

28 variables: g, r

$$\tilde{c}_1, \tilde{c}_2, \tilde{d}_1, \tilde{d}_2, \tilde{\lambda}_{BC}, \tilde{\lambda}_{TFP}, \lambda_{ER}, L_a, L_{1,y}, L_{2,y}, \tilde{K}_1, \tilde{K}_2, \tilde{w}$$

$$\tilde{c}'_1, \tilde{c}'_2, \tilde{d}'_1, \tilde{d}'_2, \tilde{\lambda}'_{BC}, \tilde{\lambda}'_{TFP}, \lambda'_{ER}, L'_a, L'_{1,y}, L'_{2,y}, \tilde{K}'_1, \tilde{K}'_2, \tilde{w}'$$

28 equations:

$$g_{t+1} = 1 + \mu(L_{a,t}^\Omega + L'_{a,t}^\Omega)$$

$$r_t = \frac{\tilde{c}'_{1,t}}{\tilde{d}_{2,t}}$$

$$\tilde{c}_{1,t} + \tilde{c}_{2,t} + \frac{\tilde{d}_{1,t} + \tilde{d}_{2,t}}{r_t} + \tilde{K}_{1,t+1} + \tilde{K}_{2,t+1} + \pi'[\tilde{K}_{2,t+1}g_{t+1} - (1 - \delta)\tilde{K}_{2,t}]^2 + \phi'\tilde{c}'_{1,t} + \tilde{w}'_t L'_{1,y,t} =$$

$$L_{1,y,t}^\alpha \tilde{K}_{1,t}^{1-\alpha} + L'_{1,y,t}{}^\alpha \tilde{K}_{2,t}^{1-\alpha} + (1 - \delta)(\tilde{K}_{1,t} + \tilde{K}_{2,t}) + \phi \frac{\tilde{d}_{2,t}}{r_t} + \frac{\tilde{w}_t L_{2,y,t}}{r_t}$$

$$r_t(\tilde{c}'_{1,t} + \tilde{c}'_{2,t}) + \tilde{d}'_{1,t} + \tilde{d}'_{2,t} + \tilde{K}'_{1,t+1} + \tilde{K}'_{2,t+1} + \pi[\tilde{K}'_{1,t+1}g_{t+1} - (1 - \delta)\tilde{K}'_{1,t}]^2 + \phi\tilde{d}_{2,t} + \tilde{w}_t L_{2,y,t} =$$

$$L_{2,y,t}^\alpha \tilde{K}'_{1,t}{}^{1-\alpha} + L'_{2,y,t}{}^\alpha \tilde{K}'_{2,t}{}^{1-\alpha} + (1 - \delta)(\tilde{K}'_{1,t} + \tilde{K}'_{2,t}) + r_t\phi'\tilde{c}'_{1,t} + r_t\tilde{w}'_t L'_{1,y,t}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{c}'_{1,t}{}^{\nu-1} = \tilde{\lambda}_{BC,t}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{c}'_{2,t}{}^{\nu-1} = \tilde{\lambda}_{BC,t}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{d}'_{1,t}{}^{\nu-1} = \frac{\tilde{\lambda}_{BC,t}}{r_t}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{d}'_{2,t}{}^{\nu-1} + \frac{\tilde{\lambda}_{BC,t}\phi}{r_t} = \frac{\tilde{\lambda}_{BC,t}}{r_t} + \frac{r_t\lambda_{ER,t}}{\tilde{d}_{2,t}}$$

$$\chi[L_{a,t}^\epsilon + (L_{1,y,t} + L_{2,y,t})^\epsilon]^\frac{\eta}{\epsilon}-1 L_{a,t}^{\epsilon-1} = \tilde{\lambda}_{TFP,t}\mu\Omega L_{a,t}^{\Omega-1}$$

$$\chi[L_{a,t}^\epsilon + (L_{1,y,t} + L_{2,y,t})^\epsilon]^\frac{\eta}{\epsilon}-1 (L_{1,y,t} + L_{2,y,t})^{\epsilon-1} = \tilde{\lambda}_{BC,t}\alpha L_{1,y,t}^{\alpha-1} \tilde{K}_{1,t}^{1-\alpha}$$

$$\chi[L_{a,t}^\epsilon + (L_{1,y,t} + L_{2,y,t})^\epsilon]^\frac{\eta}{\epsilon}-1 (L_{1,y,t} + L_{2,y,t})^{\epsilon-1} = \frac{\tilde{\lambda}_{BC,t}\tilde{w}_t}{r_t}$$

$$\tilde{w}'_t = \alpha L'_{1,y,t}{}^{\alpha-1} \tilde{K}'_{2,t}{}^{1-\alpha}$$

$$\frac{\tilde{\lambda}_{BC,t+1}}{g_{t+1}} [(1 - \alpha)L_{1,y,t+1}^\alpha \tilde{K}_{1,t+1}^{-\alpha} + 1 - \delta] = \tilde{\lambda}_{BC,t}$$

