$$
\begin{aligned}
r_\tau^* &= \frac{1-\beta}{\beta} E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} r_{i-1} \\
&= \frac{1-\beta}{\beta} \left[\beta r_\tau + E_\tau \sum_{i=(\tau+1)+1}^{\infty} \beta^{i-\tau} r_{i-1} \right] \\
&= \frac{1-\beta}{\beta} \left[\beta r_\tau + E_\tau E_{\tau+1} \beta \sum_{i=(\tau+1)+1}^{\infty} \beta^{i-(\tau+1)} r_{i-1} \right] \\
&= (1-\beta)\, r_\tau + \beta E_\tau \frac{1-\beta}{\beta} E_{\tau+1} \sum_{i=(\tau+1)+1}^{\infty} \beta^{i-(\tau+1)} r_{i-1} \\
&= (1-\beta)\, r_\tau + \beta E_\tau r_{\tau+1}^*
\end{aligned}
$$