X and L_t^N are hours worked in the exportable sector and the non tradable sector respectively. Importantly, we assume no labor market frictions, so labor is perfectly mobile and equilibrium implies that real wages must be the same in both sectors.

Household's resource constraint in period t is,

$$P_t C_t + D_t^* + \frac{\Phi_D}{2}\left(D_{t+1}^* - \bar{D^*}\right)^2 = W_t L_t + \frac{D_{t+1}^*}{1+r_t} + \Omega_t \tag{8}$$

where P_t is the price of the consumption bundle, D_t^* is the stock of international debt at the beginning of the period, W_t is real wages, r_t is the interest rate, and Ω_t includes any profit coming from firms. The term

$$\frac{\Phi_D}{2}\left(D_{t+1}^* - \bar{D^*}\right)^2 \tag{9}$$

represents a portfolio-adjustment cost, which we introduce to "close" this open economy model.

Besides consumption demands equation (1) to (6), the budget constraint and a No-Ponzi game condition, households behavior is described by the following standard first order conditions,

$$\begin{aligned} U'_{C,t} &= P_t \lambda_t \\ -U'_{l,t} &= W_t \lambda_t \\ \frac{\lambda_t}{1+r_t} + \lambda_t \Phi_D \left(D_{t+1}^* - \bar{D^*}\right) &= \beta E_t \left\{\lambda_{t+1}\right\} \end{aligned}$$

where $\beta^t \lambda_t$ denotes the Lagrange Multiplier associated with the resource constraint in period t.

1.2 Production

There are representative firms in each sector of the economy. Both of these representative firms have Cobb-Douglas production functions,

$$Y_t^j = A_t^j \left(K_t^j\right)^{\alpha_j} \left(L_t^j\right)^{1-\alpha_j}$$

for $j \in \{X, N\}$, K^j is capital, L^j is labor, and α_j is a parameter governing capital intensity in each sector.

Firms accumulate capital according to equation,

$$K_{t+1}^j = (1-\delta) K_t^j + I_t^j - I_t^j \frac{\phi}{2}\left(\frac{I_t^j}{I_{t-1}^j} - 1\right)^2 \tag{10}$$

where δ is the depreciationrate, $I^j$is sectorial investment and the last term represents capital adjustment costs, with parameter ϕ governing the size of those costs.

Firms choose labor and capital to maximize the expected present value of profits, that each period t are given by

$$P_t^j Y_t^j - W_t L_t^j - P_t^I I_t^j$$

Since households own the firms, profits are discounted by $\beta^t \lambda_t$. Let $\beta^t \lambda_t Q_t^j$ denote the Lagrange multiplier for equation (10). Firms behavior is described by the law of motion for sectoral capital, (10), a set of no-Ponzi game conditions and the following first order conditions

$$P_t^j \left(1 - \alpha_j\right) \frac{Y_t^j}{L_t^j} = W_t$$

$$\begin{aligned}
Q_t^j &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[P_{t+1}^j \alpha_j \frac{Y_{t+1}^j}{K_{t+1}^j} + Q_{t+1}^j \left(1 - \delta\right) \right] \right\} \\
P_t^I &= Q^j - \frac{\phi}{2} \left(\frac{I_{t+1}^j}{I_{t-1}^j} - 1 \right)^2 - \phi \frac{I_{t+1}^j}{I_{t-1}^j} \left(\frac{I_{t+1}^j}{I_{t-1}^j} - 1 \right) \\
&\quad + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^j \left[\phi \left(\frac{I_{t+1}^j}{I_{t-1}^j} \right)^2 \left(\frac{I_{t+1}^j}{I_{t-1}^j} - 1 \right) \right] \right\}
\end{aligned}$$

where P^j is the price of good j and $P^I$is the price of new capital.

1.3 Investment Basket

New capital in both sectors is obtained as a combination of importable and non-tradable goods. We assume that new investment goods are produced according to the following production function

$$I_t = \left(\frac{IC_t^N}{\gamma} \right)^\gamma \left(\frac{IC_t^M}{1 - \gamma} \right)^{1-\gamma}$$

where

$$I_t = I_t^N + I_t^M$$

is total investment, $IC_t^N$and IC_t^M are the amount of non-tradable and importable goods used by the investment sector, $\gamma \in (0, 1)$ is parameter that determines the share of non-tradable expenditures in the aggregate investment basket. Under perfect competition in this sector, cost minimization generates the following demands for non-tradable and importable goods

$$\begin{aligned}
IC_t^N &= \gamma \left(\frac{P_t^I}{P_t^N} \right) I_t \\
IC_t^M &= (1 - \gamma) \left(P_t^I \right) I_t
\end{aligned}$$

1.4 Market Clearing Conditions

We next describe the aggregate equilibrium conditions. Note that we have already stated that production factor markets are in equilibrium. Equilibrium in the non tradable sector implies that,

$$Y_t^N = C_t^N + IC_t^N$$

Next, we define the trade balance as

$$TB_t = P^X \left(Y_t^X - C_t^X\right) - \left(C_t^M + IC_t^M\right)$$

It's usually assumed that P_t^X is fixed by exogenous conditions. The balance of payments implies that the current account should be equal to the change in the international net investment position, that is,

$$D_t^* + \frac{\Phi_D}{2}\left(D_{t+1}^* - \bar{D}^*\right)^2 = \frac{D_{t+1}^*}{1+r_t} + TB_t$$

We are going to use the following definitions

$$\begin{aligned} GDP_t &= Y_t^N + Y_t^X \\ RER_t &= \frac{1}{P_t} \end{aligned}$$

1.5 Functional Forms and Exogenous Process

For the Bernoulli utility function we choose a GHH specification,

$$U\left(C_t, L_t\right) = \frac{\left[C_t - \zeta\nu^{-1}L_t^{\nu}\right]^{1-\theta}}{1-\theta}$$

This specification has been widely used for emerging countries (specially analyzing the impact of interest rates). Note that this specification implies that labor supply decisions are not subject to wealth effects. For the interest rate, we are going to assume

$$r_t = \bar{r} + r_t^w$$

where $\bar{r}$ denotes the steady state value of the interest rate and r_t^w is going to represent deviations. For both r_t^w and $\log\left(P_t^X\right)$ we are going to assume $AR(1)$ processes.

1.6 Steady State

We are going to take as a given the debt to GDP ratio and the total hours worked in the economy, i.e. D^*/GDP^N and L. Next, we are going to have certain "free" steady state values

$$r = \beta^{-1} - 1$$

$$P^T = \left(P^X\right)^{\chi}$$

$$TBY = \left(\frac{TB}{GDP^N}\right) = \frac{D^*}{GDP^N}\frac{r}{1+r}$$

Next, given that productivity is going to shift in our simulations, we are going to try