$$\frac{\tilde{\lambda}_{BC,t+1}}{g_{t+1}} \left((1 - \alpha)L_{1,y,t+1}^\alpha \tilde{K}_{2,t+1}^{-\alpha} + 1 - \delta + 2(1 - \delta)\pi'[\tilde{K}_{2,t+2}g_{t+2} - (1 - \delta)\tilde{K}_{2,t+1}] \right) =$$

$$\tilde{\lambda}_{BC,t} \left(1 + 2\pi'[\tilde{K}_{2,t+1}g_{t+1} - (1 - \delta)\tilde{K}_{2,t}] \right)$$

$$\frac{\tilde{\lambda}_{BC,t+1}}{g_{t+1}} \left(\alpha L_{1,y,t+1}^\alpha \tilde{K}_{1,t+1}^{1-\alpha} + \alpha L'_{1,y,t+1}{}^\alpha \tilde{K}'_{2,t+1}{}^{1-\alpha} + \pi'[\tilde{K}_{2,t+2}g_{t+2} - (1 - \delta)\tilde{K}_{2,t+1}]^2 \right) = \tilde{\lambda}_{TFP,t}$$

$$\tilde{\lambda}_{BC,t} \left(\frac{\tilde{d}_{1,t} + \tilde{d}_{2,t} - \phi \tilde{d}_{2,t} - \tilde{w}_t L_{2,y,t}}{r_t^2} \right) = \lambda_{ER,t}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{c}'_{1,t}{}^{\nu-1} + \tilde{\lambda}'_{BC,t} r_t \phi' = r_t \tilde{\lambda}'_{BC,t} - \frac{r_t \lambda'_{ER,t}}{\tilde{c}'_{1,t}}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{c}'_{2,t}{}^{\nu-1} = r_t \tilde{\lambda}'_{BC,t}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{d}'_{1,t}{}^{\nu-1} = \tilde{\lambda}'_{BC,t}$$

$$(\tilde{c}'_{1,t} + \tilde{c}'_{2,t} + \tilde{d}'_{1,t} + \tilde{d}'_{2,t})^{-1} \tilde{d}'_{2,t}{}^{\nu-1} = \tilde{\lambda}'_{BC,t}$$

$$\chi[L'_{a,t}{}^\epsilon + (L'_{1,y,t} + L'_{2,y,t})^\epsilon]^\frac{\eta}{\epsilon}-1 L'_{a,t}{}^{\epsilon-1} = \tilde{\lambda}'_{TFP,t} \Omega L'_{a,t}{}^{\Omega-1}$$

$$\chi[L'_{a,t}{}^\epsilon + (L'_{1,y,t} + L'_{2,y,t})^\epsilon]^\frac{\eta}{\epsilon}-1 (L'_{1,y,t} + L'_{2,y,t})^{\epsilon-1} = \tilde{\lambda}'_{BC,t} \tilde{w}'_t r_t$$

$$\chi[L'_{a,t}{}^\epsilon + (L'_{1,y,t} + L'_{2,y,t})^\epsilon]^\frac{\eta}{\epsilon}-1 (L'_{1,y,t} + L'_{2,y,t})^{\epsilon-1} = \tilde{\lambda}'_{BC,t} \alpha L'_{2,y,t}{}^{\alpha-1} \tilde{K}'_{2,t}{}^{1-\alpha}$$

$$\tilde{w}_t = \alpha L'_{2,y,t}{}^{\alpha-1} \tilde{K}'_{1,t}{}^{1-\alpha}$$

$$\frac{\tilde{\lambda}'_{BC,t+1}}{g_{t+1}} \left((1 - \alpha) L'_{2,y,t+1}{}^\alpha \tilde{K}'_{1,t+1}{}^{1-\alpha} + 1 - \delta + 2(1 - \delta) \pi [\tilde{K}'_{1,t+2} g_{t+2} - (1 - \delta) \tilde{K}'_{1,t+1}] \right) =$$

$$\tilde{\lambda}'_{BC,t} \left(1 + 2\pi [\tilde{K}'_{1,t+1} g_{t+1} - (1 - \delta) \tilde{K}'_{1,t}] \right)$$

$$\frac{\tilde{\lambda}'_{BC,t+1}}{g_{t+1}} [(1 - \alpha) L'_{2,y,t+1}{}^\alpha \tilde{K}'_{2,t+1}{}^{1-\alpha} + 1 - \delta] = \tilde{\lambda}'_{BC,t}$$

$$\frac{\tilde{\lambda}'_{BC,t+1}}{g_{t+1}} \left(\alpha L'_{2,y,t+1}{}^\alpha \tilde{K}'_{1,t+1}{}^{1-\alpha} + \alpha L'_{2,y,t+1}{}^\alpha \tilde{K}'_{2,t+1}{}^{1-\alpha} + \pi [\tilde{K}'_{1,t+2} g_{t+2} - (1 - \delta) \tilde{K}'_{1,t+1}]^2 \right) = \tilde{\lambda}'_{TFP,t}$$

$$\tilde{\lambda}'_{BC,t} (\phi' \tilde{c}'_{1,t} + \tilde{w}'_t L'_{1,y,t} - \tilde{c}'_{1,t} - \tilde{c}'_{2,t}) = \lambda'_{ER,t}$$